Western and Northern Canadian Protocol (WNCP)
Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings

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Executive Summary

In 2004 and 2005, at the request of Western and Northern Canadian Protocol (WNCP) Assistant Deputy Ministers, WNCP jurisdictions collaborated to consult with executives and faculty representatives of post-secondary institutions as well as with representatives from business and industry in an effort to determine the desired student mathematical skills and competencies necessary for students to make a smooth transition from secondary mathematics studies to post-secondary programs and the world of work.

A broad cross-section of responses was sought in order to inform the revision of the WNCP Common Curriculum Framework (CCF) for Grades 10-12 Mathematics, based on a clear understanding of the expectations of the secondary and post-secondary systems as well as business and industry. It is hoped that the outcome would be increased acceptance of each high school mathematics pathway for admission into related post-secondary programs, through improved articulation of the pre-requisite mathematics competencies between secondary mathematics courses and post-secondary programs at all levels, as well as to prepare students for entry into the work force upon completing secondary studies.

TABLE A: Post-secondary faculty respondents

Province	Count of Responses	Count of People
Alberta	322	1087
British Columbia	107	258
Manitoba	61	142
Nunavut	3	3
Saskatchewan	70	563
Yukon	2	2
Total	**565**	**2055**

Of these post-secondary faculty respondents who indicated their faculty or subject area, 285 were reporting on calculus-based programs (i.e. those that require the completion of at least one calculus program for program graduation) and 272 were reporting on other (non-calculus-based) programs.

TABLE B: Business respondents

Province	Count of Responses	Count of People
Alberta	50	187
British Columbia	1	1
Northwest Territories	1	1
Saskatchewan	2	102
Total	**54**	**291**

Research questions and findings

The study attempts to answer six questions, which are discussed in the following sections.

Can each post-secondary program be assigned to one of a few categories of programs based on commonality of needs for high school mathematical competencies?

In the B.C. study (completed in 2004) that was the forerunner of the current study, the organizing construct was the separation of "calculus-based" and "non-calculus-based" post secondary programs. The separation was based on whether or not the post-secondary program included a required calculus course. This led to the conclusion that while the outcomes in the WNCP pathway labeled either Principles, Pre-Calculus or Pure Mathematics (in different WNCP provinces/territories) (and abbreviated to "PM" in the rest of this report) provide a good and well-accepted preparation for the calculus-based courses, the WNCP pathway variously labeled Applications or Applied Mathematics (and abbreviated to "AM" in the rest of this report) does not seem to have the outcomes needed for the non-calculus programs (there was little agreement among faculty members from these programs on what is needed).

The current WNCP study avoided an a priori assignment of post-secondary programs to two (or whatever number of) groups. Instead analysis attempted to determine if groups are suggested by how respondents identified "master", "expose" and "not applicable" for the various outcomes. The following table shows the groupings that resulted from a cluster analysis of the responses, with group names chosen to generally represent the nature of the included subject areas.

TABLE C: Groupings of major subject areas

Sciences	**Non-sciences**	**Trades**
Mathematics	Social Sciences	Trades
Sciences	Health	Agriculture
Information Technology	Business	
Engineering	Humanities	
Engineering Technology	Education	
	Physical Education	
	Fine Arts	

The major areas in the first cluster ("science") account for 90% of the "calculus-based" respondents in the WNCP survey, which in total include 285 respondents. The second cluster (Non-Sciences) is the major subset of the "non-calculus-based" group. It includes 198 of the 272 members of that group. The third cluster (Trades) includes 36 respondents, all within the "non-calculus" group.

What topics tend to group together, based on the relative importance attached to them by post-secondary respondents?

The following three topic groupings were suggested by similarity of responses (master, expose and not applicable) to the outcomes within the topic:

Group 1:
Algebra, Conics, Characteristics of functions and equations, Linear functions and equations, Polynomial functions and equations, Exponential and logarithmic functions, Trigonometric functions and equations, Geometry, Matrices, Measurement, Properties of number systems, Systems of equations, Sequences and series, Transformations, Vectors

Group 2:
Logic, Matrices, Probability, Statistics

Group 3:
Career project, Finance, Measurement, Technological applications

It was considered possible that these three groups would largely correspond to the main topic areas in each of the pathways to be identified later. As it happened, this proved to be largely the case for the first two, and was true to a degree for the third.

<u>What are the desired student mathematical competencies for entry into various post-secondary programs?</u>

An assumption was made that, since major subject areas clustered well into three groups, and that initial identification of high school pathways could be based on those outcomes which were required for each of the three groups, the pathways could be structured as follows:

Pathway 1 – if an outcome was identified as "master" by at least 50% of "calculus-based" respondents and/or as "master" or "expose" by at least 70% of such respondents, it was included herein.

Pathway 2 - if an outcome was identified as "master" by at least 40% of the "Non-science/non-calculus-based" respondents and/or as "master" or "expose" by at least 60% of such respondents, it was included herein. The lower threshold was used here to be even more certain that the pathway would meet stated requirements (given issues with the current second pathway).

Pathway 3 - if an outcome was identified as "master" by at least 50% of "Trades" respondents and/or as "master" or "exposure" by at least 70% of such respondents, it was included herein.

This resulted in the following table, which shows the numbers of outcomes being included for each topic in each pathway, as compared to the total number of WNCP outcomes for each topic:

TABLE D: Numbers of outcomes included in each pathway

Topic	Total	Pathway 1	Pathway 2	Pathway 3
Algebra	14	14	8	7
Career Project	4	0	0	0
Conics	3	0	0	0
Finance	17	0	0	6
Characteristics of Functions and Relations	16	16	7	2
Linear Functions and Equations	17	7	6	2
Polynomial Functions and Equations	8	8	0	0
Exponential and Logarithmic Functions	7	7	0	0
Trigonometric Functions and Equations	19	18	0	0

Topic	Total	Pathway 1	Pathway 2	Pathway 3
Geometry	18	17	0	17
Logic	5	5	5	0
Matrices	4	0	0	0
Measurement	22	14	9	21
Properties of Number Systems	16	15	11	13
Probability	19	6	7	0
Systems of Equations	9	6	0	0
Statistics	25	7	23	0
Sequences and Series	10	2	0	0
Technological Applications	12	0	4	0
Transformations	6	6	0	0
Vectors	5	5	0	0
Total	**246**	**152**	**80**	**68**

What are the desired student mathematical competencies for entry directly into the world of work upon completion of secondary studies?

There were 54 business and industry responses, from a wide cross-section of industry, ranging from small businesses to major banks and oil companies. A few large public sector employers were also included. Of these respondents, 41 provided at least some responses to the outcomes questions. They were asked to identify the level of mastery of each outcome required for their employees who they would be recruiting from high school. Required outcomes were selected based on the same criteria as for post-secondary faculty respondents for Pathways 1 and 3, that is if an outcome was identified as "master" by at least 50% of business and industry respondents and/or as "master" or "expose" by at least 70% of such respondents, it was included. The following table shows the number of required outcomes, by topic:

TABLE E: Numbers of outcomes by topic

Topic	Number of Outcomes required
Finance	4
Logic	1
Measurement	10
Properties of Number Systems	10
Probability	7
Technological Applications	12

Based on general comments made by the business and industry respondents, the following general skills are in high demand, with the most often requested skills listed in order:

- Add/subtract/multiply/divide
- Accounting/financial
- "Basic" high school math (the term "basic" is in quotation marks because here, and elsewhere in the report, the term is used because it is a direct quote from respondents; there is no attempt in this report to define what constitutes "basic" mathematics)
- Fractions/decimals

Combining these two sets of responses from business, it appears that for entry level positions recruited from high school, the "basic" mathematics skills, which are largely found within the

measurement and properties of number systems topics, are of the most interest to employers, with exposure to finance, probability and the use of technology also considered useful. The skills that are included in the elementary and junior high curriculum are also of importance to this group.

The impact of this on the pathways that have been proposed is that it seems that if pathway 3 were to include some logic outcomes, some basic probability and some exposure to technology, it would meet the expressed needs of business for entry level positions.

What revisions to the Grades 10-12 Common Curriculum Framework should be considered, from a post-secondary/business and industry perspective?

In summary, the matching of sets of outcomes to sets of programs resulted in an assignment of each of the three clusters of post-secondary major areas to a proposed high school pathway (each with a proposed set of outcomes). Moving to such a pathway structure would require significant change to existing programs in at least some of the provinces and territories.

The proposed Pathway 1 would have considerably more outcomes than the current PM pathway, especially in the areas of measurement, properties of number systems, systems of equations; and many fewer in probability. It would be a major change. Pathway 2, while having a similar number of outcomes to that of the current AM, has them distributed quite differently – more in algebra, logic, properties of number systems and statistics, and fewer in finance, trigonometry, geometry and vectors. Again, this is a significant change. Pathway 3 is quite different in intent than is the WNCP Third Path (labeled "TP" in the remainder of this report) and, as expected, is quite different in structure. One of the key points to note in comparing the two pathway structures is that the proposed pathways are strictly based on meeting post-secondary entrance requirements in different sets of program areas; whereas the current PM/AM/TP structure was designed to meet the needs of a variety of educational stakeholders.

Reflections on pathway structure based on survey results

1. The survey analysis suggests that, based on expressed requirements of respondents, there are three distinct program area groupings in the post-secondary system – the Science/calculus-based group, the Non-Science professional group and the Trades/Agriculture group. Business/industry respondents, especially those reporting on skills required for entry-level positions, come closest in requirements to the Trades group but have additional "expose" requirements.
2. Each group identified a set of required outcomes, using specified thresholds for percent of the group requiring mastery and/or exposure to the outcome. The resulting three pathways have some overlap, but each has a fairly different primary focus in terms of the main topics included. Out of about 200 outcomes found in any pathway, about 60% are found in only one of the pathways. The primary foci of the three pathways are as follows (shaded topics are those that have a weaker but still significant requirement):

Pathway 1	Pathway 2	Pathway 3
Algebra	Logic	Measurement
Characteristics of Functions	Statistics	Geometry
Polynomial Functions	Properties of Number Systems	Properties of Number Systems
Exponential/Logarithmic	Algebra	Algebra
Logic		
Transformations		
Vectors		
Trigonometric Functions		
Geometry		
Properties of Number Systems		
Systems of Equations		
Measurement		

3. Pathway 1 has too many outcomes to be taught in the 100 hours per year of instructional time recommended by the WNCP. It has 152 outcomes, as compared to the 115 outcomes currently included in PM, which itself is considered too heavy. Other options will need to be explored for delivering these outcomes. One possibility is to have students aiming for the Sciences take two mathematics courses in one or more of the high school grades, and the Sciences major areas would ask for completion of both Pathway 1 Grade 12 courses for admission. Another possibility is to divide the outcomes into those that will be taught in high school and those that will be taught in the first post-secondary year.

4. Based on survey response, it is clear that most post-secondary program areas do not require most of the outcomes taught in the current PM. Their stated requirements are for a much narrower set of topics and outcomes, and it would be possible to craft two new pathways that would tackle these topics in some depth. Clearly, based on expressed requirements alone, the Principles/Pre-Calculus/Pure pathway is NOT required for successful entry into the majority of post-secondary programs, as long as other pathways can be created to meet their specific needs.

What will be necessary to align post-secondary program admission standards more appropriately with the revised high school mathematics pathways?

Assuming that the WNCP Common Curriculum Framework is re-designed to meet the needs of post-secondary programs, as expressed in this survey, and the three pathways are set up to meet these expressed needs, the institutions should consider re-aligning their program entrance requirements appropriately.

Views from post-secondary institutions

In the two sets of opportunities for input (meetings held with individual institutions before the survey and in the feedback that institutions provided to the preliminary version of this report), a number of important observations were made. Some of these questioned the methodology of the report, and these have been considered in a revision of the calculations, the addition of a limitations section, and an examination of what pathway structure might look like with different outcome inclusion criteria (See Appendix A).

Other comments focused on some key topic areas, as follows, with a brief indication given of the predominant nature of the commentary.

Modifying the composition of Pathway 1

Many respondents made suggestions regarding the modification of content within Pathway 1, nearly all proposing that one or more topic be removed from it. Those which were most frequently recommended for removal were Measurement, Vectors, Probability and Statistics. Only a few institutions indicated support for either adding a second mathematics course in high school or for having a bridging course in post-secondary.

Covering "the basics"

The main expressed concern was students' lack of a strong grasp of "basic" mathematics. Some respondents indicated a need for the strong grounding in the "basics" that should be completed by the end of junior high.

The use of technology

In the earlier meetings, there was an oft-repeated concern that technology had taken the place of mental mathematics and that students were able to do work only if they were given a formula and/or a calculator. Some of the institutions providing feedback to the report echoed this comment, indicating that technology use should be severely limited in high school.

The need to provide flexibility for students

A number of respondents were concerned about the apparent lack of mobility from one pathway to another, indicating that students would not be well served by being locked into one path in Grade 10, from which it would be difficult to move if interests changed during high school. A few of these respondents suggested that the pathways should diverge later.

Support for the proposed pathway structure

A number of institutions stated that the proposed structure was appropriate and would meet the needs of post-secondary institutions (generally, and particularly their own) quite well.

The use of mathematics in admission requirements

A few respondents expressed concern about the role of mathematics as "gatekeeper" to post-secondary, especially university entrance. There is a related concern that this forces students to take mathematics courses that may not be appropriate to their needs, in order not to "close doors."

NOTE: It is important to know that, while survey respondents were spread across program areas, feedback respondents were largely from mathematics departments (e.g. 35% of survey respondents were from the Social Sciences, Health, Business, Humanities and Fine Arts areas, but none of the feedback respondents were from these areas).

Conclusions

The results of this survey suggest that three high school pathways, with indicated content, should satisfy stated admission requirements for the appropriate sets of programs. This should:

- ensure that Pathway 1 satisfies the entrance requirements of the calculus-based (and similar) post secondary programs, with the fewest possible outcomes;
- ensure that Pathway 2 satisfies the entrance requirements of most of the remainder of the non-calculus-based programs;
- ensure that Pathway 3 satisfies the entrance requirements of the trades and agriculture, and with a few additions, would meet the needs of business and industry for positions for which they recruit from high schools.

Specific conclusions about pathway composition and mechanisms for addressing issues raised in the study are as follows:

1. Reduce the number of outcomes in Pathway 1 by considering all of the proposed modifications, including those suggested within post-secondary feedback to the preliminary report, and the changes that would result if thresholds were changed as noted in Appendix A. The most likely candidates for exclusion would appear to be:
    - Measurement – reduction in the number of outcomes from 14 to perhaps 8 (possibly taught instead in other courses and/or in earlier grades)
    - Probability – reduction from 4, or full removal (possibly taught instead in post-secondary)
    - Statistics – reduction from 5, or full removal (possibly taught instead in post-secondary)
    - Vectors – reduction from 5, or full removal (possibly taught instead in Physics)
2. Provide more flexibility for students by beginning the different pathways in Grade 11 rather than in Grade 10.
3. Re-examine the use of technology, especially graphing calculators, in high school.
4. Engage in further discussion with trades groups to ensure that Pathway 3 meets the needs of at least the large majority of trades.
5. Ensure that the revised pathways each have a logical sequence of topics, and that prerequisites for each topic are covered.

Recommendations

1. Consider the findings of this report in the review of the WNCP Common Curriculum Framework for Mathematics. In this review, recognize that this report considers pathway requirements only from the viewpoint of post-secondary respondents and a small number of business/industry respondents – it does not consider other drivers, such as the need for all post-secondary students and those in the working world to have a reasonable understanding of finance.
2. Discuss the report findings and any consequent changes to the high school program with key post-secondary institution senior administrators (e.g. academic vice-presidents), to ensure that the best possible exchange of views occurs on the topic of the validity of each pathway as an appropriate admission standard for the sets of programs to which each is intended to align.

Background

In 2004 and early 2005, at the request of Western and Northern Canadian Protocol (WNCP) Assistant Deputy Ministers, WNCP jurisdictions collaborated to consult with executives and faculty representatives of post-secondary institutions as well as with representatives from business and industry in an effort to determine the desired student mathematical skills and competencies necessary for students to make a smooth transition from secondary mathematics studies to post-secondary programs and the world of work.

A broad cross-section of responses was sought in order to inform the revision of the WNCP Common Curriculum Framework (CCF) for Grades 10-12 Mathematics, based on a clear understanding of the expectations of the secondary and post-secondary systems as well as business and industry. It was hoped that the outcome would be increased acceptance of all high school mathematics courses for admission into post-secondary programs through improved articulation of the pre-requisite mathematics competencies between secondary mathematics courses and post-secondary programs at all levels, as well as to prepare students for entry into the work force upon completion of secondary studies.

This project was led by Basic Learning staff at Alberta Education along with WNCP partners, with support from the System Improvement Group (SIG) of Alberta Education.

Methodology

1. An on-line survey, first used in British Columbia (B.C.), in 2004, was modified to represent the grades 10-12 mathematics content within all WNCP jurisdictions. The survey was created by the WNCP Mathematics Consultant Team (from 7 provinces and territories) with advice from SIG on design.
2. SIG posted the WNCP on-line survey on the Alberta Education website for post-secondary, business and industry response from across the WNCP.
3. Across the WNCP, Ministry personnel undertook consultations with post-secondary institutions to introduce them to the survey and how it will be used to inform curriculum revisions. Sessions were arranged via a letter to the appropriate senior administrator (e.g. VP Academic) in each institution and involved as much variety as possible in terms of program areas represented.
4. Representatives at these meetings were encouraged to speak with colleagues about the survey and provide responses. Interested parties were given until January 31, 2005 to respond to the survey. It was important that the key post-secondary faculties were represented and that those who attended could speak for their faculties, not just for themselves, for it was necessary to seek agreement on entrance requirements in order to not repeat the current Applied Mathematics situation, in which students with that mathematics qualification have limited access to post-secondary programs. As a result, there was time provided for the survey to make its way around the post-secondary institutions.
5. Data from across western and northern Canada was submitted using the on-line survey. Collection of demographic data from each respondent allowed for separation of the data by jurisdiction and institution.
6. A selected group of business and industry employers was also asked to complete the survey, as their views are also of importance in the choice of topics and levels of treatment, within each pathway.
7. Post-secondary faculty and senior administrators (e.g. academic vice-presidents) had an opportunity, from May to October 2005, to provide feedback on the preliminary English

version of this report, in order to clarify understandings and eliminate any errors. As no French translation of the report was done, at least one francophone institution was unable to respond.

8. SIG compiled the data, with each WNCP province/territory included, and created this final report.
9. The report will be shared with post-secondary senior administrators (e.g. academic vice-presidents (and staff selected by the senior administrator) in follow up sessions in 2006.
10. The report will be reviewed and validated by high school mathematics teachers during revision of the WNCP Common Curriculum Framework (CCF) for Grades 10-12 Mathematics. This will involve significant consultations with WNCP stakeholders.

Report limitations

There are some limitations that should be noted in using this report as input to deliberations on the future directions for the WNCP high school mathematics curriculum. These include, but may not be limited to:

1. The report largely presents the views of post-secondary respondents. It received a small amount of business response, and did not solicit general public response. It is possible that members of the public, especially parents, may have some quite different expectations of the high school mathematics program. Some of these points are noted in the section, later in this report, which discusses post-secondary feedback.
2. Provincial response rates were not proportional to their relative levels of post-secondary enrolment.
3. One respondent was concerned about the use of the terms "master", "expose" and "not applicable". The meaning of these terms is somewhat subjective: what one person thinks of as exposure could easily be mastery to someone else. Further, it may be true that gaps between the scale points are not equal or interval, and thus less than perfectly amenable to averaging techniques. It should be noted that this typology was used in the earlier survey and report done for B.C. by Holdfast Consulting.

Survey respondents

On February 5, 2005, an excerpt of the online database was taken. This database was augmented by all the records sent from B.C. (as captured in its earlier survey). The objective of this study was to analyze survey response by province, institution, faculty/subject and major area. The latter is a grouping of faculty/subjects into areas roughly similar to that used in Alberta Advanced Education's enrollment reporting system. A detailed frequency distribution of respondents by faculty/subject is presented in Appendix B. The major area grouping is used in the body of this report.

In this and other tables, there are two columns: count of responses and count of people. The latter is based on the number of persons that a response is given for. In all, 59 responses (of 650) covered more than 10 people each.

In preparing this database for analysis, it was noted that there were a number of responses that were largely completed, but had not been officially submitted. It was decided that these responses should be considered **valid** in the response analysis, even if they had only completed a fraction of the outcome responses. These records were screened to ensure that double counting did not occur, i.e. by not including partial responses provided by a person who subsequently tried again and completed a response.

The following two tables compare the number of responses, and the total number of people covered by the responses, using both sets of records – just those that had been officially submitted, and then all that have been selected for our analysis:

Total responses (from all WNCP provinces and territories

TABLE 1: Submitted responses only (post-secondary, business and industry included)

Province	Count of Responses	Count of People
Alberta	286	878
British Columbia	108	258
Manitoba	46	84
Northwest Territories	1	1
Saskatchewan	59	440
Yukon	3	3
Total	**503**	**1664**

TABLE 2: All valid responses (post secondary, business and industry included)

Province	Count of Responses	Count of People
Alberta	391	1287
British Columbia	108	258
Manitoba	64	154
Northwest Territories	1	1
Nunavut	3	3
Saskatchewan	80	702
Yukon	3	3
Total	**650**	**2408**

The reminder of the tables in this section consider all valid responses.

TABLE 3: Post-secondary respondents (faculty and non-faculty)

Province	Count of Responses	Count of People
Alberta	341	1106
British Columbia	107	258
Manitoba	64	154
Nunavut	3	3
Saskatchewan	78	602
Yukon	3	3
Total	**596**	**2126**

TABLE 4: Post-secondary faculty respondents

Province	Count of Responses	Count of People
Alberta	322	1087
British Columbia	107	258
Manitoba	61	142
Nunavut	3	3
Saskatchewan	70	563
Yukon	2	2
Total	**565**	**2055**

TABLE 5: Business respondents

Province	Count of Responses	Count of People
Alberta	50	187
British Columbia	1	1
Northwest Territories	1	1
Saskatchewan	2	102
Total	**54**	**291**

TABLE 6: Post-secondary faculty respondents by major area

Major area	Count of Responses	Count of People
Agriculture	17	87
Architecture	3	27
Business	35	191
Communications	3	3
Education	14	18
Engineering	25	110
Engineering Technology	17	107
Fine Arts	11	22
Health	60	235
Hospitality	7	26
Humanities	25	44
Information Technology	36	115
Mathematics	94	295
Medicine and Dentistry	4	29
Physical Education	13	13
Renewable Resources	7	27
Sciences	85	246
Social and Community Services	11	44
Social Sciences	66	233
Trades	21	172
Transportation	1	1
Upgrading	1	1
(no information provided)	9	9
Total	**565**	**2055**

The following table divides post-secondary faculty respondents into those reporting on calculus-based programs (i.e. those that require the completion of at least one calculus program for program graduation) and those reporting on other (non-calculus-based) programs.

TABLE 7: Respondents by program calculus requirement

Province	Non-calculus-based	Calculus-based	Total
Alberta	160	156	316
British Columbia	51	57	108
Manitoba	27	34	61
Nunavut	2	1	3
Saskatchewan	32	36	68
Yukon	0	1	1
Total	**272**	**285**	**557**

Note that a few post-secondary faculty respondents did not specify either their faculty or subject areas, so it was not possible to assign them to either the "non-calculus-based or "calculus-based" categories.

Responses by province

Alberta

TABLE 8: Alberta responses by institution (faculty and non-faculty)

Institution	Count of responses	Count of people
Alliance University College	1	1
Athabasca University	4	20
Bow Valley College	3	22
Canadian University College	2	2
DeVry Institute of Technology-Calgary	6	6
Grande Prairie Regional College	1	1
Grant MacEwan College	16	25
Keyano College	5	29
Lakeland College	9	9
Lethbridge Community College	5	9
Medicine Hat College	12	14
Mount Royal College	28	70
NorQuest College	9	81
Northern Alberta Institute of Technology	36	106
Olds College	10	36
Red Deer College	18	59
SAIT	6	9
Southern Alberta Institute of Technology	34	85
St. Mary University College	3	3
The King University College	5	17
University of Alberta	48	236
University of Calgary	50	226
University of Lethbridge	19	29
Unknown	10	10
Total	**341**	**1106**

TABLE 9: Alberta post-secondary faculty responses by major area

Major area	Count of Responses	Count of People
Agriculture	13	42
Architecture	1	25
Business	24	33
Education	6	10
Engineering	15	60
Engineering Technology	13	57
Fine Arts	2	2
Health	43	205
Hospitality	4	23
Humanities	11	20
Information Technology	22	74
Mathematics	48	18
Medicine and Dentistry	3	6
Physical Education	10	10
Renewable Resources	2	6
Sciences	45	82
Social and Community Services	3	14
Social Sciences	35	164
Trades	14	58
Transportation	1	1
Upgrading	1	1
(no information provided)	6	6
Total	**322**	**1087**

Faculty members in the major Alberta universities responded as follows:

TABLE 10: University of Alberta responses

Major area	Count of Responses	Count of People
Business	1	1
Education	2	4
Engineering	7	7
Health	2	10
Humanities	1	1
Information Technology	3	3
Mathematics	9	113
Medicine and Dentistry	1	1
Physical Education	2	2
Sciences	11	11
Social Sciences	8	82
Total	**47**	**235**

TABLE 11: University of Calgary responses

Major area	Count of Responses	Count of People
Architecture	1	25
Engineering	1	40
Health	3	3
Humanities	6	6
Information Technology	3	55
Mathematics	7	7
Physical Education	6	6
Sciences	6	35
Social Sciences	14	46
(no information provided)	1	1
Total	**48**	**224**

British Columbia

TABLE 12: British Columbia responses by institution

Institution	Count of Responses	Count of People
British Columbia Institute of Technology	7	124
Camosun College	7	7
Capilano College	1	12
Douglas College	7	7
Kwantlen University College	18	35
Langara College	1	1
Northwest Community College	1	1
Okanagan University College	4	5
Selkirk College	1	1
Simon Fraser University	1	1
The University of British Columbia	39	42
University College of the Fraser Valley	7	7
University of Northern British Columbia	6	6
University of Victoria	7	7
Total	**107**	**258**

TABLE 13: British Columbia post-secondary faculty responses by major area

Major area	Count of Responses	Count of People
Agriculture	1	1
Architecture	1	1
Business	5	5
Communications	2	2
Education	6	6
Engineering	7	7
Engineering Technology	1	1
Fine Arts	6	6

Major area	Count of Responses	Count of People
Health	7	7
Hospitality	1	1
Humanities	6	6
Information Technology	1	1
Mathematics	23	75
Physical Education	1	1
Renewable Resources	1	1
Sciences	15	15
Social and Community Services	5	5
Social Sciences	12	15
Trades	5	101
(no information provided)	1	1
Total	**107**	**258**

TABLE 14: UBC responses

Major area	Count of Responses	Count of People
Agriculture	1	1
Business	1	1
Education	5	5
Engineering	6	6
Fine Arts	3	3
Health	3	3
Humanities	4	4
Information Technology	1	1
Mathematics	1	1
Renewable Resources	1	1
Sciences	6	6
Social Sciences	6	9
(no information provided)	1	1
Total	**39**	**42**

Manitoba

TABLE 15: Manitoba responses by institution (faculty and non-faculty)

Institution	Count of Responses	Count of People
Assiniboine Community College	2	4
Canadian Mennonite University	3	3
Collège universitaire de Saint-Boniface	23	35
Red River College	4	69
University of Manitoba	28	39
University of Winnipeg	4	4
Total	**64**	**154**

TABLE 16: Manitoba post-secondary faculty responses by major area

Major area	Count of Responses	Count of People
Agriculture	1	12
Business	3	3
Engineering	1	25
Engineering Technology	1	42
Fine Arts	1	1
Health	3	3
Hospitality	2	2
Humanities	4	4
Information Technology	8	8
Mathematics	10	10
Renewable Resources	2	4
Sciences	14	14
Social and Community Services	1	4
Social Sciences	10	10
Total	**61**	**142**

TABLE 17: University of Manitoba responses

Major area	Count of Responses	Count of People
Agriculture	1	12
Health	1	1
Hospitality	1	1
Information Technology	7	7
Mathematics	8	8
Renewable Resources	1	1
Sciences	4	4
Social Sciences	3	3
Total	**26**	**37**

Nunavut

TABLE 18: Nunavut responses by institution

Institution	Count of Responses	Count of People
Nunavut Arctic College	3	3
Total	**3**	**3**

TABLE 19: Nunavut post-secondary faculty responses by major area

Major area	Count of Responses	Count of People
Humanities	1	1
Mathematics	1	1
Renewable Resources	1	1
Total	**3**	**3**

Saskatchewan

TABLE 20: Saskatchewan responses by institution (faculty and non-faculty)

Institution	Count of Responses	Count of People
Briercrest Family of Schools	2	2
First Nations University of Canada	1	1
Luther College	1	1
Private Vocational Schools	4	4
SIAST (Kelsey Campus)	11	35
SIAST (Palliser Campus)	6	6
SIAST (Wascana Campus)	4	22
SIAST (Woodland Campus)	2	19
University of Regina	19	139
University of Saskatchewan	28	373
Total	**78**	**602**

TABLE 21: Saskatchewan post-secondary faculty responses by major area

Major area	Count of Responses	Count of People
Agriculture	2	32
Architecture	1	1
Business	3	150
Communications	1	1
Education	2	2
Engineering	2	18
Engineering Technology	2	7
Fine Arts	2	13
Health	7	20
Humanities	3	13
Information Technology	4	31
Mathematics	11	20
Medicine and Dentistry	1	23
Physical Education	2	2
Renewable Resources	1	15
Sciences	11	135
Social and Community Services	2	21
Social Sciences	9	44
Trades	2	13
(no information provided)	2	2
Total	**70**	**563**

Faculty members in the major universities in Saskatchewan responded as follows:

TABLE 22: University of Saskatchewan responses

Major area	Count of Responses	Count of People
Agriculture	2	32
Business	2	149
Engineering	1	10
Fine Arts	2	13
Health	3	3
Humanities	2	12
Information Technology	1	25
Mathematics	2	2
Medicine and Dentistry	1	23
Sciences	4	36
Social and Community Services	1	20
Social Sciences	4	14
Total	**25**	**339**

TABLE 23: University of Regina responses

Major area	Count of Responses	Count of People
Communications	1	1
Education	1	1
Engineering	1	8
Mathematics	3	3
Physical Education	1	1
Sciences	5	93
Social and Community Services	1	1
Social Sciences	4	29
(no information provided)	1	1
Total	**18**	**138**

Yukon

TABLE 24: Yukon responses by institution (faculty and non-faculty)

Institution	Count of Responses	Count of People
Yukon College	3	3
Total	**3**	**3**

TABLE 25: Yukon post-secondary faculty responses by major area

Major area	Count of Responses	Count of People
Information Technology	1	1
Mathematics	1	1
Total	**2**	**2**

There were no post-secondary respondents from the Northwest Territories to the initial survey; however, Aurora College did provide feedback to the preliminary report.

Research questions

The study attempts to answer the following questions, and this report discusses the related results of the study:

1. Can each post-secondary program be assigned to one of a few categories of programs based on commonality of needs for high school mathematical competencies?
2. What topics tend to group together, based on the relative importance attached to them by post-secondary respondents?
3. What are the desired student mathematical competencies for entry into various post-secondary programs (with as broad a representation as possible from the full array of programs)?
4. What are the desired student mathematical competencies for entry directly into the world of work upon completion of secondary studies?
5. What revisions to the Grades 10-12 Common Curriculum Framework should be considered, from a post-secondary/business and industry perspective?
6. What will be necessary to align post-secondary program admission standards more appropriately with the revised high school mathematics pathways, based on the answers to the above two questions?

Information obtained from survey responses, as well as from the consultations with post-secondary institutions, has been used to attempt to answer the above questions.

Can each post-secondary program be assigned to one of a few categories of programs based on commonality of needs for high school mathematical competencies?

In the B.C. study that was the forerunner of the current study, the organizing construct was the separation of “calculus-based” and “non-calculus-based” post secondary programs. The separation was based on whether or not the post-secondary program included a required calculus course. This led to the conclusion that while the outcomes in the WNCP pathway labeled either Principles, Pre-Calculus or Pure Mathematics (in different WNCP provinces/territories) (and abbreviated to “PM” in the rest of this report) provide a good and well-accepted preparation for the calculus-based courses, the WNCP pathway variously labeled Applications or Applied Mathematics (and abbreviated to “AM” in the rest of this report) does not seem to have the outcomes needed for the non-calculus programs (there was little agreement among faculty members from these programs on what is needed).

The current WNCP study avoided an a priori assignment of post-secondary programs to two (or whatever number of) groups. Instead analysis attempted to determine if groups are suggested by how respondents identified “master”, “expose” and “not applicable” for the various outcomes.

This analysis resulted in the following possible groupings of post-secondary programs based on commonality of responses (i.e. if respondents in one major area responded similarly to those in another major area, then those two areas would likely be found in the same cluster). In attempting to cluster cases or groups, it is possible to select in advance how many clusters are to be determined and then review statistics to decide which structure to use. In this study, each of the cluster approaches (e.g. two-cluster; three-cluster; etc.) had about the same degree of associative strength.

TABLE 26: Cluster analysis results

Two cluster approach

Sciences	**Non-sciences**
Mathematics	Social Sciences
Sciences	Health
Information Technology	Business
Engineering	Humanities
Engineering Technology	Education
	Physical Education
	Fine Arts
	Trades
	Agriculture

Three cluster approach

Sciences	**Non-sciences**	**Trades**
Mathematics	Social Sciences	Trades
Sciences	Health	Agriculture
Information Technology	Business	
Engineering	Humanities	
Engineering Technology	Education	
	Physical Education	
	Fine Arts	

Four cluster approach

Sciences	**Social Sciences**	**Humanities**	**Trades**
Mathematics	Social Sciences	Humanities	Trades
Sciences	Health	Education	Agriculture
Information Tech.	Business	Fine Arts	
Engineering	Physical Education		
Engineering Tech.			

Five cluster approach

Sciences	**Mathematics**	**Social Sciences**	**Humanities**	**Trades**
Sciences	Mathematics	Social Sciences	Humanities	Trades
Info. Tech.		Health	Education	Agriculture
Engineering		Business	Fine Arts	
Eng. Tech.		Physical Ed.		

In choosing between alternative cluster approaches, since all of them have about the same level of strength, the method that was used was keep increasing the number of clusters until all pairs that have non-significant correlations with one another have been separated into different clusters. Under this rule, it is necessary to separate trades and agriculture from the other non-sciences.

Another general rule is the fewer clusters, the better, mainly to make results more practical to use. When we consider each of these criteria, the **three-cluster approach** is the most compelling.

The three-cluster approach has some similarities to the "calculus-based"/"non-calculus-based" approach taken by Holdfast Consulting in its analysis of the 2004 B.C. survey. The major areas in the "science" cluster account for 90% of the "calculus-based" respondents in the WNCP survey. The other "calculus-based" respondents are mainly in the Business (generally the university business programs) and the Social Sciences (mainly in the economics programs) major areas. In subsequent analysis, we will bring together into one group not just the 254 respondents in the five major Science cluster major areas, but all 285 members of the "calculus-based" group (including the Business, Social Sciences and other respondents who are classified therein).

The second cluster (Non-Sciences) is the major subset of the "non-calculus-based" group. It includes 198 of the 277 members of that group. The third cluster (Trades) includes 36 respondents, all within the "non-calculus" group. The 38 other "non-calculus" respondents are divided between a number of major groups with small numbers in each, and will not be separately analyzed in this report.

What topics tend to group together, based on the relative importance attached to them by post-secondary respondents?

In this analysis, an average score for each topic was calculated for each respondent. For example, if the respondent had scored half of the Algebra outcomes as "master (3)" and the other half as "expose (2)", then that respondent's average Algebra score would be 2.5. The factor analysis groups topics are based on commonality of response. For example, if all respondents who gave Algebra a '1" (NA – "not applicable") also gave Conics a "1", all who gave Algebra a '2" also gave Conics a "2" and all who gave Algebra a '3" also gave Conics a "3", then Algebra and Conics would group together perfectly, with a score of 1.00. Topics which were normally scored differently by respondents would get a low score and not group together.

The factor analysis identified three groups, as shown in the following table (showing the score which identifies how closely the topic aligns with the overall group):

TABLE 27: Factor analysis results

Group 1:

Topic	Score
Algebra	0.815
Conics	0.718
Characteristics of functions and equations	0.716
Linear functions and equations	0.754
Polynomial functions and equations	0.889
Exponential and logarithmic functions	0.811
Trigonometric functions and equations	0.902
Geometry	0.851
Matrices (also found in Group 2)	0.582
Measurement (also found in Group 3)	0.498

Topic	Score
Properties of number systems	0.676
Systems of equations	0.753
Sequences and series	0.668
Transformations	0.768
Vectors	0.788

Group 2:

Topic	Score
Logic	0.571
Matrices (also found in Group 1)	0.505
Probability	0.807
Statistics	0.833

Group 3:

Topic	Score
Career project	0.787
Finance	0.844
Measurement (also found in Group 1)	0.520
Technological applications	0.648

The above analysis was not used to determine pathway structure, but it was considered possible that these three groups would largely correspond to the main topic areas in each of the pathways to be identified later. As it happened, this was largely the case for the first two, and was true to a certain degree for the third. As a result, this analysis lends corroboration to the pathway structure that was developed later.

What are the desired student mathematical competencies for entry into various post-secondary programs (with as broad a representation as possible from the full array of programs)?

An assumption was made that, since major areas clustered well into three groups, and that initial identification of high school pathways could be based on those outcomes which were required for each of the three groups, the pathways could be structured as follows:

Pathway 1 – if an outcome was identified as “master” by at least 50% of “calculus-based” respondents and/or as “master” or “expose” by at least 70% of such respondents, it was included herein.

Pathway 2 - if an outcome was identified as “master” by at least 40% of the “Non-science/’non-calculus” respondents and/or as “master” or “expose” by at least 60% of such respondents, it was included herein. The lower threshold was used here to be even more certain that the pathway would meet stated requirements (given the issues with the current second pathway).

Pathway 3 - if an outcome was identified as “master” by at least 50% of “Trades” respondents and/or as “master” or “exposure” by at least 70% of such respondents, it was included herein.

This resulted in the following numbers of outcomes being included for each topic in each pathway:

TABLE 28: Numbers of outcomes included in each pathway

Topic	**Total**	**Pathway 1**	**Pathway 2**	**Pathway 3**
Algebra	14	14	8	7
Career Project	4	0	0	0
Conics	3	0	0	0
Finance	17	0	0	6
Characteristics of Functions and Relations	16	16	7	2
Linear Functions and Equations	17	7	6	2
Polynomial Functions and Equations	8	8	0	0
Exponential and Logarithmic Functions	7	7	0	0
Trigonometric Functions and Equations	19	18	0	0
Geometry	18	17	0	17
Logic	5	5	5	0
Matrices	4	0	0	0
Measurement	22	14	9	21
Properties of Number Systems	16	15	11	13
Probability	19	6	7	0
Systems of Equations	9	6	0	0
Statistics	25	7	23	0
Sequences and Series	10	2	0	0
Technological Applications	12	0	4	0
Transformations	6	6	0	0
Vectors	5	5	0	0
Total	**246**	**152**	**80**	**68**

Determining pathway structure

From the analyses noted above, it appears that there is a sound basis for the development of three pathways, which could be described as follows:

Pathway 1 – which includes the topics most closely associated with the WNCP PM pathway and which are required for admission into post-secondary programs requiring calculus.

Pathway 2 – which includes the topics required by the large majority of the non-calculus post-secondary programs. Its basic topics are logic, probability and statistics, with some specific topics from algebra, characteristics of functions and equations, linear functions and equations, geometry, measurement and properties of number systems.

Pathway 3 – which includes the topics required for success in the trades, agriculture and entry directly into the workforce. The basic topics are finance, geometry, measurement and properties of number systems, with some algebra topics as well.

The following table provides a list of all outcomes, indicating into which pathway(s) they would be placed, as well as indicating those outcomes which would not be included in the curriculum, based on not being needed in any pathway. Note, of course, that this is based solely on expressed needs of self-selected faculty respondents from post-secondary institutions.

TABLE 29: All outcomes showing pathway membership

Topic/Description	P1	P2	P3	Out
Algebra				
Use words and algebraic expressions to describe the data and the interrelationships in a table.	1	1		
Communicate a set of instructions used to solve an arithmetic problem.	1	1	1	
Perform arithmetic operations on irrational numbers, using appropriate decimal approximations.	1	1	1	
Explain and apply the exponent laws for powers of numbers and for variables with rational exponents.	1			
Factor polynomial expressions of the form $ax^2 + bx + c$, $a^2x^2 - b^2y^2$, $a^3x^3 - b^3y^3$, $a^3x^3 + b^3y^3$	1			
Find the product of polynomials.	1			
Divide a polynomial by a binomial, and express the result in the forms: • $\frac{P}{D} = Q + \frac{R}{D}$ • P = DQ + R • P(x) = D(x)$Q(x)$ + R.	1			
Substitute numbers for variables in expressions, and graph and analyze the relation.	1	1	1	
Translate between an oral or written expression and an equivalent algebraic expression.	1	1	1	
Use formulas to solve problems	1	1	1	
Determine the non-permissible values for the variable in rational expressions.	1			
Perform the operations of addition, subtraction, multiplication and division on rational expressions.	1	1	1	
Find and verify the solutions of rational equations.	1	1	1	
Perform operations on irrational numbers of monomial and binomial form, using exact values.	1			
Career Project				
Determine what factors are important in analysing careers				1
Describe two specific career opportunities				1
Identify mathematical educational requirements for two careers				1
Compare two careers in terms of salary, working hours, training time and cost, cost of living, and benefits				1
Conics				
Classify conic sections according to shape.				1
Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only).				1
Convert a given equation of a conic section from general to standard form and vice versa.				1
Finance				
Analyze car or house insurance needs and premiums, using such concepts as			1	

Topic/Description	P1	P2	P3	Out
loss, probability of loss, compulsory coverage, optional coverage, deductible and claims record.				
Solve consumer problems, including: • wages earned in various situations • property taxation • exchange rates • unit prices.			1	
Reconcile financial statements including: • cheque books with bank statements • cash register tallies with daily receipts.			1	
Solve budget problems, using graphs and tables to communicate solutions.			1	
Solve investment and credit problems involving simple and compound interest.			1	
Name and describe various types of commonly used consumer bank accounts and services				1
Solve problems involving the acquisition and operation of a vehicle.			1	
Prepare income tax forms.				1
Prepare a business plan to own and operate a business.				1
Determine the costs involved in purchasing a home, including gross debt service ratio				1
Solve problems involving different types of mortgages				1
Describe government expenditures including the amounts spent on social welfare benefits, social security, education, health care, policing, armed forces, and employee wages and salaries				1
Solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties)				1
Calculate provincial taxes, (e.g., PST, corporation capital, licenses, gasoline)				1
Determine how selected municipal taxes are calculated (e.g., property)				1
Compare and contrast different investment vehicles in terms of risk factors, rates of return, costs, and lengths of term				1
Describe a variety of sales promotion techniques and their financial implications for the consumer				1
Characteristics of Functions and Relations				
Generalize a pattern arising from a problem-solving context, using mathematical expressions and equations, and verify by substitution.	1	1		
Interpret the graph of a relation and describe it in words	1	1	1	
Construct a graph of a relation from its description in words	1	1	1	
Graph relations, analyze the result and draw a conclusion from a pattern.	1	1		
Create and modify tables from both recursive and non-recursive situations.	1	1		
Represent data, using function models.	1	1		
Use a graphing tool to draw the graph of a function from its equation.	1			
Describe a function in terms of: • ordered pairs • a rule, in word or equation form • a graph.	1	1		
Use function notation to evaluate and represent functions.	1			
Determine the domain and range of a relation from its graph.	1			
Plot and analyse examples of direct variation, partial variation, and inverse	1			

Topic/Description	P1	P2	P3	Out
variation				
Perform operations on functions and compositions of functions.	1			
Determine the inverse of a function.	1			
Describe, graph and analyze polynomial and rational functions, using technology.	1			
Determine if a relation is a function using a variety of methods.	1			
Formulate and apply strategies to solve absolute value equations, radical equations, rational equations and inequalities.	1			
Linear Functions and Equations				
Solve and verify one-step linear equations, using a variety of techniques including concrete materials and diagrams.	1	1	1	
Solve and verify two-step, single-variable, first-degree equations, using concrete materials, diagrams, informal algebraic methods, or formal algebra in the form: • x + a = b • ax = b • $\frac{x}{a} = b$ • ax + b = c where *a*, *b* and *c* are integers.	1	1	1	
Plot linear and nonlinear data, using appropriate scales.	1	1		
Express a linear relation of the form $y = mx + b$ • in words • as a formula • with a table of values • as a graph	1	1		
Determine the following characteristics of the graph of a linear function, given its equation: 1. intercepts 2. slope 3. domain 4. range.	1	1		
Use direct variation and arithmetic sequences as applications of linear functions.	1			
Interpolate and extrapolate values from the graph of a linear relation	1	1		
Polynomial Functions and Equations				
Determine the following characteristics of the graph of a quadratic function: • vertex • domain and range • axis of symmetry • intercepts.	1			
Connect algebraic and graphical transformations of quadratic functions, using completing the square as required.	1			
Model real-world situations, using quadratic functions.	1			
Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: • factoring • the quadratic formula • graphing	1			

Topic/Description	P1	P2	P3	Out
Determine the characteristics of the real and non-real roots of a quadratic equation, using: • the discriminant in the quadratic formula • graphing.	1			
Determine that the sum of the roots of a quadratic equation $ax^2+bx+c=0$ equals b/a and that the product of the roots equals c/a	1			
Solve nonlinear equations: 1. by factoring 2. graphically.	1			
Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials.	1			
Exponential and Logarithmic Functions and Equations				
Explain the relationship between the laws of logarithms and the laws of exponents.	1			
Change functions from exponential form to logarithmic form and vice versa.	1			
Graph and analyze logarithmic functions with and without technology.	1			
Use logarithms to model practical problems.	1			
Model, graph and apply exponential functions to solve problems	1			
Solve exponential equations having bases that are powers of one another.	1			
Solve and verify exponential and logarithmic equations and identities.	1			
Trigonometric Functions and Equations				
Apply ratio and proportion in similar triangles	1			
Use the trigonometric ratios sine, cosine, and tangent in solving right triangles	1			
Solve problems involving two right triangles.	1			
Apply the sine and cosine laws, excluding the ambiguous case, to solve problems.	1			
Solve problems involving ambiguous case triangles in 3-D and 2-D.	1			
Predict results from graphs that represent periodic events.	1			
Describe periodic events, including sinusoidal curves, using correct terminology.	1			
Collect sinusoidal data; sketch the graph of the data; and, using degrees, represent the data with an equation of the form: • $y = a \sin(kt) + c$ **OR** • $y = a \cos(kt) + c$.	1			
Use trigonometric functions to model and solve problems.	1			
Distinguish between degree and radian measure, and solve problems, using both.	1			
Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position.	1			
Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.	1			
Solve first and second degree trigonometric equations over a domain of length 2π: • *algebraically* • *graphically*.	1			
Determine the general solutions to trigonometric equations where the domain	1			

Topic/Description	P1	P2	P3	Out
is the set of real numbers.				
Verify trigonometric identities: *1. numerically for any particular case* *2. algebraically for general cases* 3. *graphically*.	1			
Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions.	1			
To determine sin nx where n is a natural number	1			
Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for: *1. amplitude, if defined* *2. period* *3. domain and range* *4. asymptotes, if any* 5. *behaviour under* transformations.	1			
Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for: *1. period* *2. domain and range* *3. asymptotes* 4. *behaviour under transformations.*				1
Geometry				
Estimate, measure and draw angles, using a protractor.	1		1	
Determine the measures of angles in a diagram.	1		1	
Use concrete materials and diagrams to verify the Pythagorean relationship.	1		1	
Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications.	1		1	
Measure the diameters, radii and circumferences of circles, and establish the relationships among them.	1		1	
Calculate the area of a circle.	1		1	
Solve problems involving the radii, diameters, area and circumferences of circles.	1		1	
Use properties of circles and polygons to solve design and layout problems.	1		1	
Solve problems involving distances between points in the coordinate plane.	1		1	
Solve problems involving midpoints of line segments.	1		1	
Solve problems involving rise, run and slope of line segments.	1		1	
Determine the equation of a line, given information that uniquely determines the line.	1		1	
Solve problems involving distances between points and lines.	1		1	
Verify and prove assertions in plane geometry, using coordinate geometry.	1		1	
Solve problems using slopes of: • *parallel lines* • *perpendicular* lines.	1		1	
Use technology and measurement to confirm and apply the following properties to particular cases: • the *perpendicular from the centre of a circle to a chord bisects the chord* • *the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc*				1

Topic/Description	P1	P2	P3	Out
• *the inscribed angles subtended by the same arc are congruent* • *the angle inscribed in a semicircle is a right angle* • *the opposite angles of a cyclic quadrilateral are supplementary* • *a tangent to a circle is perpendicular to the radius at the point of tangency* • *the tangent segments to a circle, from any external point, are congruent* • *the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord* • *the sum of the interior* angles of an n-sided polygon is (2n – 4) right angles.				
Prove the following general properties, using established concepts and theorems: *1.* the *perpendicular bisector of a chord contains the centre of the circle* *2.* *the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle)* 3. *the inscribed angles* subtended by the same arc are congruent *4.* the *angle inscribed in a semicircle is a right angle* *5.* *the opposite angles of a cyclic quadrilateral are supplementary* *6.* *a tangent to a circle is perpendicular to the radius at the point of tangency* *7.* *the tangent segments to a circle from any external point are congruent* *8.* *the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord* 9. *the sum of the interior* angles of an n-sided polygon is (2*n* – 4) right angles.	1		1	
Solve problems, using a variety of circle properties, and justify the solution strategy used.	1		1	
Logic				
Differentiate between inductive and deductive reasoning.	1	1		
Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems.	1	1		
Use examples and counterexamples to analyze conjectures.	1	1		
Distinguish between an “if–then” proposition, its converse and its contrapositive.	1	1		
Prove assertions in a variety of settings, using direct and indirect reasoning.	1	1		
Matrices				
Show an understanding of matrices and perform the operations of addition, scalar multiplication and matrix multiplication.				1
Solve problems, using the operations of addition, subtraction, scalar multiplication and matrix multiplication on matrices.				1
Use matrices and matrix operations to model and to solve consumer, network and schedule problems.				1
Determine the inverse of a 2x2 matrix				1
Measurement				
Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes.	1	1	1	
Design an appropriate measuring process or device to solve a problem.	1	1	1	
Estimate measurements of objects in SI and Imperial systems including: • *length*	1	1	1	

Topic/Description	P1	P2	P3	Out
• *area* • *volume* • *mass*				
Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy.	1	1	1	
Develop a sense of approximate conversions between SI and imperial units through investigations.	1	1	1	
Perform basic conversions within and between the Imperial and SI systems, using technology as appropriate	1	1	1	
Solve problems involving length, area, volume, time, mass and rates derived from these.	1	1	1	
Use dimensions and unit prices to solve problems involving perimeter, area and volume.	1		1	
Interpret drawings, and use the information to solve problems.	1	1	1	
Enlarge or reduce a dimensioned object, according to a specified scale.	1		1	
Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes.	1		1	
Solve problems involving percentage error when input variables are expressed with percentage errors.	1	1	1	
Solve problems involving estimation and costing for objects, shapes or processes when a design is given.			1	
Design an object, shape, layout or process within a specified budget.			1	
Use simplified models to estimate the solutions to complex measurement problems.			1	
Calculate the volume and surface area of a sphere, using formulas that are provided.	1		1	
Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects.	1		1	
Analyze objects shown in "exploded" format			1	
Draw objects in "exploded" format			1	
Draw top, front and side views for both 3-D rod or block objects and their sketches			1	
Sketch and build 3-D designs using isometric dot paper				1
Complete a project that includes a 2-D plan and a 3-D model of some physical structure			1	
Properties of Number Systems				
Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.	1	1	1	
Use estimation strategies to justify or assess the reasonableness of calculations.	1	1	1	
Apply order of operations to solve problems, using paper and pencil or a calculator.	1	1	1	
Demonstrate and explain the meaning of fractions, concretely, pictorially and symbolically.	1	1	1	
Express rates and ratios in equivalent forms to solve problems.	1	1	1	
Demonstrate and explain the meaning of ratio concretely, pictorially and symbolically.	1	1	1	
Represent and apply fractions as per cents, and percents in fraction or decimal form.	1	1	1	
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing decimals (for more than 2-digit divisors or	1	1	1	

Topic/Description	P1	P2	P3	Out
multipliers, the use of technology is expected).				
Demonstrate an understanding of integers and use arithmetic operations to solve problems involving integers.	1	1	1	
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing fractions concretely, pictorially and symbolically.	1	1	1	
Estimate and calculate percentages.	1	1	1	
Use approximate representations of irrational numbers.	1		1	
Define and illustrate complex numbers	1			
Express complex numbers in the form a + b*i*	1			
Add, subtract, multiply and divide complex numbers	1		1	
Divide complex numbers using conjugates				1
Probability				
Express probabilities as ratios, fractions, decimals, percents, and in words	1	1		
Use probability to predict the result in a given situation	1	1		
Determine the odds for and against a particular event occurring	1	1		
Compare experimental observations with theoretical predictions	1	1		
Use probabilities to calculate expected gains and losses		1		
Communicate and justify solutions to probability problems	1	1		
Solve pathway problems, interpreting and applying any constraints.				1
Use the fundamental counting principle to determine the number of different ways to perform multistep operations.				1
Construct a sample space for two or three events.				1
Classify events as independent or dependent.	1	1		
Solve problems, using the probabilities of mutually exclusive and complementary events.				1
Solve decision-making problems involving expected values, and communicate the solutions.				1
Determine the number of permutations of n different objects taken *r* at a time, and use this to solve problems.				1
Determine the number of permutations of n objects arranged in a circle.				1
Determine the number of combinations of n different objects taken *r* at a time, and use this to solve problems.				1
Determine the conditional probability of two events (Bayes' law).				1
Solve probability problems involving permutations, combinations and conditional probability.				1
Solve problems, using the binomial theorem where *N* belongs to the set of natural numbers.				1
Solve probability problems, using the binomial distribution as applied to small samples.				1
Systems of Equations				
Design and solve linear and nonlinear systems, in two variables, to model problem situations.	1			
Solve systems of linear equations, in two variables: • *algebraically (elimination and substitution)* • *graphically*.	1			
Recognize the characteristics of linear equations in two variables with graphs that are inconsistent, consistent-dependent, or consistent-independent	1			

Topic/Description	P1	P2	P3	Out
Solve nonlinear equations, using a graphing tool.	1			
Solve systems of linear equations, in three variables: • *algebraically* • *with technology*.	1			
Determine the solution to a system of nonlinear equations, using technology as appropriate				1
Graph linear inequalities, in two variables.	1			
Solve, graphically, systems of linear inequalities, in two variables, using technology.				1
Apply linear programming to find optimal solutions to decision-making problems.				1
Statistics				
Determine measures of central tendency and variability for a set of data: • *mode* • *median* • *mean* • *range.*	1	1		
Determine and use the most appropriate measure of central tendency in a given context.	1	1		
Read and interpret graphs	1	1		
Extract information from given graphs of discrete or continuous data, using: • time series • glyphs (custom pictorial representations) • continuous data • contour lines.	1	1		
Display and analyze data on a line plot	1	1		
Use suitable graph types to display data (by hand or using technology)	1	1		
Critique ways in which statistical information and conclusions are presented by the media and other sources		1		
Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation.	1	1		
Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data.		1		
Describe issues to be considered when collecting data (e.g., appropriate language, ethics, cost, privacy, cultural sensitivity)		1		
Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population.		1		
Defend or oppose inferences and generalizations about populations, based on data from samples.		1		
Determine the equation of a line of best fit, using: • *estimate of slope and one point* • *median–median method* • *least squares method* with technology.		1		
Use technological devices to determine the correlation coefficient *r*.		1		
Describe the relationship between the correlation coefficient and two data sets				1
Interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots.		1		
Find the population standard deviation of a data set or a probability distribution, using technology.		1		
Use z-scores and z-score tables to solve problems.				1

Topic/Description	P1	P2	P3	Out
Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples.		1		
Describe and illustrate normal and skewed distributions using real-world examples		1		
Explain the variability of data using standard deviation and the normal curve		1		
Use standard deviation to describe the variability within a set of data		1		
Use sample data to make predictions and decisions		1		
Determine the percentile rank of an item in a set of sample data		1		
Distinguish between percent and percentile rank		1		
Sequences and Series				
Identify sequences that appear to be: • *divergent* • *convergent* • *oscillating* • *static*.				1
Generate number patterns exhibiting arithmetic growth.				1
Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems.	1			
Relate arithmetic sequences to linear functions defined over the natural numbers.	1			
Generate number patterns exhibiting geometric growth.				1
Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems.				1
Connect geometric sequences to exponential functions over the natural numbers.				1
Estimate values of expressions for infinite geometric processes.				1
Construct a fractal pattern by repeatedly applying a procedure to a geometric figure.				1
Use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns.				1
Technological Applications				
Create a spreadsheet using various formatting options		1		
Use a spreadsheet template to solve problems		1		
Create a spreadsheet using formulas and functions		1		
Use a spreadsheet to answer "what-if" questions				1
Identify where spreadsheets could be effectively used		1		
Use spreadsheets to analyze renting or buying an increasing asset (home) under different sets of circumstances.				1
Use spreadsheets to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances.				1
Use spreadsheet(s) to analyze an investment or life insurance portfolio, applying such concepts as capital gains, interest rate, inflation rate, risk, total rate of return and after-tax rate of return.				1
Design or modify a financial spreadsheet template to allow users to input their own variables.				1
Use and modify a spreadsheet template to model recursive situations				1
Solve problems involving combinations of tables, using:				1

Topic/Description	P1	P2	P3	Out
1. addition or subtraction of two tables 2. multiplication of a table by a real number 3. spreadsheet functions and templates.				
Use technology to generate and graph finite or infinite sequences whose recursive definition may or may not be given.				1
Transformations				
Describe how various translations of functions affect graphs and their related equations: • $y = f(x - h)$ • $y - k = f(x)$.	1			
Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: • $y = af(x)$ • $y = f(kx)$.	1			
Describe how reflections of functions in both axes and in the line y = x affect graphs and their related equations: • $y = f(-x)$ • $y = -f(x)$ • $y = f^{-1}(x)$.	1			
Using the graph and/or the equation of f(x), describe and sketch $\frac{1}{f(x)}$.	1			
Using the graph and/or the equation of f(x), describe and sketch |f(x)|.	1			
Describe and perform single transformations and combinations of transformations on functions and relations.	1			
Vectors				
Use and give 3-D and 2-D examples of vector terminology and notation, including: • *vector (direction, magnitude)* • *scalar* • *unit vector* • *collinear vectors* • *opposite vectors* • *parallel vectors* • *resultant vectors.*	1			
Assign meaning to the multiplication of a vector by a scalar.	1			
Perform vector additions and subtractions, using triangle or parallelogram methods.	1			
Determine the magnitude and direction of a resultant vector, using triangle, parallelogram or component methods.	1			
Use vector diagrams and trigonometry to analyze and solve practical problems in 3-D and 2-D.				

In the above table, it should be noted that some outcomes are included in more than one pathway and that some outcomes are not included in any pathway. The following counts were determined:

TABLE 30: Outcome counts

	Count	Also in Pathway 1	Also in Pathway 2	Also in Pathway 3	Also in both other pathways	Not in other pathways
Pathway 1	152		28	24	31	69
Pathway 2	80	28		0	31	21
Pathway 3	68	24	0		31	13

Outcomes not in any pathway	59
Outcomes in exactly one pathway	103
Outcomes in exactly two pathways	52
Outcomes in all three pathways	31

What are the desired student mathematical competencies for entry directly into the world of work upon completion of secondary studies?

Business responses

As noted earlier, there were 54 business responses. These were from a wide cross-section of industry, ranging from small businesses to major banks and oil companies. A few large public sector employers were also included. Of these respondents, 41 provided at least some responses to the outcomes questions. They were asked to identify the level of mastery of each outcome required for their employees who they would be recruiting from high school. The following table shows the occupation of the employees that they reported for, and it should be noted that a number are in positions more senior than the entry level expected from high school:

TABLE 31: Business respondent occupations

Occupation	**Count**
"All"	1
Entry level	14
Manager	4
None provided	16
Professional	1
Technical	5
Total	**41**

There were a relatively small number of outcomes (44) that were seen as required for the entry level positions, and they are as follows:

TABLE 32: Outcomes requested for entry level positions by business respondents

Finance
Solve consumer problems, including: • wages earned in various situations • property taxation • exchange rates unit prices.
Reconcile financial statements including:

• cheque books with bank statements cash register tallies with daily receipts.
Solve budget problems, using graphs and tables to communicate solutions.
Solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties)
Logic
Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems.
Measurement
Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes.
Design an appropriate measuring process or device to solve a problem.
Estimate measurements of objects in SI and Imperial systems including: • *length* • *area* • *volume* • *mass*
Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy.
Perform basic conversions within and between the Imperial and SI systems, using technology as appropriate
Solve problems involving length, area, volume, time, mass and rates derived from these.
Use dimensions and unit prices to solve problems involving perimeter, area and volume.
Interpret drawings, and use the information to solve problems.
Enlarge or reduce a dimensioned object, according to a specified scale.
Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes.
Properties of Number Systems
Apply order of operations to solve problems, using paper and pencil or a calculator.
Demonstrate and explain the meaning of fractions, concretely, pictorially and symbolically.
Express rates and ratios in equivalent forms to solve problems.
Demonstrate and explain the meaning of ratio concretely, pictorially and symbolically.
Represent and apply fractions as per cents, and percents in fraction or decimal form.
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing decimals (for more than 2-digit divisors or multipliers, the use of technology is expected).
Demonstrate an understanding of integers and use arithmetic operations to solve problems involving integers.
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing fractions concretely, pictorially and symbolically.
Estimate and calculate percentages.
Add, subtract, multiply and divide complex numbers
Probability
Express probabilities as ratios, fractions, decimals, percents, and in words
Use probability to predict the result in a given situation
Determine the odds for and against a particular event occurring
Use probabilities to calculate expected gains and losses
Communicate and justify solutions to probability problems
Solve pathway problems, interpreting and applying any constraints.
Use the fundamental counting principle to determine the number of different ways to perform multistep operations.
Technological Applications
Create a spreadsheet using various formatting options
Use a spreadsheet template to solve problems
Create a spreadsheet using formulas and functions

Use a spreadsheet to answer "what-if" questions
Identify where spreadsheets could be effectively used
Use spreadsheets to analyze renting or buying an increasing asset (home) under different sets of circumstances.
Use spreadsheets to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances.
Use spreadsheet(s) to analyze an investment or life insurance portfolio, applying such concepts as capital gains, interest rate, inflation rate, risk, total rate of return and after-tax rate of return.
Design or modify a financial spreadsheet template to allow users to input their own variables.
Use and modify a spreadsheet template to model recursive situations
Solve problems involving combinations of tables, using: 4. addition or subtraction of two tables 5. multiplication of a table by a real number 6. spreadsheet functions and templates.
Use technology to generate and graph finite or infinite sequences whose recursive definition may or may not be given.

The finance, probability and technological applications outcomes were selected largely due to a significant number of "expose" answers – mastery was required only for the other areas noted above (logic, measurement and properties of number systems).

Of the business respondents, 37 also made comments on the requirements for specified types of positions. Considering the positions that could be obtained with high school graduation (or less), the following were the main skill areas noted:

- Add/subtract/multiply/divide – 13 respondents mentioned
- Accounting/financial – 7
- "basic" high school math – 5
- Fractions/decimals – 4
- Ratio, percent – 3
- Some algebra – 3
- Some measurement – 2
- Metric and imperial measures and conversion – 2
- Volume, area, radius – 1
- Geometry – 1

Three respondents indicated that no math skills were required.

Considering these responses from business, it appears that for entry level positions recruited from high school, the "basic" mathematics skills, which are largely found within the measurement and properties of number systems topics, are of the most interest to employers, with exposure to finance, probability and the use of technology also considered useful. The skills that are included in the elementary and junior high curriculum are also of importance to this group.

The impact of this on the pathways that have been proposed is that it seems that if Pathway 3 were to add some logic outcomes, some basic probability and some exposure to technology, it would meet the expressed needs of business for entry level positions. The outcomes in such a modified Pathway 3 would be as follows, with the two right-most columns identifying which of the two areas requires the outcome:

TABLE 33: Combined outcomes for Pathway 3 and business

Topic	P3	Bus.
Algebra		
Communicate a set of instructions used to solve an arithmetic problem.	1	
Perform arithmetic operations on irrational numbers, using appropriate decimal approximations.	1	
Substitute numbers for variables in expressions, and graph and analyze the relation.	1	
Translate between an oral or written expression and an equivalent algebraic expression.	1	
Use formulas to solve problems	1	
Perform the operations of addition, subtraction, multiplication and division on rational expressions.	1	
Find and verify the solutions of rational equations.	1	
Finance		
Analyze car or house insurance needs and premiums, using such concepts as loss, probability of loss, compulsory coverage, optional coverage, deductible and claims record.	1	
Solve consumer problems, including: • wages earned in various situations • property taxation • exchange rates • unit prices.	1	1
Reconcile financial statements including: • cheque books with bank statements • cash register tallies with daily receipts.	1	1
Solve budget problems, using graphs and tables to communicate solutions.	1	1
Solve investment and credit problems involving simple and compound interest.	1	
Solve problems involving the acquisition and operation of a vehicle.	1	
Solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties)		1
Characteristics of Functions and Relations		
Interpret the graph of a relation and describe it in words	1	
Construct a graph of a relation from its description in words	1	
Linear Functions and Equations		
Solve and verify one-step linear equations, using a variety of techniques including concrete materials and diagrams.	1	
Solve and verify two-step, single-variable, first-degree equations, using concrete materials, diagrams, informal algebraic methods, or formal algebra in the form: • x + a = b • ax = b • $\frac{x}{a} = b$ • ax + b = c where *a*, *b* and *c* are integers.	1	
Geometry		
Estimate, measure and draw angles, using a protractor.	1	
Determine the measures of angles in a diagram.	1	

Topic	P3	Bus.
Use concrete materials and diagrams to verify the Pythagorean relationship.	1	
Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications.	1	
Measure the diameters, radii and circumferences of circles, and establish the relationships among them.	1	
Calculate the area of a circle.	1	
Solve problems involving the radii, diameters, area and circumferences of circles.	1	
Use properties of circles and polygons to solve design and layout problems.	1	
Solve problems involving distances between points in the coordinate plane.	1	
Solve problems involving midpoints of line segments.	1	
Solve problems involving rise, run and slope of line segments.	1	
Determine the equation of a line, given information that uniquely determines the line.	1	
Solve problems involving distances between points and lines.	1	
Verify and prove assertions in plane geometry, using coordinate geometry.	1	
Solve problems using slopes of: • *parallel lines* • *perpendicular* lines.	1	
Prove the following general properties, using established concepts and theorems: *1.* the *perpendicular bisector of a chord contains the centre of the circle* *2.* *the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle)* 3. *the inscribed angles* subtended by the same arc are congruent *4.* the *angle inscribed in a semicircle is a right angle* *5.* *the opposite angles of a cyclic quadrilateral are supplementary* *6.* *a tangent to a circle is perpendicular to the radius at the point of tangency* *7.* *the tangent segments to a circle from any external point are congruent* *8.* *the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord* 9. *the sum of the interior* angles of an n-sided polygon is $(2n - 4)$ right angles.	1	
Solve problems, using a variety of circle properties, and justify the solution strategy used.	1	
Logic		
Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems.		1
Measurement		
Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes.	1	1
Design an appropriate measuring process or device to solve a problem.	1	1
Estimate measurements of objects in SI and Imperial systems including: • *length*	1	1

Topic	P3	Bus.
• *area* • *volume* • *mass*		
Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy.	1	1
Develop a sense of approximate conversions between SI and imperial units through investigations.	1	
Perform basic conversions within and between the Imperial and SI systems, using technology as appropriate	1	1
Solve problems involving length, area, volume, time, mass and rates derived from these.	1	1
Use dimensions and unit prices to solve problems involving perimeter, area and volume.	1	1
Interpret drawings, and use the information to solve problems.	1	1
Enlarge or reduce a dimensioned object, according to a specified scale.	1	1
Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes.	1	1
Solve problems involving percentage error when input variables are expressed with percentage errors.	1	
Solve problems involving estimation and costing for objects, shapes or processes when a design is given.	1	
Design an object, shape, layout or process within a specified budget.	1	
Use simplified models to estimate the solutions to complex measurement problems.	1	
Calculate the volume and surface area of a sphere, using formulas that are provided.	1	
Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects.	1	
Analyze objects shown in "exploded" format	1	1
Draw objects in "exploded" format	1	
Draw top, front and side views for both 3-D rod or block objects and their sketches	1	
Complete a project that includes a 2-D plan and a 3-D model of some physical structure	1	
Properties of Number Systems		
Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.	1	
Use estimation strategies to justify or assess the reasonableness of calculations.	1	
Apply order of operations to solve problems, using paper and pencil or a calculator.	1	1
Demonstrate and explain the meaning of fractions, concretely, pictorially and symbolically.	1	1
Express rates and ratios in equivalent forms to solve problems.	1	1
Demonstrate and explain the meaning of ratio concretely, pictorially and symbolically.	1	1
Represent and apply fractions as per cents, and percents in fraction or decimal form.	1	1
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing decimals (for more than 2-digit divisors or multipliers, the use of technology is expected).	1	1
Demonstrate an understanding of integers and use arithmetic operations	1	1

Topic	P3	Bus.
to solve problems involving integers.		
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing fractions concretely, pictorially and symbolically.	1	1
Estimate and calculate percentages.	1	1
Use approximate representations of irrational numbers.	1	
Add, subtract, multiply and divide complex numbers	1	1
Probability		
Express probabilities as ratios, fractions, decimals, percents, and in words		1
Use probability to predict the result in a given situation		1
Determine the odds for and against a particular event occurring		1
Use probabilities to calculate expected gains and losses		1
Communicate and justify solutions to probability problems		1
Solve pathway problems, interpreting and applying any constraints.		1
Use the fundamental counting principle to determine the number of different ways to perform multistep operations.		1
Technological Applications		
Create a spreadsheet using various formatting options		1
Use a spreadsheet template to solve problems		1
Create a spreadsheet using formulas and functions		1
Use a spreadsheet to answer "what-if" questions		1
Identify where spreadsheets could be effectively used		1
Use spreadsheets to analyze renting or buying an increasing asset (home) under different sets of circumstances.		1
Use spreadsheets to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances.		1
Use spreadsheet(s) to analyze an investment or life insurance portfolio, applying such concepts as capital gains, interest rate, inflation rate, risk, total rate of return and after-tax rate of return.		1
Design or modify a financial spreadsheet template to allow users to input their own variables.		1
Use and modify a spreadsheet template to model recursive situations		1
Solve problems involving combinations of tables, using: 7. addition or subtraction of two tables 8. multiplication of a table by a real number 9. spreadsheet functions and templates.		1
Use technology to generate and graph finite or infinite sequences whose recursive definition may or may not be given.		1

There are a total of 89 outcomes in the above list, 44 from Pathway 3 alone, 21 from Business alone and 24 from both areas. It may be possible to concentrate the “Pathway 3 alone” items in the Grade 12 course, thus permitting students who complete only the Grade 10 and Grade 11 courses to still emerge from high school meeting basic workplace entry requirements. Perhaps these two courses would be sufficient for diploma qualification but post-secondary trades and agriculture programs would still require completion of the Pathway 3 Grade 12 course for admission.

What revisions to the Grades 10-12 Common Curriculum Framework should be considered, from a post-secondary/business and industry perspective?

The process of matching sets of outcomes to sets of programs resulted in an assignment of each of the three clusters of post-secondary major areas to a proposed high school pathway (each with a proposed set of outcomes). Moving to such a pathway structure would require significant change to existing programs in at least some of the provinces and territories.

The following table shows the number of outcomes, for each topic, that are included in the current pathways, as defined by the WNCP provinces and territories (PM, AM and TP (Third Path)), and in the proposed related pathways.

TABLE 34: Outcome counts by pathway and current WCNP program

Topic	PM	Pathway 1	AM	Pathway 2	TP	Pathway 3
Algebra	11	14	3	8	4	7
Career Project	0	0	0	0	4	0
Conics	3	0	0	0	0	0
Finance	4	0	5	0	17	6
Characteristics of Functions	11	16	6	7	6	2
Linear Functions	3	7	3	6	5	2
Polynomial Functions	7	8	1	0	1	0
Exponential/Logarithmic Functions	7	7	1	0	0	0
Trigonometric Functions	13	18	5	0	6	0
Geometry	10	17	7	0	7	17
Logic	5	5	0	5	0	0
Matrices	0	0	3	0	1	0
Measurement	2	14	14	9	14	21
Properties of Number Systems	2	15	2	11	14	13
Probability	13	6	5	7	7	0
Systems of Equations	5	6	6	0	1	0
Statistics	5	7	11	23	17	0
Sequences and Series	7	2	3	0	0	0
Technological Applications	1	0	7	4	5	0
Transformations	6	6	0	0	0	0
Vectors	0	5	5	0	0	0
Total	**115**	**152**	**87**	**80**	**109**	**68**

From the above table, it can be seen that the proposed Pathway 1 would have considerably more outcomes than the current PM pathway, especially in the areas of measurement, properties of number systems, systems of equations; and significantly fewer in probability. It would be a major change. Pathway 2, while having a similar number of outcomes to that of the current AM, has them distributed quite differently – more in algebra, logic, properties of number systems and statistics, and fewer in finance, trigonometry, geometry and vectors. Again, this is a significant change. Pathway 3 is quite different in intent than is TP and, as expected, is quite different in

structure. One of the key points to note in comparing the two pathway structures is that the proposed pathways are strictly based on meeting post-secondary entrance requirements in different sets of program areas; whereas the current PM/AM/TP structure was designed to meet the needs of a variety of educational stakeholders.

Outcome requirements for the four respondent groups

The four respondent groups - the three post-secondary program areas groups and the business and industry group - had significantly different levels of requirement for the mathematics outcomes. The following table provides the percentage of answers to all outcome questions, for all respondents in each of the groups:

TABLE 35 – Responses by group

Group	Master (3)	Expose (2)	NA (1)
Calculus-based	45 %	26 %	29 %
Non-science, non-calculus	28 %	22 %	50 %
Trades	30 %	24 %	46 %
Business	22 %	28 %	50 %

The difference in requirement levels between the calculus-based program areas, and those of each of the other groups, is striking.

Reflections on pathway structure based on survey results

1. The survey analysis suggests that, based on expressed requirements of respondents, there are three distinct program area groupings in the post-secondary system – the Calculus-based group, the Non-Science Non-Calculus group and the Trades group. Business/industry respondents, especially those reporting on skills required for entry-level positions, come closest in requirements to the Trades group but have additional “expose” requirements.
2. Each group had identified a set of required outcomes, using specified thresholds for percent of the group requiring mastery and/or exposure to the outcome. The resulting three pathways have some overlap, but each has a fairly different primary focus in terms of the main topics included. Out of about 200 outcomes found in any pathway, about 60% are found in only one of the pathways. The primary foci of the three pathways are as follows (shaded topics are those that have a weaker but still significant requirement).

Pathway 1	Pathway 2	Pathway 3
Algebra	Logic	Measurement
Characteristics of Functions	Statistics	Geometry
Polynomial Functions	Properties of Number Systems	Properties of Number Systems
Exponential/Logarithmic	Algebra	Algebra
Logic		
Transformations		
Vectors		
Trigonometric Functions		
Geometry		
Properties of Number Systems		
Systems of Equations		
Measurement		

3. Pathway 1 has too many outcomes to be taught in the 100 hours per year of instructional time recommended by the WNCP. It has 152 outcomes, as compared to the 115 outcomes currently included in PM, which itself is considered too heavy. Other options will need to be explored for delivering these outcomes. One possibility is to have students aiming for the Sciences take two mathematics courses in one or more of the high school grades, and the Sciences major areas would ask for completion of both Pathway 1 Grade 12 courses for admission. Another possibility is to divide the outcomes into those that will be taught in high school and those that will be taught in the first post-secondary year.

4. Based on survey response, it is clear that most post-secondary program areas do not require most of the outcomes taught in the current PM program. Their stated requirements are for a much narrower set of topics and outcomes, and it would be possible to craft two new pathways that would tackle these topics in some depth. Clearly, based on expressed requirements alone, the PM program is NOT required for successful entry into the majority of post-secondary programs, as long as other pathways can be created to meet their specific needs.

What will be necessary to align post-secondary program admission standards more appropriately with the revised high school mathematics pathways?

Assuming that the WNCP Common Curriculum Framework is re-designed to meet the needs of post-secondary programs, as expressed in this survey, and the three pathways are set up to meet these expressed needs, the institutions should consider re-aligning their program entrance requirements appropriately.

Views of post-secondary institutions

Post-secondary institutions in the WNCP provinces and territories were provided with two windows of opportunity to comment on the survey and its results. The first was a series of meetings that provincial mathematics teams held with numbers of individual post-secondary institutions. The second was an opportunity provided to a senior administrator (e.g. academic vice-president) (and to any faculty to whom the administrator distributed the feedback invitation) of each post-secondary institution in the four provinces and three territories to provide written feedback on the preliminary English version of this report. As no French translation of the report was done, at least one francophone instution was unable to respond. The main themes and critical points are described in the following two sections.

Key themes from discussions with post-secondary institutions

The following comments are drawn from notes taken at the meetings of the Alberta Education mathematics team with the various Alberta post-secondary institutions. Much valuable input was obtained on a wide variety of topics relating to improving high school graduate mathematics competency. A subset of these related directly to the Program of Studies, and it is these that are summarized below.

1. The main expressed concern was students' lack of a strong grasp of "basic" mathematics. Some respondents indicated a need for the strong grounding in the "basics" that should be completed by the end of junior high.

2. Many respondents asked more focus on specific topics such as, in descending order of frequency of times mentioned:
   - Algebra
   - Fractions and decimals
   - Geometry
3. Other topics mentioned by one or two respondents included:
   - Calculus
   - Formal logic
   - Theory
   - Pattern recognition
   - Measurement
   - Proofs
   - Manual manipulation
   - Formulas

In summary, the post-secondary representatives were concerned about the lack of "basic" mathematical skills, even in terms of basic manipulation. There was an oft-repeated concern that technology had taken the place of mental mathematics and that students were able to do work only if they were given a formula and/or a calculator, as students are proficient in using technology, not in understanding and applying the concepts.

Summary of feedback received from post-secondary institutions

Respondent characteristics

By institution

A total of 21 post secondary institutions responded, 12 from British Columbia, 7 from Alberta and 1 each from Saskatchewan and the Northwest Territories. The level of institutional response can be categorized as follows:

TABLE 36A: Level of institutional feedback response

Level of response	**Total Number**
Mathematics department and/or its chair only	13
- directly (10 responses)	
- with a cover letter from Dean, Arts & Science (2 responses)	
- with a cover letter from VP Academic (1 response)	
Mathematics dept/chair and one other program (Education, Forestry or Trades) (all with a cover letter, with a few comments, by the VP or Associate VP, Academic)	3
VP Academic	1
Associate VP, Academic	1
Upgrading and GTS Coordinator	1
VP Learning plus articulation representative	1
7 respondents (Education, Adult Ed, Pre-Technology) coordinated by Manager, Program development	1

The large majority of response was from the Mathematics departments. In only three cases did a more senior official (the VP's Academic of the U of A and the University of Regina, and the Acting VP, Education, BCIT) provide even a few summary comments.

By respondent

There were a total of 33 responses from the 21 institutions, categorized as follows:

TABLE 36B: Level of institutional feedback response

Position of respondent	**Total Number**
Mathematics department chair	12
Mathematics professor/instructor	4
Mathematics Department response committee	2
Education instructor	2
Adult education instructor	2
Secondary education chair	1
Associate VP Academic	1
Upgrading and GTS Coordinator	1
Forestry instructor	1
Articulation Representative	1
Provost and VP Learning	1
Academic VP	1
Trades Access instructor	1
Head, Business Math	1
Head, Pre-Technology Program	1
Manager, Program development	1

The majority of the respondents who provided feedback on the preliminary report were in a mathematics position (19), with 5 in education, 3 in VP positions and the others divided among disciplines. There were no respondents from the Social Sciences, Health, Business, Humanities or Fine Arts.

Discussion of responses

The respondents to the request for report feedback have a dramatically different composition from that of survey respondents. Among survey respondents, there were 66 from Social Sciences (12%), 60 from Health (11%), 35 from Business (6%), 25 from Humanities (4) and 11 from Fine Arts (2%) – a total of 197 such respondents (35% of the total). There was no feedback on the preliminary report from anyone in any of these program areas.

A comparison of survey respondents and authors of responses to the preliminary report showed that very few individuals placed their names on responses to both inquiries, although some people may have contributed to group responses to one inquiry or to the other.

While the request for post-secondary response was made in order to obtain *institutional* response rather than to further solicit views of individual departments or faculty members, it is largely the latter result that has been obtained. Only in a few cases, perhaps including the Universities of Regina, Alberta and Calgary and, to a lesser degree, UBC and BCIT, were any substantive

comments received from officials at the VP level. Instead, it was largely the views of the mathematics departments that were received, which might be viewed as simply additional survey responses (albeit in a condensed and focused version).

Although this may be seen as less representative of the post-secondary viewpoint than were the survey findings, it may actually be more reflective of how institutions make decisions regarding admissions standards. In deciding the role that high school mathematics qualifications play in standards for admission to institutions and to their suites of programs, senior officials may turn to the mathematics department more often than they would solicit a joint response from a multi-faculty committee.

Summary comment

The lack of substantive feedback from post-secondary institution senior administrators (e.g. academic vice-presidents) will require a separate round of discussion with these officials once the report has been finalized and reviewed by the WNCP mathematics consultants, their Directors and Assistant Deputy Ministers, and once the WNCP has developed an approach to Common Curriculum Framework construction that considers all influential factors, with the survey report being just one of them.

Impact of feedback on the final survey report

The feedback has impacted the report in two ways:

1. Some comments have altered the current content – e.g. the suggestion that some programs included in the "non-science non-calculus" grouping in fact often do require calculus led to a re-examination of the grouping methodology; and the various ideas for how to deal with the perceived requirement for a large number of outcomes in Pathway 1 were considered in modifying report recommendations.
2. Other comments were considered in forming report recommendations, both for pathway structure and in general.

Summary of responses that altered report content

Please note in this section that sometimes direct quotes were taken from respondents, thus the occasional use of the first person singular.

Comments on the categorization of post-secondary programs

A number of respondents (from 8 different institutions) questioned the classification of some program groups into the "non-science non-calculus" grouping, indicating that at least some of the programs within given groups do indeed require calculus, and generally include at least one post-secondary calculus course in their program requirements. The programs mentioned (showing frequency of mention) were:

Business and Health (at least some included occupations) – 4 mentions each
Social sciences and Kinesiology – 2 mentions each
Agriculture, Psychology and Economics – 1 mention each

Current calendars for the programs of all respondents coded to these program areas were examined to see if a calculus course was a program requirement. It should be noted that this had

been done for Alberta institutions prior to initial coding, with the pattern of those results imputed to similar programs in similar type of institutions in the other provinces and territories, so there were already some respondents in these program areas who were coded as "calculus" respondents. The results of the re-examination, which covered respondents from all provinces and territories, were as follows:

TABLE 37 – Changes in counts by category

Program area	Original counts		Counts after re-examination	
	Calculus	Non-science	Calculus	Non-science
Business	5	30	11	24
Health	0	61	3	58
Kinesiology	0	7	1	6
Social Sciences	13	53	8	58
Total	18	151	23	146

In summary, the re-examination re-coded 10 respondents from "non-science" to "calculus" and re-coded 5 respondents from "calculus" to "non-science", leading to a net increase of 5 respondents in the "calculus" category. Within the program areas, the following patters emerged (at least to some degree):

- Business – university or university transfer programs were the ones requiring calculus
- Health – a few (not all) of the programs focusing on pharmacology or physiology require calculus
- Social Sciences – calculus is required in economics programs in all institutions, and, in a few institutions, statistics programs

The calculations in the report have been re-done using the re-coded categorizations.

Comments on the choice of thresholds

Respondents from 2 institutions suggested that the method of determining whether an outcome should be included in a pathway, i.e. the threshold percentages of respondents asking for "mastery" and/or "exposure", should be re-examined. The suggested changes were:

- If an outcome was identified as "expose" by 70% of respondents from the calculus-based group (even if 30% of respondents identified it as "not applicable"), then that outcome was considered to be required by this group. There is little debate among post-secondary educators in this group that there is too much content in the current PM program, yet this interpretation has led to a proposed increase in the number of outcomes in this pathway from 115 to 153. Respondents from calculus-based programs are unlikely to identify many topics as "not applicable", and so it is not surprising that 153 outcomes fit the specified threshold for Pathway 1. It is recommended that higher thresholds are satisfied by the responses before including a given outcome in Pathway 1, as the proposed list of outcomes seems overly large. Alternatively, outcomes that are identified as "master" with higher frequency could be designated as core outcomes, while other outcomes could be designated as additional outcomes from which material could be selected to supplement the core curriculum.
- The notion of "expose" is of little value. It is much better for students to have "mastered" a smaller number of critical topics than to be exposed to a large number of topics that they cannot remember when they arrive in post-secondary classes.

The impact, on the number of Pathway 1 outcomes, of performing these two variations on the threshold approach is dramatic:

TABLE 38: Impact of threshold changes on outcomes to include

Approach	Number of outcomes
Original	152
Using 50% at mastery (and not considering exposure)	105
Using 60% at mastery or 80% at either mastery or exposure	96

Details can be found in Appendix A.

Comments on survey methodology

Although it is too late to change the methodology, the following criticism was included in the "limitations" section of the final report:

- I'm concerned about the use of the terms "master", "expose" and "not applicable". The meaning of these terms is somewhat subjective: what I think of as exposure could easily be mastery to someone else. I think it is hard to draw many conclusions based on this rough a measure.

Summary of responses that did not alter report content but influenced recommendations

Comments on the limited applicability of survey results

Respondents from at least 2 institutions argued that decisions about the high school mathematics curriculum should not be solely based on the expressed needs of post secondary institutions. One respondent eloquently stated this concern as follows, "I am concerned with this study since it appears that its sole focus is on access to post-secondary institutions and work training programs. I would like to assert that mathematics is far too important in Canada (and the world) today to base curricular decisions only on the needs of students who will go on to post-secondary education. We have a much larger concern: how we educate our youth to fully and meaningfully participate in society. This report is deliberately biased towards conceptualizing high school mathematics for the purpose of post-secondary education. In my opinion this has the potential to reinforce the position of mathematics as a gatekeeper rather than presenting mathematics as an educational endeavour that contributes to the growth of the child in society."

Suggestions for modifying the composition of Pathway 1

Many respondents made suggestions regarding the modification of content within Pathway 1, nearly all proposing that one or more topic be removed from it. The report provided the following summary of how many outcomes within each topic should be included, and the following table also shows how many, of the 9 institutions whose respondents made a comment focused on this issue, either affirmed the need for the topic, or advocated its full or more substantial removal from Pathway 1:

TABLE 39: Proposed modifications to Pathway 1

Topic	Total	Pathway 1	Affirmed	Advocated full/ partial removal
Algebra	14	14	3	
Career Project	4	0		
Conics	3	0		1
Finance	17	0		1
Characteristics of Functions and Relations	16	16	2	
Linear Functions and Equations	17	7	1	
Polynomial Functions and Equations	8	8	1	
Exponential and Logarithmic Functions	7	7	1	
Trigonometric Functions and Equations	19	18	3	
Geometry	18	18	3	
Logic	5	5	1	3
Matrices	4	0		
Measurement	22	14	1	7
Properties of Number Systems	16	15	1	2
Probability	19	4	1	5
Systems of Equations	9	7	1	2
Statistics	25	5	1	8
Sequences and Series	10	4		3
Technological Applications	12	0		
Transformations	6	6	1	1
Vectors	5	5		5
Total	**246**	**153**		

The topics included in Pathway 1 which were most frequently recommended for removal were Measurement, Vectors, Probability and Statistics. In the case of Measurement, the reason cited most often is that this should be covered in earlier grades and/or in other courses in which measurements are made e.g. science. Those commenting on Vectors suggested that that it is best covered in Physics and/or left to universities. Comments about Statistics included that it would be better covered in depth in a first year university course.

Suggestions on covering "the basics"

At least 6 institutions supported the comment often heard during visits by the study team to Alberta post-secondary institutions, which could be phrased as "ensure that the basics are covered". To the degree that respondents were specific about what, in their view, "the basics" include, the following were offered:

- basic computational and algebraic skills;
- "basic" mathematics skills and the manipulation of algebraic equations;
- basic algebra and the ability to deal with functions.

A number of other respondents advocated the theme of "teach fewer topics in greater depth".

Comments regarding the addition of a second math course

Five institutions that saw value in adding a second math course to divide the large number of Pathway 1 outcomes into two courses. There were 2 such respondents who indicated that replacing the calculus in Math 31 with the extra Pathway 1 content would be preferable, as they could teach the calculus in post-secondary institutions. On the other hand, 2 respondents felt that adding a second math course would take away from a high school student's ability to pursue a broad education. There was little support for offering the extra content in post-secondary institutions via a special introductory course, although, as noted in the previous section, some respondents were amenable to deferring study of statistics, vectors and/or calculus to post-secondary institutions.

One respondent pointed out that if the 153 outcomes indicated in the report were implemented, the addition of a second course would leave the number of outcomes per course at the same number as it is in the current 3-course PM program, which is considered too heavy.

Comments regarding the use of technology

The preliminary report includes a section on the comments made by Alberta post-secondary institutions when Alberta Education met with them. One of the themes was summarized as follows: "In summary, the post-secondary representatives were concerned about the lack of "basic" mathematical skills, even in terms of basic manipulation. There was an oft-repeated concern that technology had taken the place of mental mathematics and that students were able to do work only if they were given a formula and/or a calculator." Some of the institutions providing feedback to the report echoed this comment, 6 of them indicating that technology use should be severely limited in high school. However, 2 other respondents felt that use of a graphing calculator, at least, was beneficial.

Comments on the need to provide flexibility for students

A number of respondents (representing 8 post-secondary institutions) were concerned about the apparent lack of mobility from one pathway to another, indicating that students would not be well served by being locked into one pathway in Grade 10, from which it would be difficult to move if interests changed during high school. A few of these respondents suggested that the pathways should diverge later, either in Grade 11 or 12.

Comments in support of the proposed pathway structure

There were 6 institutions (2 universities, 1 technical institution and 3 colleges) that stated that the proposed structure was appropriate and would meet the needs of post-secondary institutions (generally, and particularly their own) quite well.

Suggestions for changes to the pathway structure

Respondents from 5 institutions proposed structures that were different from the three-pathway approach identified in the preliminary report. They include:

- Two who advocated a single pathway, with fewer topics and with applications appropriately infused.
- One who suggested combining pathways 1 and 2 and then introducing a second course in Grade 12 to cover the specific pre-calculus requirements.

- One who proposed four pathways: "Basic" math – for basic trades, clerk typist (basic algebra); Basic technical math – for basic technologies and electrical trade (basic and intermediate algebra, basic and intermediate trig; Utilitarian math – for management studies, social sciences, arts, some biology (basic and intermediate algebra, basic trig); Advanced math – for physical, chemical sciences, engineering, technology (basic, intermediate and advanced algebra, basic, intermediate and advanced trigonometry).
- One who suggested that Pathway 2 be reconstructed to contain the basic parts of each outcome in Pathway 1. The key ideas would still be taught but the expectations would be at a lower level. This would allow more time for students in Pathway 2 to gain confidence and master the "basics" through the use of applications and examples. High achievement in such a pathway would then permit crossover to Pathway 1 and/or entry to post-secondary programs.
- One who suggested that grouping pathways according to destination makes no attempt to differentiate the differences among learners. An approach in line with the "old" academic, technical, and business streams of the 70's and 80's would be more in-line with learner abilities and post-secondary needs.

Comments on Pathway 3

Two institutions commented on this pathway, one indicating that some trades require more complex topics, and the other, from a Trades department, suggested that their main concern was the "students" lack of a strong grasp of "basic mathematics". The Trades department indicated a desire for students with mastery in "the basics", including the following content areas – arithmetic, basic algebra, geometry (plane and 3-dimensional), measurement (including the ability to check the reasonableness of their answers) and word problem applications including percent and geometry problems. They also preferred that their students had exposure to basic trigonometric functions, properties of number systems, common polynomial functions and simple vectors.

Focus on skills rather than outcomes

One institution had concerns with the focus on acquisition of outcomes rather than skills. The response indicated that an emphasis on specific outcomes does not address some of the important (non-outcome) skills that prospective students need when they enter post-secondary institutions. These skills include multistage reasoning, critical thinking, problem solving, number sense, and interdependence of concepts.

Comments on Pathway 2

Respondents from calculus-based programs clearly stated that Pathway 2 would not provide the skills needed to succeed in their programs. This was appropriate given that Pathway 2 is not intended to meet their needs.

However, one university mathematics department went further in stating that Pathway 2 would not meet university needs in general. An applicable comment from this department was "although it is conceivable that eventually some weakened high-school program in mathematics might be viable for university admission, as far as we can tell there is no acceptable proposal in sight. The present AM course is not of acceptable quality, and the proposed Pathway 2 seems to be largely a course with little intellectual content, failing to emphasize that understanding of material is at least as important as recipes."

In addition, one university academic vice-president stated that "Pathway 2 seems to be somewhat limited."

Comments on the use of mathematics in admission requirements

At least 2 respondents expressed concern about the role of mathematics as "gatekeeper" to post-secondary studies, especially university entrance. One respondent noted that even those students who take Applied Mathematics have taken 12 years of mathematics in school, and this provides a stronger foundation for success in related areas than do the much smaller amounts of time spent in some other subjects that are also entrance requirements to university. One of her related comments was that, "Students who take this course (Applied Mathematics 30) have studied mathematics for 12 years. They have spent far more time and are far more advanced in mathematics than students who have studied in a particular content area only in high school. Some such courses (for example, Drama 30, Applied Graphic Arts 35, Commercial Art 35 or any language other than English 30/31/35) can be used for entry to university (see group A and B courses)."

There is a related concern that the use of Pure Mathematics as gatekeeper forces students to take mathematics courses that may not be appropriate to their needs, in order not to "close doors."

Other substantive comments

These comments, and the response to or the recommended responding action, are shown below:

TABLE 40: Other responses

Respondent comment	Response
In my view there is a validity issue with the way in which numbers of participants are counted. On one hand it is important to know that there was some kind of collective response, but using those numbers to show how many people participated in the process is worrisome. This could be taken out of context and have undue influence.	Each response was counted equally in the analysis of responses to outcomes. Consideration was given to reducing the weight of separate responses from two or more members of the same faculty in the same institution, but there were few of those.
It would be worthwhile for the Ministry to compare the findings in their report with the outcomes of an earlier report, to be found at http://members.shaw.ca/bccupms/document/Mathematics%20Proficiencies%20Project.pdf or http://members.shaw.ca/bccupms/mathprof/mprofs.htm This report, on mathematics proficiencies required for success in university mathematics courses, did much to answer many of the questions that the Ministry is now dealing with. It could have been used as a starting point or partial replacement for what the Ministry has now done.	The WNCP team developing the revised 10-12 curriculum will be advised of this report and may wish to consider it as input, just as they will consider the survey final report as input, to their work.

Respondent comment	Response
How does one determine what mathematics is needed for the world of work? This is such a broad domain that the methodology of this particular review simply could not access it with any validity. One suggestion is to turn to reports done by other agencies, such as the *Conference Board of Canada*, for insight.	The WNCP team developing the revised 10-12 curriculum will be advised of this resource and may wish to consider it as input, just as they will consider the survey final report as input, to their work.
Pathway 3 wants students to understand compound interest without being able to explain laws of exponentiation, or knowing anything about exponential functions.	The WNCP team developing the revised 10-12 curriculum will ensure that the revised program of studies include topics in a logical sequence.
Pathway 2 wants a lot of statistics, but no z-scores. How are students to understand how to calculate with the normal distribution without z-scores?	The WNCP team developing the revised 10-12 curriculum will ensure that the revised program of studies include topics in a logical sequence.
Under number systems, addition and subtraction of complex numbers will be covered in Pathway 3 without any apparent background in irrational or complex numbers.	The WNCP team developing the revised 10-12 curriculum will ensure that the revised program of studies include topics in a logical sequence.

Conclusions

The results of this survey suggest that three high school pathways, with indicated content, should satisfy stated admission requirements for the appropriate sets of programs. This should:

1. Ensure that Pathway 1 satisfies the entrance requirements of the calculus-based (and similar) post secondary programs with the fewest possible outcomes.
2. Ensure that Pathway 2 satisfies the entrance requirements of most of the remainder of the non-calculus-based programs.
3. Ensure that Pathway 3 satisfies the entrance requirements of the trades and agriculture, and with a few additions, meets the needs of business and industry for positions for which they recruit from high schools.

Specific conclusions about pathway composition and mechanisms for addressing issues raised in the study are as follows:

1. Reduce the number of outcomes in Pathway 1 by considering all of the proposed modifications, including those suggested within post-secondary feedback to the preliminary report, and the changes that would result if thresholds were changed as noted in Appendix A. The most likely candidates for exclusion would appear to be:
   - Measurement – reduction in the number of outcomes from 14 to perhaps 8 (possibly taught instead in other courses and/or in earlier grades)
   - Probability – reduction from 4, or full removal (possibly taught instead in post-secondary)
   - Statistics – reduction from 5, or full removal (possibly taught instead in post-secondary)
   - Vectors – reduction from 5, or full removal (possibly taught instead in Physics).
2. Provide more flexibility for students by beginning the different pathways in Grade 11 rather than in Grade 10.

3. Re-examine the use of technology, especially graphing calculators, in high school.
4. Engage in further discussion with trades groups to ensure that Pathway 3 meets the needs of at least the large majority of trades.
5. Ensure that the revised pathways each have a logical sequence of topics, and that prerequisites for each topic are covered.

Recommendations

1. Consider the findings of this report in the review of the WNCP Common Curriculum Framework for Mathematics. In this review, recognize that this report considers pathway requirements only from the viewpoint of post-secondary respondents and a small number of business/industry respondents – it does not consider other drivers, such as the need for all post-secondary students and those in the working world to have a reasonable understanding of finance.
2. Discuss the report findings and any consequent changes to the high school program with key post-secondary institution senior administrators (e.g. academic vice-presidents), to ensure that the best possible exchange of views occurs on the topic of the validity of each pathway as an appropriate admission standard for the sets of programs to which each is intended to align.

Appendix A: Modification of outcomes inclusion criteria for Pathway 1

In allocating outcomes to Pathway 1, the report used the criterion that the outcome needed to be identified as “master” by at least 50% of “calculus-based” respondents and/or as “master” or “expose” by at least 70% of such respondents. This resulted in allocation of 152 outcomes to Pathway 1, and this is a much larger number of outcomes than can be accommodated in the current time allotment for one mathematics pathway in high school.

It was decided to examine the impact of applying stricter criteria for Pathway 1 inclusion, and the results are presented below. Two variations were examined:

1. Remove the "expose" part of the Pathway 1 criteria, and only include outcomes where 50% or more of the calculus-based respondents indicated that mastery was required. Results of this change are shown in the columns labeled 1A in the following two tables.
2. Raise the master/expose threshold to 60% / 80%. Results of this change are shown in the columns labeled 1B in the following two tables.

Thus, the summary of outcomes, using the various methods, is as follows:

Pathway 1 (P1) – if an outcome was identified as “master” by at least 50% of “calculus-based” respondents and/or as “master” or “expose” by at least 70% of such respondents, it was included herein. This is the approach taken in this report.

Pathway 1A (P1A) - if an outcome was identified as “master” by at least 50% of “calculus-based” respondents, it was included herein.

Pathway 1B (P1B) - if an outcome was identified as “master” by at least 60% of “calculus-based” respondents and/or as “master” or “expose” by at least 80% of such respondents, it was included herein.

Pathway 2 (P2) - if an outcome was identified as “master” by at least 40% of the “Non-science non-calculus” respondents and/or as “master” or “expose” by at least 60% of such respondents, it was included herein. The lower threshold was used here to be even more certain that the pathway would meet stated requirements (given issues with the current second pathway).

Pathway 3 (P3) - if an outcome was identified as “master” by at least 50% of “Trades” respondents and/or as “master” or “exposure” by at least 70% of such respondents, it was included herein.

This resulted in the following numbers of outcomes being included for each topic in each pathway:

Topic	Total	P1	P1A	P1B	P2	P3
Algebra	14	14	12	14	8	7
Career Project	4	0	0	0	0	0
Conics	3	0	0	0	0	0
Finance	17	0	0	0	0	8
Characteristics of Functions and Relations	16	16	13	14	9	2
Linear Functions and Equations	7	7	6	7	6	2
Polynomial Functions and Equations	8	8	6	6	0	0
Exponential and Logarithmic Functions	7	7	7	7	0	0
Trigonometric Functions and Equations	19	18	10	8	0	0
Geometry	18	17	13	13	0	17
Logic	5	5	4	5	5	0
Matrices	4	0	0	0	0	0
Measurement	22	14	8	8	8	21
Properties of Number Systems	16	15	12	12	11	13
Probability	19	6	2	0	9	0
Systems of Equations	9	6	3	2	0	0
Statistics	25	7	4	0	24	0
Sequences and Series	10	2	0	0	0	0
Technological Applications	12	0	0	0	4	0
Transformations	6	6	3	0	0	0
Vectors	5	5	2	0	0	0
Total	**246**	**152**	**105**	**96**	**84**	**70**

The following table shows the original assignment of mathematics outcomes to pathways, as presented in the original report. The two columns following P1 identify outcomes which would be allocated to Pathway 1 if allocation criteria were changed as noted in discussion of the above table. The detailed assignment of outcomes to pathways, using the various methods, is as follows:

Topic/Description	P1	1A	1B	P2	P3	Out
Algebra						
Use words and algebraic expressions to describe the data and the interrelationships in a table.	1	1	1	1		
Communicate a set of instructions used to solve an arithmetic problem.	1	1	1	1	1	
Perform arithmetic operations on irrational numbers, using appropriate decimal approximations.	1	1	1	1	1	
Explain and apply the exponent laws for powers of numbers and for variables with rational exponents.	1	1	1			
Factor polynomial expressions of the form $ax^2 + bx + c$, $a^2x^2 - b^2y^2$, $a^3x^3 - b^3y^3$, $a^3x^3 + b^3y^3$	1	1	1			
Find the product of polynomials.	1	1	1			
Divide a polynomial by a binomial, and express the result in the forms:	1		1			

Topic/Description	P1	1A	1B	P2	P3	Out
• $\frac{P}{D} = Q + \frac{R}{D}$ • P = DQ + R • P(x) = D(x)$Q(x)$ + R.						
Substitute numbers for variables in expressions, and graph and analyze the relation.	1	1	1	1	1	
Translate between an oral or written expression and an equivalent algebraic expression.	1	1	1	1	1	
Use formulas to solve problems	1	1	1	1	1	
Determine the non-permissible values for the variable in rational expressions.	1	1	1			
Perform the operations of addition, subtraction, multiplication and division on rational expressions.	1	1	1	1	1	
Find and verify the solutions of rational equations.	1	1	1	1	1	
Perform operations on irrational numbers of monomial and binomial form, using exact values.	1		1			
Career Project						
Determine what factors are important in analysing careers						1
Describe two specific career opportunities						1
Identify mathematical educational requirements for two careers						1
Compare two careers in terms of salary, working hours, training time and cost, cost of living, and benefits						1
Conics						
Classify conic sections according to shape.						1
Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only).						1
Convert a given equation of a conic section from general to standard form and vice versa.						1
Finance						
Analyze car or house insurance needs and premiums, using such concepts as loss, probability of loss, compulsory coverage, optional coverage, deductible and claims record.					1	
Solve consumer problems, including: • wages earned in various situations • property taxation • exchange rates • unit prices.					1	
Reconcile financial statements including: • cheque books with bank statements • cash register tallies with daily receipts.					1	
Solve budget problems, using graphs and tables to communicate solutions.					1	
Solve investment and credit problems involving simple and compound interest.					1	
Name and describe various types of commonly used consumer bank accounts and services						1
Solve problems involving the acquisition and operation of a vehicle.					1	
Prepare income tax forms.						

Topic/Description	P1	1A	1B	P2	P3	Out
Prepare a business plan to own and operate a business.						
Determine the costs involved in purchasing a home, including gross debt service ratio						1
Solve problems involving different types of mortgages						1
Describe government expenditures including the amounts spent on social welfare benefits, social security, education, health care, policing, armed forces, and employee wages and salaries						1
Solve problems involving the calculation of selected federal taxes (e.g., GST, excise tax and duties)						1
Calculate provincial taxes, (e.g., PST, corporation capital, licenses, gasoline)						1
Determine how selected municipal taxes are calculated (e.g., property)						1
Compare and contrast different investment vehicles in terms of risk factors, rates of return, costs, and lengths of term						1
Describe a variety of sales promotion techniques and their financial implications for the consumer						1
Characteristics of Functions and Relations						
Generalize a pattern arising from a problem-solving context, using mathematical expressions and equations, and verify by substitution.	1	1	1	1		
Interpret the graph of a relation and describe it in words	1	1	1	1	1	
Construct a graph of a relation from its description in words	1	1	1	1	1	
Graph relations, analyze the result and draw a conclusion from a pattern.	1	1	1	1		
Create and modify tables from both recursive and non-recursive situations.	1			1		
Represent data, using function models.	1	1	1	1		
Use a graphing tool to draw the graph of a function from its equation.	1		1			
Describe a function in terms of: • ordered pairs • a rule, in word or equation form • a graph.	1	1	1	1		
Use function notation to evaluate and represent functions.	1	1	1			
Determine the domain and range of a relation from its graph.	1	1	1			
Plot and analyse examples of direct variation, partial variation, and inverse variation	1	1	1			
Perform operations on functions and compositions of functions.	1	1	1			
Determine the inverse of a function.	1	1	1			
Describe, graph and analyze polynomial and rational functions, using technology.	1					
Determine if a relation is a function using a variety of methods.	1	1	1			
Formulate and apply strategies to solve absolute value equations, radical equations, rational equations and inequalities.	1	1	1			
Linear Functions and Equations						
Solve and verify one-step linear equations, using a variety of techniques including concrete materials and diagrams.	1	1	1	1	1	
Solve and verify two-step, single-variable, first-degree equations, using concrete materials, diagrams, informal algebraic methods, or	1	1	1	1	1	

Topic/Description	P1	1A	1B	P2	P3	Out
formal algebra in the form: • x + a = b • ax = b • $\frac{x}{a} = b$ • ax + b = c where *a*, *b* and *c* are integers.						
Plot linear and nonlinear data, using appropriate scales.	1	1	1	1		
Express a linear relation of the form $y = mx + b$ • in words • as a formula • with a table of values • as a graph	1	1	1	1		
Determine the following characteristics of the graph of a linear function, given its equation: 5. intercepts 6. slope 7. domain 8. range.	1	1	1	1		
Use direct variation and arithmetic sequences as applications of linear functions.	1		1			
Interpolate and extrapolate values from the graph of a linear relation	1	1	1	1		
Polynomial Functions and Equations						
Determine the following characteristics of the graph of a quadratic function: • vertex • domain and range • axis of symmetry • intercepts.	1	1	1			
Connect algebraic and graphical transformations of quadratic functions, using completing the square as required.	1	1	1			
Model real-world situations, using quadratic functions.	1	1	1			
Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using: • factoring • the quadratic formula • graphing	1	1	1			
Determine the characteristics of the real and non-real roots of a quadratic equation, using: • the discriminant in the quadratic formula • graphing.	1	1	1			
Determine that the sum of the roots of a quadratic equation $ax^2+bx+c=0$ equals b/a and that the product of the roots equals c/a	1					
Solve nonlinear equations: 3. by factoring 4. graphically.	1	1	1			
Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials.	1					
Exponential and Logarithmic Functions and Equations						
Explain the relationship between the laws of logarithms and the	1	1	1			

Topic/Description	P1	1A	1B	P2	P3	Out
laws of exponents.						
Change functions from exponential form to logarithmic form and vice versa.	1	1	1			
Graph and analyze logarithmic functions with and without technology.	1	1	1			
Use logarithms to model practical problems.	1	1	1			
Model, graph and apply exponential functions to solve problems	1	1	1			
Solve exponential equations having bases that are powers of one another.	1	1	1			
Solve and verify exponential and logarithmic equations and identities.	1	1	1			
Trigonometric Functions and Equations						
Apply ratio and proportion in similar triangles	1	1	1			
Use the trigonometric ratios sine, cosine, and tangent in solving right triangles	1	1	1			
Solve problems involving two right triangles.	1	1	1			
Apply the sine and cosine laws, excluding the ambiguous case, to solve problems.	1	1	1			
Solve problems involving ambiguous case triangles in 3-D and 2-D.	1					
Predict results from graphs that represent periodic events.	1					
Describe periodic events, including sinusoidal curves, using correct terminology.	1	1	1			
Collect sinusoidal data; sketch the graph of the data; and, using degrees, represent the data with an equation of the form: • $y = a \sin(kt) + c$ **OR** • $y = a \cos(kt) + c$.	1	1				
Use trigonometric functions to model and solve problems.	1	1	1			
Distinguish between degree and radian measure, and solve problems, using both.	1	1	1			
Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position.	1	1				
Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.	1	1	1			
Solve first and second degree trigonometric equations over a domain of length 2π: • *algebraically* • *graphically*.	1					
Determine the general solutions to trigonometric equations where the domain is the set of real numbers.	1					
Verify trigonometric identities: *4. numerically for any particular case* *5. algebraically for general cases* 6. *graphically.*	1					
Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions.	1					

Topic/Description	P1	1A	1B	P2	P3	Out
To determine sin nx where n is a natural number	1					
Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for: 6. *amplitude, if defined* 7. *period* 8. *domain and range* 9. *asymptotes, if any* 10. *behaviour under* transformations.	1					
Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for: 5. *period* 6. *domain and range* 7. *asymptotes* 8. *behaviour under transformations.*						1
Geometry						
Estimate, measure and draw angles, using a protractor.	1	1	1		1	
Determine the measures of angles in a diagram.	1	1	1		1	
Use concrete materials and diagrams to verify the Pythagorean relationship.	1				1	
Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications.	1	1	1		1	
Measure the diameters, radii and circumferences of circles, and establish the relationships among them.	1	1	1		1	
Calculate the area of a circle.	1	1	1		1	
Solve problems involving the radii, diameters, area and circumferences of circles.	1	1	1		1	
Use properties of circles and polygons to solve design and layout problems.	1				1	
Solve problems involving distances between points in the coordinate plane.	1	1	1		1	
Solve problems involving midpoints of line segments.	1	1	1		1	
Solve problems involving rise, run and slope of line segments.	1	1	1		1	
Determine the equation of a line, given information that uniquely determines the line.	1	1	1		1	
Solve problems involving distances between points and lines.	1	1	1		1	
Verify and prove assertions in plane geometry, using coordinate geometry.	1	1	1		1	
Solve problems using slopes of: • *parallel lines* • *perpendicular* lines.	1	1	1		1	
Use technology and measurement to confirm and apply the following properties to particular cases: • the *perpendicular from the centre of a circle to a chord bisects the chord* • *the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc* • *the inscribed angles subtended by the same arc are congruent* • *the angle inscribed in a semicircle is a right angle* • *the opposite angles of a cyclic quadrilateral are*	1					

Topic/Description	P1	1A	1B	P2	P3	Out
supplementary • *a tangent to a circle is perpendicular to the radius at the point of tangency* • *the tangent segments to a circle, from any external point, are congruent* • *the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord* • *the sum of the interior* angles of an n-sided polygon is (2n – 4) right angles.						
Prove the following general properties, using established concepts and theorems: *10.* the *perpendicular bisector of a chord contains the centre of the circle* *11. the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle)* 12. *the inscribed angles* subtended by the same arc are congruent *13.* the *angle inscribed in a semicircle is a right angle* *14. the opposite angles of a cyclic quadrilateral are supplementary* *15. a tangent to a circle is perpendicular to the radius at the point of tangency* *16. the tangent segments to a circle from any external point are congruent* *17. the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord* 18. *the sum of the interior* angles of an n-sided polygon is (2*n* – 4) right angles.	1				1	
Solve problems, using a variety of circle properties, and justify the solution strategy used.	1				1	
Logic						
Differentiate between inductive and deductive reasoning.	1	1	1	1		
Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems.	1	1	1	1		
Use examples and counterexamples to analyze conjectures.	1	1	1	1		
Distinguish between an “if–then” proposition, its converse and its contrapositive.	1	1	1	1		
Prove assertions in a variety of settings, using direct and indirect reasoning.	1		1	1		
Matrices						
Show an understanding of matrices and perform the operations of addition, scalar multiplication and matrix multiplication.						1
Solve problems, using the operations of addition, subtraction, scalar multiplication and matrix multiplication on matrices.						1
Use matrices and matrix operations to model and to solve consumer, network and schedule problems.						1
Determine the inverse of a 2x2 matrix						1
Measurement						
Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes.	1	1	1	1	1	

Topic/Description	P1	1A	1B	P2	P3	Out
Design an appropriate measuring process or device to solve a problem.	1			1	1	
Estimate measurements of objects in SI and Imperial systems including: • *length* • *area* • *volume* • *mass*	1	1	1	1	1	
Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy.	1	1		1	1	
Develop a sense of approximate conversions between SI and imperial units through investigations.	1		1	1	1	
Perform basic conversions within and between the Imperial and SI systems, using technology as appropriate	1	1		1	1	
Solve problems involving length, area, volume, time, mass and rates derived from these.	1	1	1	1	1	
Use dimensions and unit prices to solve problems involving perimeter, area and volume.	1	1	1		1	
Interpret drawings, and use the information to solve problems.	1	1	1	1	1	
Enlarge or reduce a dimensioned object, according to a specified scale.	1				1	
Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes.	1				1	
Solve problems involving percentage error when input variables are expressed with percentage errors.	1		1	1	1	
Solve problems involving estimation and costing for objects, shapes or processes when a design is given.					1	
Design an object, shape, layout or process within a specified budget.					1	
Use simplified models to estimate the solutions to complex measurement problems.					1	
Calculate the volume and surface area of a sphere, using formulas that are provided.	1	1	1		1	
Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects.	1				1	
Analyze objects shown in "exploded" format					1	
Draw objects in "exploded" format					1	
Draw top, front and side views for both 3-D rod or block objects and their sketches					1	
Sketch and build 3-D designs using isometric dot paper						1
Complete a project that includes a 2-D plan and a 3-D model of some physical structure					1	
Properties of Number Systems						
Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system.	1	1	1	1	1	
Use estimation strategies to justify or assess the reasonableness of calculations.	1	1	1	1	1	
Apply order of operations to solve problems, using paper and pencil or a calculator.	1	1	1	1	1	
Demonstrate and explain the meaning of fractions, concretely, pictorially and symbolically.	1	1	1	1	1	

Topic/Description	P1	1A	1B	P2	P3	Out
Express rates and ratios in equivalent forms to solve problems.	1	1	1	1	1	
Demonstrate and explain the meaning of ratio concretely, pictorially and symbolically.	1	1	1	1	1	
Represent and apply fractions as per cents, and percents in fraction or decimal form.	1	1	1	1	1	
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing decimals (for more than 2-digit divisors or multipliers, the use of technology is expected).	1	1	1	1	1	
Demonstrate an understanding of integers and use arithmetic operations to solve problems involving integers.	1	1	1	1	1	
Demonstrate an understanding of and proficiency with adding, subtracting, multiplying and dividing fractions concretely, pictorially and symbolically.	1	1	1	1	1	
Estimate and calculate percentages.	1	1	1	1	1	
Use approximate representations of irrational numbers.	1	1	1		1	
Define and illustrate complex numbers	1					
Express complex numbers in the form a + b*i*	1					
Add, subtract, multiply and divide complex numbers	1				1	
Divide complex numbers using conjugates						1
Probability						
Express probabilities as ratios, fractions, decimals, percents, and in words	1	1		1		
Use probability to predict the result in a given situation	1	1		1		
Determine the odds for and against a particular event occurring	1			1		
Compare experimental observations with theoretical predictions	1			1		
Use probabilities to calculate expected gains and losses				1		
Communicate and justify solutions to probability problems	1			1		
Solve pathway problems, interpreting and applying any constraints.						1
Use the fundamental counting principle to determine the number of different ways to perform multistep operations.						1
Construct a sample space for two or three events.						1
Classify events as independent or dependent.	1			1		
Solve problems, using the probabilities of mutually exclusive and complementary events.						
Solve decision-making problems involving expected values, and communicate the solutions.						
Determine the number of permutations of n different objects taken *r* at a time, and use this to solve problems.						1
Determine the number of permutations of n objects arranged in a circle.						1
Determine the number of combinations of n different objects taken *r* at a time, and use this to solve problems.						1
Determine the conditional probability of two events (Bayes' law).						1
Solve probability problems involving permutations, combinations and conditional probability.						1
Solve problems, using the binomial theorem where *N* belongs to the set of natural numbers.						1

Topic/Description	P1	1A	1B	P2	P3	Out
Solve probability problems, using the binomial distribution as applied to small samples.						1
Systems of Equations						
Design and solve linear and nonlinear systems, in two variables, to model problem situations.	1	1	1			
Solve systems of linear equations, in two variables: • *algebraically (elimination and substitution)* • *graphically*.	1	1	1			
Recognize the characteristics of linear equations in two variables with graphs that are inconsistent, consistent-dependent, or consistent-independent	1	1				
Solve nonlinear equations, using a graphing tool.	1					
Solve systems of linear equations, in three variables: • *algebraically* • *with technology*.	1					
Determine the solution to a system of nonlinear equations, using technology as appropriate	1					
Graph linear inequalities, in two variables.	1					
Solve, graphically, systems of linear inequalities, in two variables, using technology.						1
Apply linear programming to find optimal solutions to decision-making problems.						1
Statistics						
Determine measures of central tendency and variability for a set of data: • *mode* • *median* • *mean* • *range.*	1	1		1		
Determine and use the most appropriate measure of central tendency in a given context.	1			1		
Read and interpret graphs	1	1		1		
Extract information from given graphs of discrete or continuous data, using: • time series • glyphs (custom pictorial representations) • continuous data • contour lines.	1			1		
Display and analyze data on a line plot	1	1		1		
Use suitable graph types to display data (by hand or using technology)	1	1		1		
Critique ways in which statistical information and conclusions are presented by the media and other sources				1		
Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation.	1			1		
Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data.				1		
Describe issues to be considered when collecting data (e.g., appropriate language, ethics, cost, privacy, cultural sensitivity)				1		
Choose, justify and apply sampling techniques that will result in an				1		

Topic/Description	P1	1A	1B	P2	P3	Out
appropriate, unbiased sample from a given population.						
Defend or oppose inferences and generalizations about populations, based on data from samples.				1		
Determine the equation of a line of best fit, using: • *estimate of slope and one point* • *median–median method* • *least squares method* with technology.				1		
Use technological devices to determine the correlation coefficient *r*.				1		
Describe the relationship between the correlation coefficient and two data sets						
Interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots.				1		
Find the population standard deviation of a data set or a probability distribution, using technology.				1		
Use z-scores and z-score tables to solve problems.						1
Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples.				1		
Describe and illustrate normal and skewed distributions using real-world examples				1		
Explain the variability of data using standard deviation and the normal curve				1		
Use standard deviation to describe the variability within a set of data				1		
Use sample data to make predictions and decisions				1		
Determine the percentile rank of an item in a set of sample data				1		
Distinguish between percent and percentile rank				1		
Sequences and Series						
Identify sequences that appear to be: • *divergent* • *convergent* • *oscillating* • *static*.						1
Generate number patterns exhibiting arithmetic growth.						
Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems.	1					
Relate arithmetic sequences to linear functions defined over the natural numbers.	1					
Generate number patterns exhibiting geometric growth.						1
Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems.						
Connect geometric sequences to exponential functions over the natural numbers.						1
Estimate values of expressions for infinite geometric processes.						1
Construct a fractal pattern by repeatedly applying a procedure to a geometric figure.						1
Use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns.						1
Technological Applications						

Topic/Description	P1	1A	1B	P2	P3	Out
Create a spreadsheet using various formatting options				1		
Use a spreadsheet template to solve problems				1		
Create a spreadsheet using formulas and functions				1		
Use a spreadsheet to answer "what-if" questions						1
Identify where spreadsheets could be effectively used				1		
Use spreadsheets to analyze renting or buying an increasing asset (home) under different sets of circumstances.						1
Use spreadsheets to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances.						1
Use spreadsheet(s) to analyze an investment or life insurance portfolio, applying such concepts as capital gains, interest rate, inflation rate, risk, total rate of return and after-tax rate of return.						1
Design or modify a financial spreadsheet template to allow users to input their own variables.						1
Use and modify a spreadsheet template to model recursive situations						1
Solve problems involving combinations of tables, using: 10. addition or subtraction of two tables 11. multiplication of a table by a real number 12. spreadsheet functions and templates.						1
Use technology to generate and graph finite or infinite sequences whose recursive definition may or may not be given.						1
Transformations						
Describe how various translations of functions affect graphs and their related equations: • $y = f(x - h)$ • $y - k = f(x)$.	1	1				
Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations: • $y = af(x)$ • $y = f(kx)$.	1	1				
Describe how reflections of functions in both axes and in the line y = x affect graphs and their related equations: • $y = f(-x)$ • $y = -f(x)$ • $y = f^{-1}(x)$.	1	1				
Using the graph and/or the equation of f(x), describe and sketch $\frac{1}{f(x)}$.	1					
Using the graph and/or the equation of f(x), describe and sketch |f(x)|.	1					
Describe and perform single transformations and combinations of transformations on functions and relations.	1					
Vectors						
Use and give 3-D and 2-D examples of vector terminology and notation, including: • *vector (direction, magnitude)* • *scalar* • *unit vector* • *collinear vectors*	1	1				

Topic/Description	P1	1A	1B	P2	P3	Out
• *opposite vectors* • *parallel vectors* • *resultant vectors.*						
Assign meaning to the multiplication of a vector by a scalar.	1					
Perform vector additions and subtractions, using triangle or parallelogram methods.	1					
Determine the magnitude and direction of a resultant vector, using triangle, parallelogram or component methods.	1	1				
Use vector diagrams and trigonometry to analyze and solve practical problems in 3-D and 2-D.						

Appendix B: Responses by Faculty or Subject

In the survey, respondents were asked to indicate their faculty and subject area. Although there was some degree of commonality in responses from persons in similar areas, there were enough variations that it was thought reasonable to combine the two fields into one, highlighting the most useful response and standardizing the spelling of some subjects. The following table shows the distribution of all post-secondary respondents across all provinces and territories by this combined field.

Faculty/Subject	Count of Responses	Count of People
Accounting	7	7
Adult Development	1	1
Agricultural Design	1	1
Agricultural Engineering	3	22
Agriculture Technology	1	10
Aircraft Maintenance	1	1
Anatomy and Physiology	1	1
Animal Science	5	31
Anthropology and Sociology	2	3
Applied Science	1	1
Archaeology	1	1
Architectural Technology	1	1
Architecture	1	1
Asian Studies	1	1
Astronomy	1	1
Automotive Repair	5	19
Basic education	1	1
Biochemistry	1	1
Biology	12	14
Biomechanics	2	2
Business	4	4
Business & Tourism	1	1
Business and Economics	1	1
Business Computer Applications	1	1
Business Math	1	1
Business Statistics	1	1
CAD	1	25
Calculus	1	1
Carpentry	2	2
Chemical Engineering	4	13
Chemical Technology	1	1
Chemistry	28	59
Child and Youth Studies	2	2
Choral Music	1	1
Civil Engineering	3	3
Civil Engineering Technology	1	1
Commerce	2	51

Faculty/Subject	Count of Responses	Count of People
Communication	1	4
Community and Health Studies	2	2
Computer Engineering Technology	1	1
Computer Science	36	113
Construction Design	1	1
Construction Geomatics	1	1
Culinary Arts	5	24
Curriculum Studies	2	2
Dental Assisting	1	4
Dentistry	2	24
Design Studies	1	1
Diagnostic Laboratory	1	1
Diesel Fuel Injection	1	1
Drama	1	12
Early Childhood Education	2	4
Earth & Ocean Science	2	2
Earth and Atmospheric Sciences	2	2
Economics	10	18
Ecotoxicology	1	1
Education	2	2
Educational Psychology	1	1
Electrical Engineering	3	42
Electrical Engineering Technology	2	13
Electronics	4	9
Electronics Engineering Technology	1	10
Elementary Math Methods	1	1
Engineering	3	3
Engineering Technology	1	42
English	4	13
Environmental Science	3	7
Environmental Design	1	25
Equine Science	1	6
Exercise Physiology	2	2
Fashion Design and Technology	1	1
Finance	3	3
Fine Arts	1	1
Food Science	1	1
Forest Resources Management	1	1
French	2	2
Genetics	1	1
Geographic Information Systems	3	3
Geography	7	39
Geology	4	44
Gerontology Recreation	2	2
Germanic Studies	1	1
Graphic Design	1	1

Faculty/Subject	Count of Responses	Count of People
Health Care Aide	1	15
History	12	20
Home Support Attendant	1	1
Hospitality Management	3	3
Human Kinetics	2	2
Humanities	3	3
Hydrogeology	1	1
Industrial Systems	1	8
Instrumentation	2	3
Interior Design	5	5
International Business	1	10
Inuktitut	1	1
Journalism	2	2
Justice Studies	3	14
Kinesiology	8	8
Laboratory Medicine	1	1
Labour Relations	1	1
Land Agent	1	1
Land and Water Management	1	3
Landscape Management	1	1
Large animal management	1	1
Law	1	20
Machinist	1	1
Management	1	99
Marketing	1	1
Materials Engineering	1	1
Mathematics	71	218
Mathematics and Computer Science	3	9
Mathematics and Sciences	7	7
Mathematics and Statistics	12	56
Mathematics Education	2	4
Mechanical Engineering	2	2
Mechanical trades	1	1
Medical diagnostics	1	1
Medical Laboratory Science	3	11
Medical Radiation Technology	2	2
Metals and Materials Engineering	2	2
Microbiology	3	3
Microcontrollers	1	1
Millwright	1	1
Motor mechanic	2	2
Music	3	3
Native Studies	1	4
Natural Resources	1	15
Nuclear Medicine Technology	2	2
Nursing	28	116

Faculty/Subject	Count of Responses	Count of People
Operations Management	1	1
Organizational Behavior	1	1
Paramedic	1	1
Petroleum Engineering	2	3
Pharmaceutical Sciences	1	1
Philosophy	9	16
Physical Education	1	1
Physics	27	36
Physiology	3	3
Plumbing	2	2
Political Science	3	13
Power Engineering	2	2
Practical Nursing	8	73
Psychology	12	19
Public Relations	1	1
Recreation Vehicle	1	18
Rehabilitation	1	1
Religion	1	1
Respiratory Therapy	4	4
Science	3	82
Sheet Metal	1	1
Social sciences	1	13
Sociology	5	88
Soil Science	2	13
Spanish	1	1
Special Needs Education	1	1
Statistics	10	14
Structural Design	1	1
Technology	1	1
Telecommunications	2	26
Theatre	2	2
Tourism	1	1
Trades	1	25
Transfusion	1	1
Translation	1	1
Transportation Trades	1	97
Ultrasound	1	1
Welding	1	1
(no information provided)	8	20
Total	**565**	**2055**