WNCP MATHEMATICS RESEARCH PROJECT:

FINAL REPORT

PREPARED FOR THE

WESTERN AND NORTHERN CANADIAN PROTOCOL

BY

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EXECUTIVE SUMMARY

From October 2003 through March 2004 Hold Fast Consultants Inc. collected information concerning the state of the Western and Northern Canadian Protocol (WNCP) Mathematics Framework from: WNCP member jurisdictions, teachers, parents, and professors and instructors from a wide range of post-secondary institutions (including universities, colleges, and trade & technical institutes). During this time staff reviewed hundreds of research journals, articles, books, theses, websites, etc. in order to compile an accurate picture of the state of mathematics education in Canada, North America, and the world at large. This report attempts to organize and summarize this wealth of information in such a way that it can be used by the WNCP member jurisdictions as the starting point for a revision of the *Common Curriculum Framework for K – 12 Mathematics (CCF).* The following is a brief summary of our findings and recommendations.

Theories of Learning

The most important concept that needs to be adhered to is that not only do the theories of learning fall on a spectrum; they overlap in a variety of ways. The perceived flaws of one learning theory may be addressed by another and the combined effect of using a broad range of approaches is much more powerful than relying on any single model.

The driving force behind learning can be boiled down to ten common "needs". The CCF must address the following student needs, identified by Sfard (2003), if effective learning is to take place:

1. The need for meaning
2. The need for structure
3. The need for repetitive action
4. The need for difficulty (i.e., challenge students appropriately)
5. The need for significance and relevance
6. The need for social interaction
7. The need for verbal-symbolic interaction
8. The need for a well-defined discourse
9. The need for belonging
10. The need for balance

A complete description of these needs can be found on page 35 of this report. Although all of these needs are important, the truth is that to adequately address all of them there must be a bit of everything in the classroom. Therefore the most important concept to incorporate into any curriculum revision is "balance". The reality is that, for effective learning to take place, classroom teachers need to include in their teaching: problem solving as well as skills practice, teamwork as well as individual learning and teacher exposition, real-life problems as well as abstract problems, learning by talking as well as silent learning (Sfard, 2003).

Conceptual vs. Procedural Knowledge

Results of the WNCP Member Survey, the Stakeholder Survey, and the BC Post-secondary Mathematics Curriculum Survey underscore the fact that there are competing expectations our society places on our students. This is a case of wanting the best of both worlds: students that have developed computational skills without compromising the conceptual understanding needed to help them learn new, more sophisticated mathematics. To accomplish this mathematics curricula and classroom instruction must include learning outcomes that focus on both of these aspects of knowledge.

It is insufficient to include procedural and conceptual outcomes in a curriculum unless both types of knowledge are assessed (by both classroom-based and large-scale assessment instruments). Correct answers are not always a safe indicator of understanding. Teachers and policy makers must ensure that curriculum and assessment are properly aligned (BC Ministry of Education, 2002).

Two other factors may also have an impact on conceptual learning – choice and connections. The problem-based instruction and open, project-based approaches to learning both provide opportunities for students to make their own choices and to form connections between their mathematical learning and real-world applications. These factors have a far greater impact on students than most educators realize.

Technology in the Mathematics Curriculum

Calculators, particularly graphing calculators, can have a positive impact on students' learning of mathematics. It appears that benefits will be maximized if calculators are used in a pedagogical role and not just for performing calculations. This has curricular implications, as it requires adaptation of both content (i.e. representations and procedures) and instructional approaches. Support in terms of both resources and inservice is essential to ensure that the technology is appropriately integrated into the classroom.

The Impact of the Teacher in Educational Change

A significant portion of the research indicates that substantive change can be achieved with appropriate professional development. This calls for both a commitment on the part of the teachers and the inservice providers. Professional development must be seen as a continuing enterprise for teachers and an integral component of any curriculum change. Thus, it is strongly felt that the WNCP member jurisdictions need to consider ways to encourage and support this important step in any recommended curriculum changes. This is particularly true of elementary teachers, who are generally non-mathematics specialists, and would benefit from ongoing in-service that integrated mathematics content and mathematics pedagogy.

Aboriginal Students and Mathematics

Many Aboriginal children struggle with mathematics. Contextualizing that mathematics and making it much more applicable to the needs of the Aboriginal population will help some students. A rethink in how mathematics is assessed and an explicit development of appropriate strategies for struggling learners to make them feel more confident in attempting mathematics test items appears to be warranted.

For the WNCP member jurisdictions to improve Aboriginal student learning, they should provide information related to learning styles and appropriate teaching strategies for aboriginal students. They should also provide professional development to assist teachers in making appropriate adjustments to their teaching and assessment practices.

Mathematical Literacy and the WNCP Curriculum

The evidence indicates that the use of the present CCF organization is consistent with many of the definitions of numeracy, or more accurately, mathematical literacy found in the literature. Regardless of the term used, the present curriculum structure includes those elements recognized throughout the world as supporting the development of mathematical literacy. In order to better support the teaching of conceptual understanding it may be advisable to either add new curriculum strands (e.g., Problem Solving and Reasoning & Proof) to the curriculum organization with the expectation that teachers would integrate these learning outcomes as they teach, or to include explicit mathematical process learning outcomes into the four content strands as appropriate.

To support teachers in ensuring that these mathematical processes are truly part of the delivered curriculum the CCF should include: additional information describing effective teaching methods; and, examples of student performance related to these as well as other learning outcomes.

Common Curriculum Framework Design & Content

Results from the various surveys, as well as research into national and international assessments support a reduction in the number of topics at all grade levels. Teachers have reported for several years their concerns about the number of topics – resulting in a superficial treatment of a large number of concepts. Post-secondary instructors (particularly those from calculus-based programs of studies) have indicated that incoming students need to have in-depth knowledge of fewer topics to have a reasonable chance of being successful. A more in-depth treatment of the mathematics curriculum will also provide teachers and students greater opportunity to attain the levels of procedural and conceptual understanding that society demands.

In order to maintain the required curriculum depth at the secondary level it is vital that three distinct programs of study be available to students. The BC post-secondary study clearly shows that the present Pure Mathematics curriculum can, with relatively minor revisions, meet the needs of the 20% of our students that go on to programs that include higher mathematics. As all jurisdictions have programs of study designed to meet the needs of the 20% - 30% of students not going on to post-secondary studies (i.e., AB - Mathematics 14/24, BC, MB, NT, NU, YT - Essentials of Mathematics or Consumer Mathematics, SK - Mathematics 11/21 & 18/28), it seems logical to pool WNCP resources to accomplish this as part of a revised CCF. Unfortunately this leaves us with a significant gap as the needs of the approximately 50% of our students going on (or eventually going on) to post-secondary studies that do not require higher mathematics are not being addressed by the Applied Mathematics curriculum. This suggests that further consultation with representatives of these programs is necessary to determine what mathematics they want these students to know.

The following recommendations are put forward for consideration by the member jurisdictions as they prepare to revise the WNCP *Common Curriculum Framework for K – 12 Mathematics*. It is recommended that:

1. The CCF not be developed using a single learning theory as its basis, but rather be developed using the common characteristics of learning theories identified in this report (see page 35).
2. The CCF consist of a single program of studies from Kindergarten to Grade 9.
3. In order for the CCF to effectively meet the needs of our secondary students, it include three distinct programs for Grades 10 to 12. Specifically, it is recommended that the secondary programs be developed to meet the needs of students who are:
    3.1. Entering post-secondary programs that require calculus (e.g., Mathematics, Sciences, Engineering, Commerce, etc.);
    3.2. Entering post-secondary programs that do not require calculus (e.g., Humanities, Fine Arts, some Trades and Technical programs, etc.); and,
    3.3. Entering the workforce, Trades or Technical programs that do not require advanced mathematics.
4. Additional consultation with post-secondary representatives of programs of study that do not include calculus be undertaken to identify what mathematics is required of students entering these types of programs.
5. The WNCP reduce the breadth of the CCF so as to increase its depth. In any revision of the CCF, consideration should be given to decreasing the breadth or spread of specific outcomes across multiple grades. Specifically, it is recommended that:
    5.1. The curriculum for Kindergarten to Grade 3 be focused on Number and Measurement only;
    5.2. The K – 3 curriculum be based upon conceptual understanding with algorithms introduced only when conceptual understanding supports them;

5.3. When developing the curriculum for K – 12 the WNCP take into consideration the Suggested Topics & Grade Groupings described in Table 19 (page 145); and,

5.4. The number of topics in each secondary program (Grades 10 – 12) be reduced by as much as one third. This is needed to allow more time for in-depth instructional experiences consistent with the identified common learning theory characteristics.

This does not mean that all content from such areas should be removed. One possibility would be to separate outcomes from applications. For example, the outcomes in grade 2 could emphasize "number" but some applications could be from "3-D geometry." However, the application would not be part of what students are expected to master. Rather, the application would be a vehicle to develop an understanding of number.

6. The CCF includes both procedural and conceptual learning outcomes. The intent of this recommendation is to better support the teaching of conceptual understanding by either:

   6.1. Adding new curriculum strands (e.g., Problem Solving and Reasoning & Proof) to the curriculum organization with the expectation that teachers would integrate these learning outcomes in their teaching; or,

   6.2. Including explicit mathematical process learning outcomes into the four content strands as appropriate.

7. The WNCP member jurisdictions develop large-scale assessment instruments that are consistent with the learning theory characteristics used to develop the CCF and thus assess students' procedural **and** conceptual understanding of mathematics. It is evident that large-scale assessments send a clear message to teachers concerning what is valued in the mathematics curriculum. The implication is that the procedural and conceptual mathematics teachers are being asked to teach in their classrooms must be reflected in any large-scale assessments used for accountability purposes.

8. For each grade level and topic, an Effective Instruction and Assessment Practices description be developed. The description should outline the classroom environment and instructional and assessment approaches that research (consistent with the identified learning theory characteristics) has identified as being most effective. Additionally, it is recommended that:

   8.1. The CCF support teachers in ensuring that mathematical processes are truly part of the delivered curriculum by including:

      8.1.1. Additional information describing effective instructional and assessment methods (including methods designed specifically for Aboriginal students); and, 8.1.2. Examples and/or descriptions of the level of performance that is expected of students.

9. The CCF incorporate the use of technology such as computers, calculators and graphing calculators in a phased manner. Specifically, it is recommended that:

    9.1. The CCF should not require the use of technology in the Grades K to 3 learning outcomes;

    9.2. The CCF should introduce technology into the Grade 4 to 7 learning outcomes as appropriate as part of the mathematical learning and problem solving processes;

    9.3. The CCF should require technology in the Grade 8 to 12 learning outcomes as appropriate as part of the mathematical learning and problem solving processes; and,

    9.4. The Effective Instruction and Assessment Practices include instructional and assessment methods related specifically to the appropriate use of such technology along with examples and/or descriptions of the level of performance that is expected of students.

10. The WNCP jurisdictions pilot the Draft CCF for one full year prior to implementation and use the information collected from the pilot sites to revise both the learning outcomes and the Effective Instruction and Assessment Practices as needed.

11. The WNCP work with publishers to ensure that learning resources consistent with the identified learning theory characteristics and the mathematical content of the CCF be developed and piloted before implementation.

12. The WNCP jurisdictions provide a significant level of targeted support for teachers during the initial implementation of the CCF. Specifically, it is recommended that:

    12.1. Jurisdictions provide ongoing inservice for teachers and school administrators for a minimum of two years from the date of first implementation. This is particularly important for elementary teachers (i.e., non-mathematics specialists), who would benefit the most from ongoing inservice that integrated mathematics content <u>and</u> mathematics pedagogy; and,

    12.2. Jurisdictions communicate and work with post-secondary institutions to ensure that accurate information concerning course acceptance is available to students, parents, teachers, counsellors, administrators and the public in general.

13. The WNCP jurisdictions, prior to revising the CCF, develop and use a program evaluation model that allows them to assess the effectiveness of the revised curriculum. Throughout this project it was evident that the WNCP did not have a plan in place for the systematic collection of information that could be used for

program evaluation purposes. Although this was not necessarily true of individual jurisdictions, the information that was collected by jurisdications was not done so in a consistent manner. This made data analysis and synthesis unnecessarily complex and less reliable than it otherwise could be. Good evaluation studies include four important criteria identified by the Joint Committee on Standards for Educational Evaluation (Gall, et al, 1996):

- Utility – study is informative, timely and useful to the affected persons;
- Feasibility – the evaluation design is appropriate to the setting in which the study is to be conducted (i.e., schools) and the design is cost effective;
- Propriety – the study is conducted legally and ethically; and,
- Accuracy – the study produces valid, reliable, and comprehensive information for making judgements of a program's worth.

INTRODUCTION

In 1993 the ministers of education of the western provinces and territories signed the Western and Northern Canadian Protocol (WNCP) for Collaboration in Basic Education with the stated intent to provide quality education for all students from Kindergarten to Grade 12. The western jurisdictions agreed to, among other things, identify means by which the parties could significantly enhance current efforts to work together in the area of mathematics curriculum.

The Western and Northern Canadian Protocol (WNCP) member jurisdictions now include the following provinces and territories:

- Alberta (lead jurisdiction for Mathematics)
- British Columbia
- Manitoba
- Northwest Territories
- Nunavut
- Saskatchewan
- Yukon

The WNCP developed and published *The Common Curriculum Framework (CCF) for K – 12 Mathematics* in two parts: *Kindergarten to Grade 9* in 1995 and *Grade 10 to Grade 12* in 1996. Since the initial release of the CCF the various member jurisdictions have used this two-part framework, to varying degrees, as the basis for their respective mathematics curricula.

Project Background

The member jurisdictions have had the opportunity to reflect on the lessons learned from the first iteration of the CCF and are now prepared to enter into the process of evergreening the CCF.

Several key issues (e.g., content, inclusion of a third pathway, transitions between grades, and evergreening (i.e., limited revision) of the CCF) have been identified and have provided further impetus for the evergreening of the CCF. It is essential that the revised CCF be responsive to the needs of jurisdictional partners and the revision process be effective, reflective and respectful of jurisdictional contexts. To these ends the member jurisdictions have embarked upon a research project concerning the CCF.

The major activities of this research project include a literature review, a survey of all WNCP partners to determine the degree of implementation of the CCF, and the identification of essential learning outcomes required for success in the world of work and for continuing education.

Project Questions

In October 2003 the member jurisdictions of the WNCP contracted Hold Fast Consultants Inc. to conduct the WNCP Mathematics Research Project. There are three components to this project that are included in this report:

Literature Review

A review of the current academic literature on numeracy and mathematical literacy, brain research as it relates to mathematics teaching and pedagogy and new developments in terms of Kindergarten to Grade 12 Mathematics.

WNCP Member Survey

Hold Fast created the survey in consultation with the WNCP member jurisdictions to:

- Identify to what degree the member jurisdictions have adopted the Mathematics CCF as part of their curriculum;
- Provide a rationale as to why the CCF was modified to meet jurisdictional needs; and,
- Estimate teachers' actual implementation of existing jurisdictional curricula.

Essential Learning Outcomes Survey

Hold Fast created the survey and administered it to stakeholders via the World Wide Web. The Literature Review and this survey were used to make recommendations concerning the essential mathematics knowledge and skills that should be included in a revised mathematics CCF.

Project Team

The WNCP Mathematics Research Project Team consists of the following individuals:

- Bruce McAskill (Project Lead and President of Hold Fast Consultants Inc.)
- Glen Holmes (Teacher & Education Consultant, S.D. #63, Saanich, BC)
- Leslee Francis-Pelton (Associate Professor of Mathematics Education, University of Victoria)
- Wayne Watt (Education Consultant, Winnipeg, MB)

Research on the Teaching and Learning of Mathematics

Over the past 20 years a great deal of research has focused on the teaching and learning of mathematics. The majority of this research examined such capabilities as counting, understanding numerical magnitudes, doing arithmetic, learning pre-algebra, algebra, geometry, and computer programming (Siegler, 2003). More recent research has extended the focus to include issues related to student and teacher characteristics as well as newer forms of technology, such as the graphing calculator. There is now a sufficient body of research to allow relatively firm conclusions to be drawn about a number of aspects of the teaching and learning of mathematics. This section will summarize findings related to the teaching and learning of mathematics in seven areas:

1. Theories of learning;
2. Common themes that cross the learning theories and their implications for mathematics education;
3. Cognitive variability and strategy choice;
4. Individual differences;
5. Relations between conceptual and procedural knowledge and instruction;
6. Technology in the Mathematics Curriculum; and
7. The impact of the teacher in educational change.

Although the research reported in this section gives insight into how children typically learn certain skills and concepts and instructional practices that support learning, it does not provide definitive answers on broad philosophical issues about the nature of students as learners or about what should actually be taught. Its main goal is to provide insight into children's learning and the teacher's role in that learning to provide supporting evidence for our recommendations to the WNCP jurisdictions.

Theories of Learning

What has been termed the reform movement has been heavily influenced by the National Council of Teachers of Mathematics, beginning with their landmark Curriculum and Evaluation Standards for School Mathematics published in 1989 (National Council of Teachers of Mathematics, 1989). The succeeding 15 years have seen other documents and support materials released, including an updated version of the Standards document, Principles and Standards for School Mathematics, published in 2000 (National Council of Teachers of Mathematics, 2000). These documents have been both praised and criticized over the years. Proponents state the need to teach mathematics with a human face - to care for both the student and the mathematics. They argue that mathematics should not be taught as a static body of knowledge and that the processes of problem solving, reasoning, representation, communication and connections are as important as the content. Critics of the reform movement contend that the mathematics that students are supposed to learn has been hurt by the movement and that the ensuing curricula have lacked the desired rigor and mastery of procedural knowledge.

There is no doubt that the Standards documents are being used to influence mathematics curricula in North America and even further afield. The controversy in their use comes, not so much in the Standards themselves, but in how they are often translated into practice. As with any reform movement, the deficiencies of one approach to teaching is often made up for by swinging the pendulum to the other pole, adopting a new theory of learning, and discounting approaches not congruent with the new theoretical base. However, any learning theory, if it is well conceived, may lend itself to support a variety of educational practices without privileging any of them (Sfard, 2003). Thus, educational theories should only suggest, not dictate. Further, multiplicity of theories does not automatically imply a contradiction. Few theories are truly contradictory. More often they are complementary, simply concerned with different aspects of the same phenomena. A combination of theoretical outlooks may not only be desirable, but also have a synergetic effect.

Various learning theories exist to describe how students gain knowledge. Some theories have their foundations in work done more than half a century ago, while others are of more recent origin. Some theories cross all content areas, while others apply to specific content areas. These theories fall roughly into two conceptual categories. The "acquisitionist" group uses cognitivist approaches to explain learning and knowledge in terms of things such as cognitive schemes, models, concept images, or misconceptions. The "participationist" category includes the theories that view learning as an activity integrating the learner with a community of practice. The two sets are not mutually exclusive. Aspects of each are usually present in every theory. However, one metaphor is usually more prominent than the other (Sfard, 1998b). A summary of some of the general theories follows (Kearsley, 2003; On Purpose Associates, 2001):

1. Algo-Heuristic Theory (L. Landa)

Definition

Landa's theory (Landa, 1976) is concerned with identifying mental processes -- conscious and especially unconscious -- that underlie expert learning, thinking and performance in any area. His methods represent a system of techniques for getting inside the mind of expert learners and performers, which enable one to uncover the processes involved. Once uncovered, they are broken down into their relative elementary components -- mental operations and knowledge units that can be viewed as a kind of psychological "atoms" and "molecules". Performing a task or solving a problem always requires a certain system of elementary knowledge units and operations.

There are classes of problems for which it is necessary to execute operations in a well structured, predefined sequence (algorithmic problems). For such problem classes, it is possible to formulate a set of precise unambiguous instructions (algorithms) as to what one should do mentally and/or physically in order to successfully solve any problem belonging to that class. There are also classes of problems (creative or heuristic problems) for which precise and unambiguous sets of instructions cannot be formulated. For such classes of problems, it is possible to formulate instructions that contain a certain degree of uncertainty (heuristics). Landa also describes semi-algorithmic and semi-heuristic problems, processes and instructions.

Main points

1. It is more important to teach algo-heuristic processes to students than prescriptions (knowledge of processes); on the other hand, teachers need to know both.

2. Processes can be taught through prescriptions and demonstrations of operations.

3. Teaching students how to discover processes is more valuable than providing them already formulated.

4. Break processes down into elementary operations of size and length suitable for each student (individualization of instruction).

Implications of Algo-Heuristic theory for Education

The theory suggests that all cognitive activities can be analyzed into operations of an algorithmic, semi-algorithmic, heuristic, or semi-heuristic nature. Once discovered, these operations and their systems can serve as the basis for instructional strategies and methods. The theory specifies that students be taught not only knowledge but the algorithms and heuristics of experts as well. They also have to be taught how to discover algorithms and heuristics on their own. Special emphasis is placed on teaching students cognitive operations, algorithms and heuristics that make up general methods of thinking (i.e., intelligence).

With respect to sequencing of instruction, Landa proposes a number of strategies, the most important of which is the "snowball" method. This method applies to teaching a system of cognitive operations by teaching the first operation, then the second that is practiced with the first, and so on.

2. Behaviorism

Definition

Behaviorism is a theory of animal and human learning that only focuses on objectively observable behaviors and discounts mental activities. Behavior theorists define learning as nothing more than the acquisition of new behavior.

Main points

Experiments by behaviorists identify **conditioning** as a universal learning process. There are two different types of conditioning, each yielding a different behavioral pattern:

1. *Classic conditioning* occurs when a natural reflex responds to a stimulus. The most popular example is Pavlov's observation that dogs salivate when they eat or even see food. Essentially, animals and people are biologically "wired" so that a certain stimulus will produce a specific response.

Behavioral or *operant conditioning* occurs when a response to a stimulus is reinforced. Basically, operant conditioning is a simple feedback system: If a reward or reinforcement follows the response to a stimulus, then the response becomes more probable in the future. For example, leading behaviorist B.F. Skinner used reinforcement techniques to teach pigeons to dance and bowl a ball in a mini-alley.

There have been many criticisms of behaviorism, including the following:

1. Behaviorism does not account for all kinds of learning, since it disregards the activities of the mind.
2. Behaviorism does not explain some learning--such as the recognition of new language patterns by young children--for which there is no reinforcement mechanism.
3. Research has shown that animals adapt their reinforced patterns to new information. For instance, a rat can shift its behavior to respond to changes in the layout of a maze it had previously mastered through reinforcements.

How Behaviorism Impacts Learning

This theory is relatively simple to understand because it relies only on observable behavior and describes several universal laws of behavior. Its positive and negative reinforcement techniques can be very effective--both in animals, and in treatments for human disorders such as autism and antisocial behavior. Teachers, who reward or punish student behaviors, often use behaviorism.

3. Brain-based Learning

Definition

This learning theory is based on the structure and function of the brain. As long as the brain is not prohibited from fulfilling its normal processes, learning will occur.

Main points

People often say that everyone **can** learn. Yet the reality is that everyone **does** learn. Every person is born with a brain that functions as an immensely powerful processor.

Traditional schooling, however, often inhibits learning by discouraging, ignoring, or punishing the brain's natural learning processes.

The core principles of brain-based learning state that:

1. The brain is a parallel processor, meaning it can perform several activities at once, like tasting and smelling.
2. Learning engages the whole physiology.
3. The search for meaning is innate.
4. The search for meaning comes through patterning.
5. Emotions are critical to patterning.
6. The brain processes wholes and parts simultaneously.
7. Learning involves both focused attention and peripheral perception.
8. Learning involves both conscious and unconscious processes.
9. We have two types of memory: spatial and rote.
10. We understand best when facts are embedded in natural, spatial memory.
11. Learning is enhanced by challenge and inhibited by threat.
12. Each brain is unique.

The three instructional techniques associated with brain-based learning are:

1. *Orchestrated immersion*--Creating learning environments that fully immerse students in an educational experience
2. *Relaxed alertness*--Trying to eliminate fear in learners, while maintaining a highly challenging environment
3. *Active processing*--Allowing the learner to consolidate and internalize information by actively processing it

How Brain-Based Learning Impacts Education

Curriculum--Teachers must design learning around student interests and make learning contextual.

Instruction--Educators let students learn in teams and use peripheral learning. Teachers structure learning around real problems, encouraging students to also learn in settings outside the classroom and the school building. Support is given for a constructivist epistemology in current brain theory (Brown & Wheatley, 1995).

Assessment--Since all students are learning, their assessment should allow them to understand their own learning styles and preferences. This way, students monitor and enhance their own learning process.

What Brain-Based Learning Suggests

How the brain works has a significant impact on what kinds of learning activities are most effective. Educators need to help students have appropriate experiences and capitalize on those experiences. As Renate and Geoffrey Caine (1997) illustrate, three interactive elements are essential to this process:

- Teachers must immerse learners in complex, interactive experiences that are both rich and real. One excellent example is immersing students in a foreign culture to teach them a second language. Educators must take advantage of the brain's ability to parallel process.
- Students must have a personally meaningful challenge. Such challenges stimulate a student's mind to the desired state of alertness.
- In order for a student to gain insight about a problem, there must be intensive analysis of the different ways to approach it, and about learning in general. This is what's known as the "active processing of experience."

Anderson and Stewart (1997) maintain that teachers who are effectively using brain-based techniques will:

- Encourage student autonomy, initiative, and leadership.
- Allow student thinking to drive and alter lesson plans.
- Ask students to elaborate on their responses.
- Allow wait time when asking questions.
- Encourage students to interact with one another and with their teachers.
- Ask thoughtful, open-ended questions.
- Encourage students to reflect on experiences and predict future outcomes.
- Ask students to articulate their theories about concepts before the teacher presents his or her understanding of the concepts.
- Look for students' alternative conceptions and design lessons to address any misconceptions.

A few other tenets of brain-based learning include:

- Feedback is best when it comes from reality, rather than from an authority figure.
- People learn best when solving realistic problems.
- The big picture can't be separated from the details.
- Because every brain is different, educators should allow learners to customize their own environments.
- The best problem solvers are those that laugh!

- Designers of educational tools **must be artistic** in their creation of brain-friendly environments. Instructors need to realize that the best way to learn is not through lecture, but by participation in realistic environments that let learners try new things safely.

4. Communities of Practice

Definition

This approach views learning as an act of membership in a "community of practice." The theory seeks to understand both the structure of communities and how learning occurs in them.

Basic Elements

The communities of practice concept was pioneered by the Institute for Research on Learning, a spin-off of the Xerox Corporation in Palo Alto, CA. The Institute pursues a cross-disciplinary approach to learning research, involving cognitive scientists, organizational anthropologists, and traditional educators. Communities of practice is based on the following assumptions:

Learning is fundamentally a social phenomenon. People organize their learning around the social communities to which they belong. Therefore, schools are only powerful learning environments for students whose social communities coincide with that school.

Knowledge is integrated in the life of communities that share values, beliefs, languages, and ways of doing things. These are called **communities of practice**. Real knowledge is integrated in the doing, social relations, and expertise of these communities.

The processes of learning and membership in a community of practice are inseparable. Because learning is intertwined with community membership, it is what lets us belong to and adjust our status in the group. As we change our learning, our identity--and our relationship to the group--changes.

Knowledge is inseparable from practice. It is not possible to **know** without **doing**. By doing, we learn.

Empowerment--or the ability to contribute to a community--creates the potential for learning. Circumstances in which we engage in real action that has consequences for both our community and us create the most powerful learning environments.

How Communities of Practice Impacts Education

This approach to learning suggests teachers understand their students' communities of practice and acknowledge the learning students do in such communities. The communities of practice theory also suggests educators structure learning opportunities that embed knowledge in both work practices and social relations--for example, apprenticeships, school-based learning, service learning, and so on. Plus, educators should create opportunities for students to solve real problems with adults, in real learning situations.

5. Constructivism

Definition

Constructivism is a philosophy of learning founded on the premise that, by reflecting on our experiences, we construct our own understanding of the world we live in. Each of us generates our own "rules" and "mental models," which we use to make sense of our experiences. Learning, therefore, is simply the process of adjusting our mental models to accommodate new experiences.

Main points

There are several guiding principles of constructivism:

- Learning is a search for meaning. Therefore, learning must start with the issues around which students are actively trying to construct meaning.
- Meaning requires understanding **wholes** as well as parts. And parts must be understood in the context of wholes. Therefore, the learning process focuses on primary concepts, not isolated facts.
- In order to teach well, we must understand the mental models that students use to perceive the world and the assumptions they make to support those models.
- The purpose of learning is for an individual to construct his or her own meaning, not just memorize the "right" answers and regurgitate someone else's meaning. Since education is inherently interdisciplinary, the only valuable way to measure learning is to make the assessment part of the learning process, ensuring it provides students with information on the quality of their learning.

How Constructivism Impacts Learning

Curriculum--Constructivism calls for the elimination of a standardized curriculum. Instead, it promotes using curricula customized to the students' prior knowledge. Also, it emphasizes hands-on problem solving.

Instruction--Under the theory of constructivism, educators focus on making connections between facts and fostering new understanding in students. Instructors tailor their teaching strategies to student responses and encourage students to analyze, interpret, and predict information. Teachers also rely heavily on open-ended questions and promote extensive dialogue among students.

Assessment--Constructivism calls for the elimination of grades and standardized testing. Instead, assessment becomes part of the learning process so that students play a larger role in judging their own progress.

6. Control Theory

Definition

This theory of motivation proposed by William Glasser contends that behavior is never caused by a response to an outside stimulus. Instead, the control theory states that behavior is inspired by what a person **wants** most at any given time: survival, love, power, freedom, or any other basic human need.

Discussion

Responding to complaints that today's students are "unmotivated," Glasser attests that all living creatures "control" their behavior to maximize their need satisfaction. According to Glasser, if students are not motivated to do their schoolwork, it's because they view schoolwork as irrelevant to their basic human needs.

Boss teachers use rewards and punishment to coerce students to comply with rules and complete required assignments. Glasser calls this "leaning on your shovel" work. He shows how high percentages of students recognize that the work they do--even when their teachers praise them--is such low-level work.

Lead teachers, on the other hand, avoid coercion completely. Instead, they make the intrinsic rewards of doing the work clear to their students, correlating any proposed assignments to the students' basic needs. Plus, they only use grades as temporary indicators of what has and hasn't been learned, rather than a reward. Lead teachers will "fight to protect" highly engaged, deeply motivated students who are doing quality work from having to fulfill meaningless requirements.

How the Control Theory Impacts Learning

Curriculum--Teachers must negotiate both content and method with students. Students' basic needs literally help shape **how** and **what** they are taught.

Instruction--Teachers rely on cooperative, active learning techniques that enhance the power of the learners. Lead teachers make sure that all assignments meet some degree of their students' need satisfaction. This secures student loyalty, which carries the class through whatever relatively meaningless tasks might be necessary to satisfy official requirements.

Assessment--Instructors only give "good grades"--those that certify quality work--to satisfy students' need for power. Courses for which a student doesn't earn a "good grade" are not recorded on that student's transcript. Teachers grade students using an absolute standard, rather than a relative "curve."

7. Conversation Theory

Definition

Conversation Theory, developed by G. Pask, is based on the fundamental idea that learning occurs through conversations about a subject matter that serve to make knowledge explicit. Conversations can be conducted at a number of different levels: natural language (general discussion), object languages (for discussing the subject matter), and metalanguages (for talking about learning/language). In order to facilitate learning, Pask argued that subject matter should be represented in the form of entailment structures that show what is to be learned. Entailment structures exist in a variety of different levels depending upon the extent of relationships displayed (e.g., super/subordinate concepts, analogies). Pask's theory has been applied most extensively to the teaching and learning of statistics and probabilities.

Pask identified two different types of learning strategies: serialists who progress through an entailment structure in a sequential fashion and holists who look for higher order relations.

Main points

1. To learn a subject matter, students must learn the relationships among the concepts.
2. Explicit explanation or manipulation of the subject matter facilitates understanding (e.g., use of teachback technique).
3. Individuals differ in their preferred manner of learning relationships (serialists versus holists).

Implications of Conversation Theory for Learning

The critical method of learning according to conversation theory is "teachback" in which one person teaches another what they have learned. Thus, discussion must be a central component of any lesson.

8. Gestalt Theory

Definition

This theory, one of the older ones, emphasizes higher-order cognitive processes in the midst of behaviorism. The focus of Gestalt theory was the idea of "grouping", i.e., characteristics of stimuli cause us to structure or interpret a visual field or problem in a certain way. The primary factors that determine grouping were: (1) proximity - elements tend to be grouped together according to their nearness, (2) similarity - items similar in some respect tend to be grouped together, (3) closure - items are grouped together if they tend to complete some entity, and (4) simplicity - items will be organized into simple figures according to symmetry, regularity, and smoothness. These factors were called the laws of organization and were explained in the context of perception and problem-solving.

Main points

1. The learner should be encouraged to discover the underlying nature of a topic or problem (i.e., the relationship among the elements).
2. Gaps, incongruities, or disturbances are an important stimulus for learning
3. Instruction should be based upon the laws of organization: proximity, closure, similarity and simplicity.

Example:

The classic example of Gestalt principles is children finding the area of parallelograms. As long as the parallelograms are regular figures, a standard procedure can be applied (making lines perpendicular from the corners of the base). However, if a parallelogram with a novel shape or orientation is provided, the standard procedure will not work and children are forced to solve the problem by understanding the true structure of a parallelogram (i.e., the figure can be bisected anywhere if the ends are joined).

9. Learning Styles

Definition

This approach to learning emphasizes the fact that individuals perceive and process information in very different ways. The learning styles theory implies that how much individuals learn has more to do with whether the educational experience is geared toward their particular style of learning than whether or not they are "smart." In fact, educators should not ask, "Is this student smart?" but rather "How is this student smart?"

Main points

The concept of learning styles is rooted in the classification of psychological types. The learning styles theory is based on research demonstrating that, as the result of heredity, upbringing, and current environmental demands, different individuals have a tendency to both perceive and process information differently. The different ways of doing so are generally classified as:

1. *Concrete and abstract perceivers*--Concrete perceivers absorb information through direct experience, by doing, acting, sensing, and feeling. Abstract perceivers, however, take in information through analysis, observation, and thinking.
2. *Active and reflective processors*--Active processors make sense of an experience by immediately using the new information. Reflective processors make sense of an experience by reflecting on and thinking about it.

Traditional schooling tends to favor abstract perceiving and reflective processing. Other kinds of learning aren't rewarded and reflected in curriculum, instruction, and assessment nearly as much.

How the Learning Styles Theory Impacts Education

Curriculum--Educators must place emphasis on intuition, feeling, sensing, and imagination, in addition to the traditional skills of analysis, reason, and sequential problem solving.

Instruction--Teachers should design their instruction methods to connect with all four learning styles, using various combinations of experience, reflection, conceptualization, and experimentation. Instructors can introduce a wide variety of experiential elements into the classroom, such as sound, music, visuals, movement, experience, and even talking.

Assessment--Teachers should employ a variety of assessment techniques, focusing on the development of "whole brain" capacity and each of the different learning styles.

10. Mathematical Problem Solving (A. Schoenfeld)

Definition

Alan Schoenfeld presents the view that understanding and teaching mathematics should be approached as a problem-solving domain. According to Schoenfeld (1985) four categories of knowledge/skills are needed to be successful in mathematics: (1) Resources - proposition and procedural knowledge of mathematics, (2) heuristics - strategies and techniques for problem solving such as working backwards, or drawing figures, (3) control - decisions about when and what resources and strategies to use, and (4) beliefs - a mathematical "world view" that determines how someone approaches a problem.

Schoenfeld's theory is supported by extensive protocol analysis of students solving problems. It also places emphasis on the importance of metacognition and the cultural components of learning mathematics (i.e., belief systems) (Schoenfeld, 1987).

Main points

Successful solution of mathematics problems depends upon a combination of resource knowledge, heuristics, control processes and belief, all of which must be learned and taught.

Example:

Schoenfeld (1985, Chapter 1) uses the following problem to illustrate his theory: Given two intersecting straight lines and a point P marked on one of them, show how to construct a circle that is tangent to both lines and has point P as its point of tangency to the lines. Examples of resource knowledge include the procedure to draw a perpendicular line from P to the center of the circle and the significance of this action.

An important heuristic for solving this problem is to construct a diagram of the problem. A control strategy might involve the decision to construct an actual circle and line segments using a compass and protractor. A belief that might be relevant to this problem is that solutions should be empirical (i.e., constructed) rather than derived.

Implications of Mathematical Problem Solving Theory for Learning

Solving problems is at the heart of mathematical learning and understanding. Therefore, curricula should be designed to give students opportunities to solve problems in many different ways. Use of a variety of heuristics should be both modelled and encouraged.

11. Multiple Intelligences

Definition

This theory of human intelligence, developed by psychologist Howard Gardner, suggests there are at least seven ways that people have of perceiving and understanding the world. Gardner labels each of these ways a distinct "intelligence"--in other words, a set of skills allowing individuals to find and resolve genuine problems they face.

Discussion

Gardner defines an "intelligence" as a group of abilities that:

- Is somewhat autonomous from other human capacities
- Has a core set of information-processing operations
- Has a distinct history in the stages of development we each pass through
- Has plausible roots in evolutionary history

While Gardner suggests his list of intelligences may not be exhaustive, he identifies the following seven:

1. *Verbal-Linguistic*--The ability to use words and language
2. *Logical-Mathematical*--The capacity for inductive and deductive thinking and reasoning, as well as the use of numbers and the recognition of abstract patterns
3. *Visual-Spatial*--The ability to visualize objects and spatial dimensions, and create internal images and pictures
4. *Body-Kinesthetic*--The wisdom of the body and the ability to control physical motion
5. *Musical-Rhythmic*--The ability to recognize tonal patterns and sounds, as well as a sensitivity to rhythms and beats
6. *Interpersonal*--The capacity for person-to-person communications and relationships
7. *Intrapersonal*--The spiritual, inner states of being, self-reflection, and awareness

How Multiple Intelligences Impact Learning

Curriculum--Traditional schooling heavily favors the verbal-linguistic and logical-mathematical intelligences. Gardner suggests a more balanced curriculum that incorporates the arts, self-awareness, communication, and physical education.

Instruction--Gardner advocates instructional methods that appeal to all the intelligences, including role playing, musical performance, cooperative learning, reflection, visualization, story telling, and so on.

Assessment--This theory calls for assessment methods that take into account the diversity of intelligences, as well as self-assessment tools that help students understand their intelligences.

12. Neuroscience

Definition

Neuroscience is the study of the human nervous system, the brain, and the biological basis of consciousness, perception, memory, and learning.

Main points

The nervous system and the brain are the physical foundation of the human learning process. Neuroscience links our observations about cognitive behavior with the actual physical processes that support such behavior. This theory is still "young" and is undergoing rapid, controversial development.

Some of the key findings of neuroscience are:

- *The brain has a triad structure*. Our brain actually contains three brains: the lower or reptilian brain that controls basic sensory motor functions; the mammalian or limbic brain that controls emotions, memory, and biorhythms; and the neocortex or thinking brain that controls cognition, reasoning, language, and higher intelligence.
- *The brain is not a computer*. The structure of the brain's neuron connections is loose, flexible, "webbed," overlapping, and redundant. It's impossible for such a system to function like a linear or parallel-processing computer. Instead, the brain is better described as a self-organizing system.
- *The brain changes with use, throughout our lifetime*. Mental concentration and effort alters the physical structure of the brain. Our nerve cells (neurons) are connected by branches called dendrites. There are about 10 billion neurons in the brain and about 1,000 trillion connections. The possible combination of connections is about ten to the one-millionth power. As we use the brain, we strengthen certain patterns of connection, making each connection easier to create next time. This is how memory develops.

How Neuroscience Impacts Education

When educators take neuroscience into account, they organize a curriculum around real experiences and integrated, "whole" ideas. Plus, they focus on instruction that promotes complex thinking and the "growth" of the brain. Neuroscience proponents advocate continued learning and intellectual development throughout adulthood.

13. Observational Learning

Definition

Observational learning, also called social learning theory, occurs when an observer's behavior changes after viewing the behavior of a model. An observer's behavior can be affected by the positive or negative consequences--called vicarious reinforcement or vicarious punishment-- of a model's behavior.

Main points

There are several guiding principles behind observational learning, or social learning theory:

1. The observer will imitate the model's behavior if the model possesses characteristics-- things such as talent, intelligence, power, good looks, or popularity-- that the observer finds attractive or desirable.
2. The observer will react to the way the model is treated and mimic the model's behavior. When the model's behavior is rewarded, the observer is more likely to reproduce the rewarded behavior. When the model is punished, an example of vicarious punishment, the observer is less likely to reproduce the same behavior.
3. A distinction exists between an observer's "acquiring" a behavior and "performing" a behavior. Through observation, the observer can acquire the behavior without performing it. The observer may then later, in situations where there is an incentive to do so, display the behavior.
4. Learning by observation involves four separate processes: *attention, retention, production and motivation*:
    - Attention: Observers cannot learn unless they pay attention to what's happening around them. This process is influenced by characteristics of the model, such as how much one likes or identifies with the model, and by characteristics of the observer, such as the observer's expectations or level of emotional arousal.
    - Retention: Observers must not only recognize the observed behavior but also remember it at some later time. This process depends on the observer's ability to code or structure the information in an easily remembered form or to mentally or physically rehearse the model's actions.

- Production: Observers must be physically and/intellectually capable of producing the act. In many cases the observer possesses the necessary responses. But sometimes, reproducing the model's actions may involve skills the observer has not yet acquired. It is one thing to carefully watch a circus juggler, but it is quite another to go home and repeat those acts.
- Motivation: In general, observers will perform the act only if they have some motivation or reason to do so. The presence of reinforcement or punishment, either to the model or directly to the observer, becomes most important in this process.

5. Attention and retention account for acquisition or learning of a model's behavior; production and motivation control the performance.
6. Human development reflects the complex interaction of the person, the person's behavior, and the environment. The relationship between these elements is called *reciprocal determinism*. A person's cognitive abilities, physical characteristics, personality, beliefs, attitudes, and so on influence both his or her behavior and environment. These influences are reciprocal, however. A person's behavior can affect his feelings about himself and his attitudes and beliefs about others. Likewise, much of what a person knows comes from environmental resources such as television, parents, and books. Environment also affects behavior: what a person observes can powerfully influence what he does. But a person's behavior also contributes to his environment.

How Observational Learning Impacts Learning:

Curriculum-- Students must get a chance to observe and model the behavior that leads to a positive reinforcement.

Instruction-- Educators must encourage collaborative learning, since much of learning happens within important social and environmental contexts.

Assessment--A learned behavior often cannot be performed unless there is the right environment for it. Educators must provide the incentive and the supportive environment for the behavior to happen. Otherwise, assessment may not be accurate.

14. Piaget’s Developmental Theory

Definition

Swiss biologist and psychologist Jean Piaget (1896-1980) is renowned for constructing a highly influential model of child development and learning. Piaget's theory is based on the idea that the developing child builds cognitive structures--in other words, mental "maps," schemes, or networked concepts for understanding and responding to physical experiences within his or her environment. Piaget further attested that a child's cognitive structure increases in sophistication with development, moving from a few innate reflexes such as crying and sucking to highly complex mental activities.

Main points

Piaget's theory identifies four developmental stages and the processes by which children progress through them. The four stages are:

1. *Sensorimotor stage (birth - 2 years old)*--The child, through physical interaction with his or her environment, builds a set of concepts about reality and how it works. This is the stage where a child does not know that physical objects remain in existence even when out of sight (object permanence).
2. *Preoperational stage (ages 2-7)*--The child is not yet able to conceptualize abstractly and needs concrete physical situations.
3. *Concrete operations (ages 7-11)*--As physical experience accumulates, the child starts to conceptualize, creating logical structures that explain his or her physical experiences. Abstract problem solving is also possible at this stage. For example, arithmetic equations can be solved with numbers, not just with objects.
4. *Formal operations (beginning at ages 11-15)*--By this point, the child's cognitive structures are like those of an adult and include conceptual reasoning.

Piaget outlined several principles for building cognitive structures. During all development stages, the child experiences his or her environment using whatever mental maps he or she has constructed so far. If the experience is a repeated one, it fits easily--or is assimilated--into the child's cognitive structure so that he or she maintains mental "equilibrium." If the experience is different or new, the child loses equilibrium, and alters his or her cognitive structure to accommodate the new conditions. This way, the child erects more and more adequate cognitive structures.

How Piaget's Theory Impacts Learning

Curriculum--Educators must plan a developmentally appropriate curriculum that enhances their students' logical and conceptual growth.

Instruction--Teachers must emphasize the critical role that experiences--or interactions with the surrounding environment--play in student learning. For example, instructors have to take into account the role that fundamental concepts, such as the permanence of objects, play in establishing cognitive structures.

15. Repair Theory

Definition

Repair theory is an attempt to explain how people learn procedural skills with particular attention to how and why they make mistakes (i.e., bugs). The theory suggests that when a procedure cannot be performed, an impasse occurs and the individual applies various strategies to overcome the impasse. These strategies (meta-actions) are called repairs. Some repairs result in correct outcomes whereas others generate incorrect results and hence "buggy" procedures (Brown & VanLehn, 1980).

Students shift back and forth among bugs, a phenomenon called bug migration. The theory's explanation for bug migration is that the student has a stable underlying procedure but that the procedure is incomplete in such a way that the student reaches impasses on some problems. Students can apply any repair they can think of. Sometimes they choose one repair and sometimes another. The different repairs manifest themselves as different bugs. So bug migration comes from varying the choice of repairs to a stable, underlying impasse. (VanLehn, 1990)p 26.

Main points

1. Bugs that cause errors in procedural tasks are systematic and can be identified.
2. Once the bugs associated with a particular task are known, they can be used to improve student performance and the examples used to teach the procedure.

Example:

If a student learns subtraction with two digit numbers and is then presented with the following problem: 365 - 109 =?, they must generate a new rule for borrowing from the left column. Unlike a two-digit problem, the left adjacent and the left-most column are different creating an impasse. To resolve the impasse, the student needs to repair their current rule (Always-Borrow-Left) by making it Always-Borrow-Left Adjacent. Alternatively, the student could skip the borrowing entirely generating a different bug called Borrow-No-Decrement-Except-Last.

Implications of Repair Theory for Learning

Repair theory assumes that people primarily learn procedural tasks by induction and that bugs occur because of biases that are introduced in the examples provided or the feedback received during practice (as opposed to mistakes in memorizing formulas or instructions). Therefore, the implication of repair theory is that problem sets should be chosen to eliminate the bias likely to cause specific bugs. Another implication is that bugs are often introduced when students try to extend procedures beyond the initial examples provided.

16. Right Brain vs. Left Brain

Definition

This theory of the structure and functions of the mind suggests that the two different sides of the brain control two different "modes" of thinking. It also suggests that each of us prefers one mode over the other.

Main points

Experimentation has shown that the two different sides, or hemispheres, of the brain are responsible for different manners of thinking. The following table illustrates the differences between left-brain and right-brain thinking:

Table 1: Differences Between Left Brain and Right Brain Thinking

Left Brain	Right Brain
Logical	Random
Sequential	Intuitive
Rational	Holistic
Analytical	Synthesizing
Objective	Subjective
Looks at parts	Looks at wholes

Most individuals have a distinct preference for one of these styles of thinking. Some, however, are more whole-brained and equally adept at both modes. In general, schools tend to favor left-brain modes of thinking, while downplaying the right-brain ones. Left-brain scholastic subjects focus on logical thinking, analysis, and accuracy. Right-brained subjects, on the other hand, focus on aesthetics, feeling, and creativity.

How Right-Brain vs. Left-Brain Thinking Impacts Learning

Curriculum--In order to be more "whole-brained" in their orientation, schools need to give equal weight to the arts, creativity, and the skills of imagination and synthesis.

Instruction--To foster a more whole-brained scholastic experience, teachers should use instruction techniques that connect with both sides of the brain. They can increase their classroom's right-brain learning activities by incorporating more patterning, metaphors, analogies, role playing, visuals, and movement into their reading, calculation, and analytical activities.

Assessment--For a more accurate whole-brained evaluation of student learning, educators must develop new forms of assessment that honor right-brained talents and skills.

17. Structural Learning Theory (J. Scandura)

Definition

According to structural learning theory, what is learned are rules that consist of a domain, range, and procedure. There may be alternative rule sets for any given class of tasks. Problem solving may be facilitated when higher order rules are used, i.e., rules that generate new rules. Higher order rules account for creative behavior (unanticipated outcomes) as well as the ability to solve complex problems by making it possible to generate (learn) new rules.

Main points

1. Whenever possible, teach higher order rules that can be used to derive lower order rules.
2. Teach the simplest solution path first and then teach more complex paths or rule sets.
3. Rules must be composed of the minimum capabilities possessed by the learners.

Example:

1. The first step involves selecting a representative sample of problems such as 9-5, 248-13, or 801-302.
2. The second step is to identify the rules for solving each of the selected problems. To achieve this step, it is necessary to determine the minimal capabilities of the students (e.g., can recognize the digits 0-9, minus sign, column and rows). Then the detailed operations involved in solving each of the representative problems must be worked out in terms of the minimum capabilities of the students. For example, one subtraction rule students might learn is the "borrowing" procedure that specifies if the top number is less than the bottom number in a column, the top number in the column to the right must be made smaller by 1.
3. The next step is to identify any higher order rules and eliminate any lower order rules they subsume. In the case of subtraction, we could replace a number of partial rules with a single rule for borrowing that covers all cases.
4. The last step is to test and refine the resulting rule(s) using new problems and extend the rule set if necessary so that it accounts for all problems in the domain. In the case of subtraction, we would use problems with varying combinations of columns and perhaps different bases.

Implications of Structural Learning Theory for Education

Structural learning prescribes teaching the simplest solution path for a problem and then teaching more complex paths until the entire rule has been mastered. The theory proposes that we should teach as many higher-order rules as possible as replacements for lower order rules. The theory also suggests a strategy for individualizing instruction by analyzing which rules a student has/has not mastered and teaching only the rules, or portions thereof, that have not been mastered.

18. Vygotsky and Social Cognition

Definition

The social cognition learning model asserts that culture is the prime determinant of individual development. Humans are the only species to have created culture, and every human child develops in the context of a culture. Therefore, a child's learning development is affected in ways large and small by the culture--including the culture of family environment--in which he or she is enmeshed.

Main points

1. Culture makes two sorts of contributions to a child's intellectual development. *First*, through culture children acquire much of the content of their thinking, that is, their knowledge. *Second*, the surrounding culture provides a child with the processes or means of their thinking, what Vygotskians call the tools of intellectual adaptation. In short, according to the social cognition learning model, culture teaches children both what to think and how to think.
2. Cognitive development results from a dialectical process whereby a child learns through problem-solving experiences shared with someone else, usually a parent or teacher but sometimes a sibling or peer.
3. Initially, the person interacting with child assumes most of the responsibility for guiding the problem solving, but gradually this responsibility transfers to the child.
4. Language is a primary form of interaction through which adults transmit to the child the rich body of knowledge that exists in the culture.
5. As learning progresses, the child's own language comes to serve as her primary tool of intellectual adaptation. Eventually, children can use internal language to direct their own behavior.
6. Internalization refers to the process of learning--and thereby internalizing--a rich body of knowledge and tools of thought that first exist outside the child. This happens primarily through language.
7. A difference exists between what child can do on her own and what the child can do with help. Vygotskians call this difference the zone of proximal development (ZPD).
8. Since much of what a child learns comes form the culture around her and much of the child's problem solving is mediated through an adult's help, it is wrong to focus on a child in isolation. Such focus does not reveal the processes by which children acquire new skills.
9. Interactions with surrounding culture and social agents, such as parents and more competent peers, contribute significantly to a child's intellectual development.

How Vygotsky Impacts Learning:

Curriculum--Since children learn much through interaction, curricula should be designed to emphasize interaction between learners and learning tasks.

Instruction--With appropriate adult help, children can often perform tasks that they are incapable of completing on their own. With this in mind, scaffolding--where the adult continually adjusts the level of his or her help in response to the child's level of performance--is an effective form of teaching. Scaffolding not only produces immediate results, but also instills the skills necessary for independent problem solving in the future.

Assessment--Assessment methods must take into account the zone of proximal development. What children can do on their own is their level of actual development and what they can do with help is their level of potential development. Two children might have the same level of actual development, but given the appropriate help from an adult, one might be able to solve many more problems than the other. Assessment methods must target both the level of actual development and the level of potential development.

Common Themes that Cross Learning Theories

Sfard (2003) has identified ten needs that appear to cross all learning theories, which she claims are the driving force behind learning and must be fulfilled if the learning is to be successful. The needs are universal, but may be expressed differently in different individuals and at different ages. These needs are:

1. The need for meaning
2. The need for structure
3. The need for repetitive action
4. The need for difficulty
5. The need for significance and relevance
6. The need for social interaction
7. The need for verbal-symbolic interaction
8. The need for a well-defined discourse
9. The need for belonging
10. The need for balance

Each of these needs will be briefly summarized with the implications for mathematics education.

The Need for Meaning

Definition: Learners look for order, logic, causal dependencies behind things, events, and experiences. They must have the role of an autonomous meaning builder.

Implications for mathematics education: Because the need for meaning motivates us to learn, instruction that focuses on it is more effective than instruction that ignores this need. All learning theories conclude in some way that an understanding of a concept can only be achieved through a student's activity with that concept. Abbott and Ryan (1999) describe learning as an active process in which an individual assimilates new facts and experiences into a pre-existing web of knowledge and understanding. This model coincides with both constructivism and researchers' growing understanding of the brain's natural learning processes. They claim that current economic and social needs require the development of skills, such as flexibility and problem solving, that are best developed through constructivist learning.

Even misconceptions are an attempt by students to search for meaning and construct their own conceptions. Many research studies report the types of misconceptions students develop in their search for meaning (Confrey, 1990; Sfard, 1992; Smith, diSessa, & Rochelle, 1993; Vinner & Dreyfus, 1989). This knowledge may be helpful in supporting children in their efforts to make meaning. Children as young as 5 years of age benefit when they solve a difficult problem incorrectly, are told the correct answer, and then are asked, "How do you think I knew that?" (Siegler, 1995). Like discovery-learning approaches, this approach requires children to actively engage in generating the meaning for themselves. However, it is also efficient in directing the student's thinking so no further time is lost investigating incorrect paths.

This does not imply that instruction that does not result in immediate understanding should be avoided. Effortless mathematics becomes trivial and uninspiring, and leads to a "watered down" curriculum (Jackson, 1997). Instead, students must be helped to develop patience, persistence, and tolerance for lack of sufficient clarity while they are developing meaning for new concepts.

The Need for Structure

Definition: The need for structure follows from the need for meaning. Meaning involves relations among concepts, not just concepts as such. Understanding of these concepts requires the ability to see structure from the relations among concepts.

Implications for mathematics education: If understanding means seeing structure, then the connections between concepts already learned and new concepts being introduced must be an integral part of the curriculum and instruction. Such connections must include not only real-world applications and relevance, but also assistance in building mathematical abstractions, so students can see how the results can be transferred from one context to another (Wu, 1997). Because students must construct their own meaning, we cannot just "show" students the structure. However, we can ask students to be more explicit in their organization of mathematics and the structures we wish them to develop. The real-life context of learning can be balanced with instruction to build an understanding of the part that abstract mathematics plays in the curriculum.

Williams (1998) examined students' conceptual understanding of functions by using concept maps as instruments for assessment. Concept maps were chosen as they are a direct method of looking at the organization and structure of an individual's knowledge and at the efficiency with which that knowledge can be used. The more connections that exist among facts, ideas, and procedures, the better the understanding. of conceptual understanding, Twenty-eight first-year calculus students, half from non-traditional sections and half from traditional sections, participated in the study. Eight professors with PhDs in mathematics also completed concept maps. These expert maps are compared with the student maps. Qualitative analysis of the maps reveals differences between the student and expert groups as well as between the two student groups. Observations included:

- many of the students' (both groups) concepts were trivial or irrelevant;
- student maps tended to be algorithmic, particularly for the traditional students. "Instead of naming concepts and the relationship connecting them, the students gave steps in a procedure" (p. 418) ;
- students from the reform group had a better understanding that functions may be used to model real-life situations;
- few student maps showed any significant hierarchical structure;
- expert maps showed no hint of algorithms but tended to have categorical groupings such as "types of functions";
- expert maps showed much more homogeneity than students maps;

Cognitive psychologists indicate that the internal representation of knowledge resembles webs of ideas that are connected and organized. If it is true that "the more connections that exist, the better the understanding", (Williams, 1998), p.414) a revised WNCP mathematics curriculum should explicitly identify many of the connections necessary for conceptual understanding in topics such as number and function.

<u>The Need for Repetitive Action</u>

Definition: A person who has created meaning and structure for a mathematical concept is aware of a repetitive, constant structure of certain actions and can see it well enough to reify it. To reify it means he is able to think and speak about the process as an object whose inner structures do not have to be remembered each time one deals with it.

Implications for mathematics education: A reasonable level of mastery of basic skills is an important element in constructing mathematics knowledge(Fuson & Briars, 1990; Fuson

& Kwon, 1992; Hiebert & Wearne, 1996; Siegler, 2003; Stevenson & Stigler, 1992). If we wish students to reflect on the processes of mathematics, they must first have a certain mastery of those processes. Children without a reasonable ability to perform basic algorithms may have nothing with which to build their further mathematics (Sfard, 2003). This does not mean a focus on rote repetition. Rather, it should be a process of reflective practice, where mastery of the action leads to reflection on the meaning of that action which leads to further understanding and learning.

The Need for Difficulty

Definition: True learning implies coping with difficulties.

Implications for mathematics education: The goal of learning is to advance a student from abilities he now possesses to those he has not yet developed. The best way to accomplish this is to present the student with tasks beyond his present developmental level but within his zone of proximal development (ZPD). Tasks must be demanding, but still within reach of the student.

The Need for Significance and Relevance

Definition: Significance is the ability to understand and appreciate the place and importance of what is to be learned within the system of concepts that are already well understood. In other words, it is an awareness of the way in which existing knowledge generates a problem at hand and necessitates its solutions.

Implications for mathematics education: Significance means linking new knowledge to existing knowledge, so again stresses the importance of helping students build connections. Recent research has shown that people tend to do significantly better in applying mathematics to real-life problems than in attempting to deal with the same mathematical content in the context of typical school problems (Nunes, Schlieman, & Carraher, 1993; Saxe, 1991; Schlieman & Carraher, 1996). This has led to programs where teaching of content, such as basic skills, is done via problem solving. For example, studies of students in Cognitively Guided Instruction (CGI) classes have shown that students had significantly higher levels of achievement in problem solving than control classes and no difference in achievement in number skills, despite less emphasis on the number skills (Carey, Fennema, Carpenter, & Franke, 1995; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter, Franke, Jacobs, & Fennema, 1998; Villasenor & Kepner, 1993).

It is important to recognize, however, that significance does not come only from the concrete and the real (Wu, 1997). It also comes from problems that are more abstract and part of a system directed at an important goal. Focusing only on real-life applications would lead to a fragmented, incomplete picture of the mathematics, particularly at the pre-calculus level.

The Need for Social Interaction

Definition: There is an inherent social nature to learning and making meaning. Bruner states that social transaction is the fundamental vehicle of education and not, so to speak, solo performance (Bruner, 1985).

Implications for mathematics education: The importance of social interaction in the mathematics class has been reported by many researchers (Cobb, 1995, 1999; O'Connor, 1996, 1998; Schoenfeld, 1996). The most obvious forms are student-teacher or student-student exchanges, but even interaction with a textbook is a form of social interaction (Sfard, 2003). Cooperative learning is another form of learning interaction that does not have the teacher in the central role. It has become popular because of research findings suggesting positive effects on student achievement of collective effort (O'Connor, 1998; Siegler, 2003; Webb, 1991; Webb & Farivar, 1994). However, cooperative learning does not always result in increased learning, and at times leads to worse learning than trying to solve problems on one's own (Levin & Druyan, 1993; Tudge, 1992). According to Ann Brown, designing effective cooperative learning situations requires at least as much engineering as standard classroom instruction does. Without careful structuring, problems of freeloading and disorganization can lead to inferior learning. The collaborative arrangements that generate the most productive instructional dialogs are those that encourage joint problem solving and discourage competition among students (Damon & Phelps, 1989). Thus, use of collaborative groups must be balanced with the need for solitary effort and teacher interventions (Sfard, 2003).

The Need for Verbal-Symbolic Interaction

Definition: Interaction in learning means communication, and communication means using both language (speech) and symbols (written language as well as special mathematics symbols) to convey thoughts.

Implications for mathematics education: If mathematics learning is to take place in an interactive setting, students must be encouraged to "talk" mathematics. Research shows that classroom discussion provides many learning opportunities (Ball, 1991; Cobb, Wood, & Yackel, 1991, 1993; Lampert, 1990; Schoenfeld, 1996). The role of the teacher is particularly important. For the discourse to be effective, students must be taught to communicate mathematically. A study by McConney (McConney, 2003) showed that there are significant differences in the type of communication that occurs in Chinese mathematics classrooms when compared to American classrooms. Teachers in both U.S. and Chinese classes used about the same number of explanations and same number of words per explanation. However, there were only an average of 23 student explanations in U.S. lessons compared to 84 explanations by Chinese students. The average number of words per explanation was five for U.S. students compared to 16 for Chinese students. Chinese students had twice as many public explanations as U.S. students and no alternative perspectives were presented when discussing solutions. This indicates U.S. students still need to work on communicating mathematically. The teacher must thus shoulder the responsibility to initiate, moderate, and coordinate discussions in the classroom.

The Need for a Well-Defined Discourse

Definition: Discourse goes beyond the idea of a conversation. It refers to all communication practices of the classroom - both written and verbal.

Implications for mathematics education: Discourse implies that the resulting communication follows specific rules and metarules. Metarules include what Cobb calls sociomathematical norms (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Sfard, 1998a). Although it appears desirable to have these rules be as close as possible to those used my mathematicians, experience has shown that this is neither simple nor practical to do (Brown, 1997). Researchers now recommend that rules be adjusted to the needs and possibilities of the learning child (Siegler, 2003). This does not mean giving up the need for rigor, but it does mean making clear to the students the metarules we are using and being careful in the choice or the rules we modify.

The Need for Belonging

Definition: The desire to belong and be counted as a member of a particular social group is a powerful force behind our actions. Learning by participation requires one to be a part of a learning community.

Implications for mathematics education: Students need to feel respected and free to speak their mind in the classroom. However, the extent to which students value belonging to a mathematical community is influenced by the value given to mathematics by the wider community culture (Comiti & Ball, 1996). Thus, it may be difficult to instill a desire to belong to the mathematical community in an environment where mathematics is not valued. The most promising directions for improvement seem to be those that incorporate historical context in the mathematics content, portray mathematics as something unique in our world, and something to be valued for its own sake (Sfard, 2003).

The Need for Balance

Definition: To meet learners' many varied needs, the pedagogy must be variegated and rich in possibilities.

Implications for mathematics education: The need for balance suggests a need to search for the good in all theories. It does not imply that old and new are mutually exclusive. For instance, the profound constructivist views of the learner building his or her own knowledge sometimes become trivialized into "teaching by telling". Similarly, the need for communication sometimes becomes interpreted to mean cooperative learning is mandatory. Problem solving as a focus becomes viewed to mean a de-emphasis on basic skills acquisition and any non-problem based instruction. The reality is that there must be a bit of everything in the classroom: problem solving as well as skills practice, teamwork as well as individual learning and teacher exposition, real-life problems as well as abstract problems, learning by talking as well a silent learning (Sfard, 2003).

A Government-commissioned inquiry into the state of mathematics in Britain recognizes this need and will report that radical measures are needed to save the subject from slipping into terminal decline in schools and universities. Professor Adrian Smith, principal of Queen Mary University, London, headed the 15-month inquiry. The report is expected to demand a major overhaul of maths lessons for 14 to 19-year-olds in an attempt to halt the decline in numbers taking A-level maths and to train the future workforce. "We need to make the material much more inspirational," says Professor Smith (Cassidy, 2004).

Mathematicians worry that their subject is trapped in a vicious circle. Many students do not enjoy maths lessons, so few study the subject at university, and even fewer graduate in mathematics, making the pool from which maths teachers can be recruited very small (It is compulsory to be a mathematics graduate in order to teach that subject in the British system.) The report is expected to call for:

- Reform of maths courses so every student learns basic numeracy
- Students to continue with maths for longer and to emerge with a meaningful qualification
- A variety of pathways which students can master at their own rates
- The exam system to become less age-fixated
- The Government to fund new conversion courses to allow graduates with some maths in their background to retrain as maths specialists
- More financial incentives for maths graduates to train as teachers

Cognitive Variability and Strategy Choice

Research in the 1980's often described children's thinking as progressing from simple strategies to more advanced strategies to even further advanced strategies. For instance, children often begin addition by counting all by ones, then by counting on from the first addend, then from the higher addend, possibly progressing to use of derived facts, till finally they master the basic facts and perform the addition from memory (Ashcraft, 1987; Carpenter & Moser, 1984).

More recent studies, however, have shown that children's thinking does not progress in quite so ordered a way. Rather than using a single strategy, children use a wide range of strategies from early learning through more advanced grades. They may use both simple and complex strategies at the same time, sometimes changing strategy even when reworking the same problem. Children, youth, and even adults continue to use slower, less efficient strategies even when faster, more accurate strategies have been mastered. (Kuhn, Garcia-Mila, Zohar, & Andersen, 1995; Schauble, 1996).

This use of multiple strategies when solving problems of the same type is a spontaneous feature of children's thinking and is usually difficult to change. Although it makes sense to discourage use of less efficient strategies, children actually learn better when they are allowed to choose the strategy they wish to use (Siegler, 2003). Immature strategies tend to decline in use as children feel comfortable using more sophisticated approaches. Also, students who use a greater variety of different strategies to solve problems tend to subsequently learn better (Chi, de Leeuw, Chiu, & LaVancher, 1994; Siegler, 1995). This better learning occurs partly because the greater variety of strategies equips the students to cope with whatever type of problem they encounter rather a narrow range of problems (Siegler, 2003). Allowing students to use their own choice of strategy coupled with instruction to help them understand why different strategies yield the same answer or why a strategy that appears reasonable is actually incorrect appears to build deeper understanding (Siegler, 2002).

Effective use of diverse strategies requires students to choose appropriately among their strategies. They must adjust to different situations and different problems. For instance, if computational speed is important, students will attempt to use recall, even if it is inaccurate. If accuracy is important, then a strategy that is more likely to produce a correct answer will be chosen. Adaptive choice involves using quick or easy strategies when they are sufficient, and using those that require more effort only when they are necessary to being correct. Greater practice and instruction in how to execute strategies may be the most useful approach to improving students' arithmetic skills (Siegler, 2003).

Individual Differences

Mathematical disabilities, as defined by poor performance on standardized tests, constitute a serious problem in the U.S. where approximately 6% of children are labeled as having such disabilities (Siegler, 2003). This is about twice the number reported as having literacy disabilities. Research has identified various factors contributing to these disabilities. As early as grade one children can be divided into three groups based on their choices of arithmetic strategies: good students, not-so-good students, and perfectionists (Siegler, 1988). The good and perfectionist students are faster, more accurate, use more advanced strategies, and perform better on standardized tests. Relative frequencies of children in each group appear stable across socio-economic groups and genders. Roughly half the students in the not-so-good group go on to be classified as having learning difficulties by grade four.

Other students identified as having mathematical disabilities can also be identified early in their education. As first-graders, these students frequently use immature counting procedures (counting all rather than from the larger addend), have limited and poor use of backup strategies, and use recall of basic facts rarely and inaccurately (Geary, 1994). Even as they progress in their use of strategies in subsequent years, they still have difficulty retrieving facts from memory. They encounter further difficulties when they encounter tasks that build on basic arithmetic.

Reasons for these difficulties point to limited exposure to numbers before entering school. Children with limited mathematical backgrounds are already far behind other children in counting skills, knowledge of numerical magnitudes, and knowledge of arithmetic facts on entering school. Early identification and intervention for these students is essential if they are to avoid becoming labeled as having a mathematical disability. Other research shows that those students labeled as mathematically disabled cannot hold as much numerical information in their memory as their age peers can (Geary, Bow-Thomas, & Yao, 1992; Koontz & Berch, 1996). Limited conceptual understanding of the arithmetic operations and counting is another factor in hindering their learning of mathematics (Geary, 1994; Hitch & McAuley, 1991). In summary, these mathematical disabilities reflect a combination of limited background knowledge, limited processing capacity, and limited conceptual understanding.

Relations Between Conceptual and Procedural Knowledge

It is widely accepted today that both procedural and conceptual knowledge are essential in mathematics. Although there are a few exceptions, procedural skill and conceptual understanding are usually highly correlated (Siegler, 2003). Asian and U.S. elementary students have shown parallel national differences in conceptual and procedural knowledge of multi-digit addition and subtraction. Korean children tested both for the procedures and conceptual understanding were able to succeed in both types of problems (Fuson & Kwon, 1992). Stevenson and Stigler (1992) found similar procedural and conceptual competence in first through fifth grade students in Japan and China.

Reys and Yang examined the understanding of number concepts or number sense of sixth and eighth grade students in Taiwan. They defined number sense as using multiple representations of number, recognizing the relative and absolute magnitudes of numbers, selecting and using benchmarks, decomposing and recomposing numbers, understanding the relative effects of operations on numbers, and flexibility and appropriately performing mental composition and estimation.

Number sense is not mentioned in the national mathematics curriculum of Taiwan. Teachers are expected to follow the national curriculum. In Taiwan, the development of computation is completed by the end of Grade 6. Students learn standard written algorithms by means of direct instruction and mastery is expected. The four basic operations on whole numbers, fractions, and decimals are addressed in elementary school. Calculators are not used in elementary school; emphasis is placed on developing competence with standard written procedures to find exact answers, and estimation is not given systematic attention.

Researchers constructed a 20-item Written Computation Test (WCT). All items were open-ended and consistent with the national curriculum for Taiwan. The same test was used for Grades 6 and 8. A 40-item Number Sense Test (NST) was also constructed by researchers. The NST included multiple choice, open-ended, and combination of multiple choice with short answer response items. The first 20 items of the NST were parallel with WCT items.

Results showed that performance on the WCT was higher and much less variable than on the NST. There were no significant gender differences at either grade level on either test. Follow-up interviews were conducted with randomly selected students from the top and middle performing levels on the exams. Interview items were designed to explore understanding of number magnitude and the use of benchmarks. The number magnitude items focused on relative number sizes, comparing and ordering numbers, and understanding the density of rational numbers. There seemed to be a sharp distinction between the levels of number sense possessed by the top- and middle-level students of each grade level. Middle-level students' initial responses used rule-based methods rather than understanding or number sense. Initial responses of most students were to search for and to apply computational algorithms.

This study provides evidence in contrast to that of the studies of Fuson and Kwon (1992) and Stevenson and Stigler (1992). Chinese students in Taiwan performed at very different levels on written computation compared to number sense. Taiwanese students were highly skilled in paper-and-pencil computation but not equally skilled in their use of non-computational approaches that rely on number sense.

In yet further contrast, studies with U.S. children from grades 2 to 5 showed they frequently lacked both conceptual and procedural knowledge of multi-digit addition and subtraction (Fuson, 1990). Lack of conceptual knowledge was shown by incorrectly identifying the place value of digits in a multi-digit number (Kouba, Carpenter, & Swafford, 1989), and lack of ability to explain or demonstrate ten-for-one trading with concrete representations (Ross, 1986). Lack of procedural knowledge was evident in the frequency of errors while solving multi-digit addition and subtraction problems using paper and pencil techniques (Fuson & Briars, 1990; Kouba et al., 1989; Stevenson & Stigler, 1992).

Studies aimed at improving competence in multi-digit addition and subtraction problems suggest that instruction that emphasizes conceptual understanding as well as procedural skill is more effective in building both kinds of competence than instruction that focuses only on procedural skills (Fuson & Briars, 1990; Hiebert & Wearne, 1996). A study by Correa, Kumar, and Sims (2003) suggests that U.S. teachers still need improvement in this type of instruction. The study looked at 10 Chinese and 10 American mathematics lessons from the start of the lesson till the first complete example or activity of the new material was completed. They found major differences in the types of lesson beginnings between the two groups. First, U.S. lessons tended to prompt students for answers. Chinese lessons used more conversation, discussion, and justification. Even after the correct answer was given teachers asked more "why" questions. The role of review was very different for the two groups. Chinese review was in depth to insure students were prepared for the new material. U.S. review was primarily to prompt students to recall facts. Finally, new material was presented in very different ways. Chinese students were responsible for suggesting ideas. In U.S. classes the teacher primarily showed the procedure and then asked students to apply it.

Rittle-Johnson and Alibali (1999) studied the impact of whether conceptual or procedural knowledge was given first. Randomly selected fifth grade students were divided into three groups. One group received conceptually oriented instruction, another received procedurally oriented instruction, and the third group was given neither. All children then practiced solving addition problems and were then given a post-test assessing both conceptual and procedural knowledge. Students who were given conceptually oriented instruction had substantial gains in both kinds of knowledge. Students who had procedurally oriented instruction had substantial gains in procedural knowledge and smaller gains in conceptual knowledge. Although this is only one study, it may suggest that conceptual instruction should be undertaken before instruction to teach procedures.

Problem-centered instruction is one approach to develop conceptual understanding. Wood and Sellers (1997) performed longitudinal analyses of the mathematical achievement and beliefs of three groups of elementary pupils: those students who had received two years of problem-centered mathematics instruction, those who had received one year, and those who had received only textbook instruction. Comparisons were made for the groups using both a standardized norm-referenced achievement test and instruments developed to measure conceptual understanding of arithmetic as well as beliefs and motivation for learning mathematics. Results indicated that after two years in problem-centered classes, students had significantly higher achievement on standardized achievement measures, better conceptual understanding, and more task-oriented beliefs for learning mathematics than did those in textbook instruction. They were not motivated by a desire to be better than others (ego orientation) but rather by a belief in the importance of finding their own ways to solve problems (task orientation). They also believed that to be successful in mathematics they should try to explain their thinking to others. They did not believe that they could be successful in mathematics by being quiet in class, or by turning in neat work—characteristics identified by those who were textbook instructed.

Comparisons of pupils in problem-centered classes for only one year revealed that after returning to textbook instruction, these students' mathematical achievement and beliefs are most similar to the textbook group. Textbook-instructed students did not believe that doing mathematics is about trying to understand but rather about solving questions using one single method. That single method was normally "the teacher's way." (p. 182)

Boaler (1998, 2002) found similar results in her comparison of two schools in the United Kingdom using contrasting instructional methods. The two schools in this research study were chosen because their teaching methods were very different but their student bodies were very similar. The schools were:

Amber Hill School—used 200 students in the study. In mathematics lessons the teachers used the School Mathematics Project (SMP) that is a content-based scheme which introduced students to mathematical procedures and techniques and then presented a range of questions for student practice. "Many of the Amber Hill students held a view that mathematics was all about memorizing a vast number of rules, formulas, and equations, and this view appeared to influence their mathematical behaviour." (Boaler, 1998, p. 46)

Phoenix Park School—110 students in the study. Students at this school were encouraged to take responsibility for their own actions and to be independent thinkers. Students worked on open-ended projects and in mixed-ability groups at all times in this school.

Students who followed a traditional approach (Amber Hill) developed an inert, procedural knowledge that they found difficult to use in anything other than textbook questions. It was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project-based environment (Phoenix Park) developed a conceptual understanding that provided them with advantages in a range of assessments and situations. It seemed that using mathematical procedures within authentic activities allowed the students to view the procedures as tools that they could use and adapt. The project students had been "apprenticed" into a system of thinking and using mathematics that helped them in both school and nonschool settings. Boaler concluded that "a traditional textbook approach that emphasizes computation, rules, and procedures, at the expense of depth of understanding, is disadvantageous to students, primarily because it encourages learning that is inflexible, school-bound, and of limited use." (Boaler, 1998, p.60)

Implications for Mathematics Education

Mathematics curricula and classroom instruction must include learning outcomes that focus on both conceptual and procedural knowledge. Assessment must also test both types of knowledge. Correct answers are not always a safe indicator of understanding. Teachers must examine more than answers to get a true picture of a student's conceptual understanding of numbers.

Two other factors may also have an impact on conceptual learning – choice and connections. The problem-based instruction and open, project-based approaches to learning both provide opportunities for students to make their own choices and to form connections between their mathematical learning and real-world applications. These factors have a far greater impact on students than most educators realize.

Technology in the Mathematics Curriculum

In *Principles and Standards for School Mathematics,* the NCTM (2000) states categorically that, "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (p. 24) Technology, as envisioned by the NCTM, includes calculators and computers, which are to be used for teaching, learning and doing mathematics. This section of the literature review will briefly look at current research on technology and mathematics education, with particular emphasis on the impact of calculators on students' mathematics achievement.

Electronic computation has been available to mathematicians for five decades, but has been available to classroom teachers for approximately half that time (Ellington, 2003; Kaput, 1992). During this period the number of calculators (basic, scientific, and graphing) has increased significantly (Waits & Demana, 2000), but their integration into school practice lags behind – it appears that teachers resist the integration of new technologies into their teaching of mathematics (Mariotti, 2002).

Advocates for the use of technology within the mathematics classroom (e.g. Barrett & Goebel, 1991; Bottino & Chiappini, 2002; Ellington, 2003; Kaput, 1992; Waits & Demana, 2000) agree that the benefits to students include:

- Reducing the drudgery of applying arithmetic and algebraic procedures when those procedures are not the focus of the lesson;
- Using dynamic geometry software to allow for investigations that lead to much better understanding;
- Helping them solve realistic applications requiring sophisticated mathematics – and thus seeing that it has value;
- Making instructional approaches that link multiple representations possible (e.g., graphical and numerical);
- Providing deeper understanding and an intuition about functions; and,
- Creating a learning environment that is more rewarding and enjoyable.

Chazan and Yerushalmy warn, however, that related changes to the curriculum or to modes of instruction are crucial to gain these benefits. In reference to graphing applications, they point out that graphical representations are not transparent. The process of learning to read such representations is complex and requires teaching and learning, In particular, more needs to be made of the differences between graphing equations in which all points on the Cartesian plane can potentially be used and graphing functions in which once a value for a particular *x* is established, no other points in that vertical line can be used.

In addition, tension often arises between the standard curriculum and graphing calculator technologies. With graphing calculators, expressions are always taken as the equation of functions in the form y = __. Thus, use of graphing calculators often examines equations in a form different than one initially given, and may involve manipulations and interpretations requiring assumptions or limitations on the equation. Although such requirements can be addressed through the curriculum and instruction, it adds yet one more wrinkle to the tasks learners face.

A number of studies have been conducted that look at the impact of technology on students' mathematics achievement. These studies (e.g., Ellington, 2003; Schacter, 1999) consistently concluded that informed use of technology in the mathematics classroom improved student achievement in mathematics. John Schacter (1999) analyzed the five largest scale studies of education technology available at that time. The studies and associated findings are summarized in Table 2.

<u>Table 2: Summary of Findings From "The Impact of Education Technology on Student Achievement: What the Most Current Research Has to Say"</u>

Author(s)	Title (Year)	Type	Summary of Results
Kulik, J.	Meta-Analytical Studies of Findings on Computer-Based Instruction (1994)	Meta-analysis	Meta-analysis of more than 500 individual studies found that: • On average, students who used computer-based instruction scored at the 64th percentile on test of achievement compared to students in the control conditions without computers who scored at the 50th percentile; • Students learn more in less time when they receive computer-based instruction; • Students in classes that include computer-based instruction have more positive attitudes
Baker, E. J., Gearhart, M., & Herman, J. L.	Evaluating the Apple Classrooms of Tomorrow (1994)	Non-random study	Assessed the impact of interactive technologies on teaching and learning in five schools (in California, Tennessee, Minnesota, and Ohio). The study concluded that ACOT had a positive impact on student attitudes and changed teacher practices (less stand-up lecturing).
Sivin-Kachala, J.	Report on the Effectiveness of Technology in Schools (1998)	Meta-analysis	Meta-analysis of 219 individual studies from 1990 to 1997 found that students in technology rich environments: • Experienced positive effects on achievement in all major subject areas; • Showed increased achievement in pre-school computer-based instruction; • Improved their attitudes toward learning

Author(s)	Title (Year)	Type	Summary of Results
Wenglinsky, H.	Does it Compute? The Relationship Between Educational Technology and Student Achievement in Mathematics (1998)	Random Study	Assessment of the effects of simulation and higher order thinking technologies on a U.S. national sample of 6,227 fourth grade and 7,146 eighth grade mathematics achievement scores on the national Assessment of Educational Progress. The study found that: • Eighth grade students who used the technologies showed gains in math scores up to 15 weeks above grade level as measured by NAEP; • Eighth grade students whose teachers received professional development on computers showed gains in math scores of up to 13 weeks above grade level; • Higher order uses of computers and professional development were positively related to students' academic achievement in mathematics for both 4th and 8th grade students
Mann, D. Shakeshaft, C., Becker, J., & Kottkamp, R.	West Virginia's basic Skills/Computer Education program: An Analysis of Student Achievement (1999)	Non-random Study	Analysis of a representative sample of 950 fifth-grade students' achievement from 18 elementary schools across the state. The study found that: • Consistent access to technology, positive attitudes towards technology and teacher training in technology led to the greatest student achievement; • Half of the teachers in the sample thought that technology had helped a lot with West Virginia's instructional goals and objectives.

In a more recent meta-analytical study Ellington (2003) identified 54 studies published between January 1983 and March 2002 that met a stringent set of selection criteria. Characteristics of the studies that were featured in the meta-analysis include:

- Publication status (Journal, Dissertation, other unpublished source);
- Test instrument (Standardized, Non-standardized (teacher made));
- Educational division (Elementary, Middle School, High School);
- Ability of students (Mixed, Low, High);
- Treatment length (Test only, 0 – 3 weeks, 4 – 8 weeks, 9 or more weeks);
- Curriculum (Traditional, Special);

- Calculator use; (Functional, Pedagogical);
- Study design (Random, Nonrandom); and,
- Sample size (1-100, 101-200, 210-1000, over 1000)

Meta-analysis of the studies resulted in the following findings:

- When calculators were available during instruction but not during testing students in grade K – 12:
  - Maintained the paper-and-pencil skills and the skills necessary to understand concepts;
  - Improved their operational skills as a result of calculator use during instruction;
  - Received the most benefit when calculators had a pedagogical role in the classroom and were not just available for drill and practice or checking work;
  - Needed to be provided long-term exposure to calculator use during instruction;
  - Improved their skills in selecting appropriate problem-solving strategies.
- When calculators were included in testing and instruction, students in grade K – 12:
  - Experienced improvement in operational skills as well as in paper-and-pencil skills and the skills necessary for understanding mathematical concepts;
  - Received the most benefit when calculators had a pedagogical role in the classroom;
  - Received the most benefit in developing conceptual skills when the calculator use was short term (0 – 3 weeks);
  - Received the most benefit in developing operational skills when the calculator use was short term (0 – 3 weeks) or long term (9 or more weeks);
- Problem-solving skills improved the most under two conditions:
  - When special curriculum materials designed for calculator use were used in the class; and,
  - When the technological tool in use was the graphing calculator.
- Students' attitudes showed the most improvement after 9 or more weeks of calculator use;
- The development of elementary students' computational skills was not hindered by calculator use during instruction for both with and without calculator use in testing;
- Operational skills benefited from all types of calculators (basic, scientific, graphing); and,
- Graphing calculators had more influence on students' attitudes when compared to other types.

Ellington (2003) concluded that the meta-analysis supported the use of calculators in all precollege mathematics classrooms. Ellington also concluded that, "because limited research has been conducted featuring the early grades, calculator use should be restricted to experimentation and concept development activities." (p. 457). Finally, Ellington concluded that calculators should be particularly emphasized during instruction of problem-based skills in middle and high school and that calculators should be available during evaluations of these students' problem-solving skills and their understanding of mathematical concepts.

Implications for WNCP Mathematics Curriculum

Calculators can have a positive impact in the learning of mathematics for students. It appears that benefits will be maximized if calculators are used in a pedagogical role and not just for performing calculations. This has curricular implications, as it requires adaptation of both content (i.e. representations and procedures) and instructional approaches. Support in terms of both resources and inservice is essential to ensure that the technology is appropriately integrated into the classroom.

The Impact of the Teacher in Educational Change

What teachers and students are able to do together in mathematics classes is at the heart of mathematics education. Research on mathematics education has called attention to the unique combination of mathematics content and the pedagogical knowledge needed to teach it effectively. Of 354 articles dealing specifically with mathematics teaching and learning published between 1986 and 1998, almost half of those articles focused on the mathematics teachers alone (Ball, Lubienski, & Mewborn, 2001). Many others included teachers' knowledge and beliefs among their central questions.

There is continuing concern with how to improve the quality of mathematics education. Amid frequent claims of how to improve teaching practice, research, policy, practice, and advocacy are all intertwined. It is insufficient for policy makers to simply recommend a course (Wilson, Peterson, Ball, & Cohen, 1996). Change in practice requires strong advocacy and support for the teachers who must initiate the change. Failures of past reform efforts to change mathematics teaching in classes has been a concern of many scholars (Cohen, 1989; Cohen & Ball, 2001; McLaughlin, 1990; Tyack & Cuban, 1995). The explanations most frequently given include misrepresentation of mathematics; culturally embedded views of knowledge, teaching, and learning; social organizations of schools and teaching; curriculum materials and assessment; and teacher education and professional development as factors that hinder progress.

Teacher Beliefs

The view that knowledge is fixed, that teaching is best done through transmission, and that the teacher is the authority in the classroom runs very deep in North American culture (Ball et al., 2001). By contrast, many Asian classrooms base their instruction around open-ended tasks, with students taking a primary role in the knowledge creation (Stigler & Hiebert, 1999). The study by Wood and Sellers (1997) noted that teachers who volunteered to participate with the problem-centered program held beliefs that were more consistent with a constructivist philosophy than those teachers using textbook instruction. They were more child-centered differing in belief from the textbook teachers in both what mathematics is and what it means to do mathematics. Teachers who were textbook-centered tended to reflect a teacher-centered view in which "the authority of the textbook and/or teacher is constituted as the sole source of mathematical knowledge" (p. 183).

Sims, Correa, and Zhou (2003) investigated the beliefs US and Chinese teachers on the teaching and learning of mathematics. Their findings were:

- When asked the question "How do students best learn mathematics?" 90% of Chinese teachers stated it was by developing student interest. In contrast, US teachers claimed students learned best by practice (78%) and hands-on learning (50%).
- Chinese teachers felt that the role of prior knowledge was to enable students to connect old knowledge to new knowledge. US teachers talked about review and spiraling.
- Chinese teachers acknowledged that differing abilities exist, but attribute them to lack of interest or motivation of the student. US teachers believe that students have different abilities and different learning styles.
- Chinese teachers group according to needs - heterogeneous – with the top students leading the others. US teachers group according to needs - homogeneous - students move at own pace.

The NCTM's Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) and Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000) both advocate change to more constructivist approaches to teaching mathematics. However, studies show that even when teachers are aware of these approaches, have policies encouraging change, and believe they are teaching in a more open way, the actual classroom practice may not have made significant change. Klein (1997) discovered that even when preservice teachers were adopting the required terminology and some of the strategies of constructivist practice, they changed neither their epistemological beliefs about the nature of mathematical knowledge nor their view of how mathematics should be taught. Holloway (1999) cites a study by Battista (1999) that further contends that teachers with incorrect conceptions of how learning occurs can distort the original ideas of a curriculum's creators. Even when teachers are supplied with new curriculum materials aligned to reform approaches, teachers' knowledge and beliefs continue to shape their interpretations and uses of those materials (Collopy, 1999; Remillard, 1999a, 1999b).

Schorr, Firestone, and Monfils (2003) found that state mandated tests based on standards are also insufficient to effect change in the classroom practice. They investigated the teaching practice of fourth-grade teachers in New Jersey, a state with a fourth-grade mathematics test designed to be aligned with state and national standards. The intent of this test is to challenge conventional practice. However, there is a lack of strong pressure to produce high test scores or effective guidance on the kinds of learning opportunities that must complement those tests in order to lead to fundamental change in teaching. Through interviews and observations of 63 teachers, they found that the teachers generally reported changes in practice in a direction consistent with state and national standards. The teachers reported four changes: having students explain their thought processes; using manipulatives; emphasizing problem solving; and working on getting students to write more about mathematics. However, observed practices identified the following results:

1. Teachers incorporated specific activities without changing their basic approach to teaching. For example, the study reported that manipulatives were used extensively but, in 81% of the lessons observed, manipulatives were used in an algorithmic manner—including telling students how to use the materials or demonstrating a particular procedure in class with the materials.
2. Teachers continued to assign the same kinds of mathematical tasks they had in the past. The following chart summarizes the findings (p. 389).

Code	Coding Options	Frequency	Percent
I. Task demand	1) Memorization only	9	7
	2) Procedures without connections	84	69
	3) Procedures with connections	25	21
	4) "Doing math"	3	2
V. Task type	1) Practice of routine procedures	96	79
	2) Non-routine types of problems	25	21
VII. Knowledge needed	1) Definition/procedural	103	85
	2) Principled/conceptual knowledge	18	15

3. Classroom discourse did not foster substantive conversations among students. The study reported that in nearly 80% of all cases the teacher rarely asked students whether their answers were reasonable (missing an excellent opportunity to encourage a sense of number) or to share multiple strategies in solving a problem. Based on the data in 73% of all cases, the teacher was interested only in correct answers.

Interviews and observation in this study suggest that a test without additional policy support is likely to have minimal impact. Teachers may talk about the importance of conceptual understanding but continue to emphasize practice operations either on paper or when using manipulatives.

> "The New Jersey case suggests that tests alone are a weak policy lever for influencing instructional practice. They can raise issues and sensitize teachers to new practices, but they are not enough to get teachers to shift their basic approaches to instruction. It also suggests that without some form of external guidance, districts are unlikely to provide the kind of professional development that . . . might change instructional practice and improve student achievement" (p. 399).

Teacher beliefs also influence their assessment and grading practices. Senk, Beckmann, and Thompson (1997) examined assessment and grading practices in 19 mathematics classes in 5 high schools in 3 states were studied. In each class the most frequently used assessment tools were tests and quizzes, with these determining about 77% of students' grades. In 12 classes other forms of assessment, such as written projects or interviews with students, were also used, with performance on such instruments counting for about 7% of students' grades averaged across all 19 classes. Test items generally were low level, were stated without reference to a realistic context, involved very little reasoning, and were almost never open-ended. Most test items were either neutral or inactive with respect to technology. Written projects usually involved more complex analyses or applications than tests did. The teachers' knowledge and beliefs, as well as the content and textbook of the course, influenced the characteristics of test items and other assessment instruments.

Because the schools used in this study were sought out and since they professed to use newer forms of assessment, to use calculators and computers, represented a variety of socioeconomic opportunities, and had good opportunities for professional development, the picture painted, although grim, is likely more optimistic than that of the entire set of high school mathematics classrooms in either the United States or Canada. Teachers' emphasis on classroom tests will likely continue. Hence, improving classroom tests may be the surest way to improve the quality of assessment in high school mathematics.

Possible actions to take include:

1. Improve teachers' knowledge of alternative assessment techniques so that they become confident in applying such techniques.
2. Provide a little professional guidance on how to balance older and newer forms of assessment.
3. Ask teachers to introduce variety into their assessment practices. Perhaps, they could use one new assessment technique each semester or year. This would be manageable and yet significant enough to represent real change.

Teacher Mathematical Knowledge

The problem of changing teacher practice is compounded when teachers lack an understanding of the mathematical content. Ma (1999) characterized US elementary mathematics teachers as lacking a profound understanding of mathematics. Other research appears to confirm this (Ball et al., 2001; Mewborn, 2003). Ball further stated that "Not only must teachers know content deeply, know it conceptually, and know the connections among ideas, but also must know the representations for, and the common student difficulties with, particular ideas" (p. 449). Ma commented that unfortunately, low-quality school mathematics education and low-quality teacher knowledge of school mathematics reinforce each other. Ambrose (2002) gave pre-service elementary teachers work of two students to evaluate - one using a standard approach and one using an alternate approach. Then they were given another problem and asked which approach would be easier for the student to use and which they would prefer their own students to use. Results showed that 36% had a stereotypical preference for the standard algorithm, even if it was not the easier approach. Baroody (2002) found that both pre-service teachers as well as students had difficulties moving between various interpretations of the whole in faction problems. Kennedy (1997) suggests that if we wish to efficiently help children develop their mathematical understanding, it makes sense to help the teachers build their own understanding of these ideas and their relationships.

The Need for Professional Development

All of the above studies support the case for the importance of professional development. Changing the curriculum alone will not change classroom teaching practices. Teachers need to be supported not only with appropriate teaching materials, but adequate professional development opportunities for change to occur. Brooks and Brooks (1999) claim that constructivist educational practice cannot be realized without the classroom teacher's autonomous, ongoing, professional judgment. State education departments could and should support development of good educational practice, but too often they do not. Ma (1999) states that possible actions to crack the self-perpetuating relationships between unsatisfactory student learning and inadequate teacher knowledge include: address teacher knowledge and students' learning at the same time; enhance the interactions between teachers' study of school mathematics and how to teach it; refocus teacher preparation; understand the role that curricular materials, including textbooks, might play in reform; and understand the key to reform, i.e. whatever the form of classroom interactions might be, they must focus on substantive mathematics.

Fortunately there are studies that show substantive change can be achieved with appropriate professional development (Arcavi & Schoenfeld, 2003; Callingham & Griffin, 2001; Schoenfeld, 2003; Sykes, 1996; Wilcox & Jones, 2003). This calls for both a commitment on the part of the teachers and the inservice providers. Professional development must be seen as a continuing enterprise for teachers and an integral component of any curriculum change. Thus, the WNCP needs to consider ways to encourage and support this important step in any recommended curriculum changes.

NUMERACY & MATHEMATICAL LITERACY

Recent initiatives have focused extensively on the need to identify and develop the skills and competencies required for a person to be a mathematically literate member of society. This section will begin with a discussion of what is meant by the terms numeracy and mathematical literacy. It will then report on the type of curriculum that best supports the development of numeracy, what is needed to promote the development of mathematical literacy, and some implications for the WNCP. Finally, it will conclude with information related to mathematical learning and numeracy of two special groups of students: children entering school, and aboriginal students.

What is meant by the term numeracy?

"Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfillment." (Alberta Learning, 1996, p.2). We must keep in mind that: "Mathematics is more than numbers just as reading is more that letters. Literacy involves placing numbers into meaningful context in daily living." (Balas, 1997) "Like literacy, numeracy is not a case of one's either being proficient or not, rather individuals' skills are "situated along a continuum of different purposes and levels of accomplishment with numbers. Numeracy includes a range of skills that are necessary for initial survival in a new country and for functioning as a fully literate person." (Ciancone, 1996)

As the world of today's students rapidly changes more and more they find themselves affected by things involving technology and mathematics. In order for today's students to be prepared to succeed as productive members of society it is essential that they become knowledgeable in the many areas of mathematics. Being literate, according to The American Heritage Dictionary of the English Language and Merriam-Webster's Online Dictionary is to have knowledge or competence. From this we might assume that to be mathematically literate is to have knowledge or competence in the area of mathematics. However, this definition lacks the clarity needed to be fully useful when considering mathematics curriculum.

Just as the term literacy, defined in the Compact Oxford English Dictionary as the ability to read and write, implies the everyday use of letters in the process of communication, mathematical literacy must involve the everyday practical use of numbers. This implies that to be mathematically literate a person must be able to use numbers in everyday activities and to understand how others use them within the societal context that the individual is found. Part of the concept of mathematical literacy would appear to be numeracy, which, according to The American Heritage Dictionary of the English Language is "the ability to think and express oneself effectively in quantitative terms." The Merriam-Webster's Online Dictionary defines numeracy as "the capacity for quantitative thought and expression." These appear to be much closer to the idea of literacy as it is defined in relation to language.

The Ontario Literacy Coalition has published three bulletins in their Best Practice and Innovations series. Their bulletin on numeracy indicates that a dictionary definition of numeracy is hard to find even in Britain where the term is most commonly used instead of mathematical literacy (Ontario Literacy Coalition, 2001). One definition put forward in this document seems to support the OECD/PISA definition with respect to the application of mathematical skills and knowledge. This definition could easily cross to represent that of mathematical literacy: "Numeracy not only incorporates the individual's abilities to use and apply mathematical skills efficiently and critically, but also requires the person to be able to interpret and communicate about mathematical information and reasoning processes"

Unlike most other jurisdictions in Canada, British Columbia has used the term numeracy rather than mathematical literacy in its "Handbook for Parents" entitled "Numeracy for Secondary Students." In this handbook the definition for numeracy developed by the British Columbia Association of Mathematics Teachers (BCAMT) is adopted. The BCAMT defines numeracy as: "...the combination of mathematical knowledge, problem solving and communication skills required by all persons to function successfully within our technological world. Numeracy is more than knowing about numbers and number operations." According to Hughes and his colleagues (Hughes, Desforges, Mitchell, & Carre, 2000) numeracy is more about the ability to use and apply rather than just knowing. Another way to consider it is: "Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical, information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value (Evans, 2000)."

What is meant by the term mathematical literacy?

The two terms, mathematical literacy and numeracy are often used interchangeably according to the work done by the Mathematics Council of the Alberta Teachers' Association (2004). However, this study found that when people were asked what they thought the term numeracy meant the most frequent response was that it meant "something about number and computation." Kees Hoogland (2003), in his work based on Eva Jablonka's chapter from the Second International Handbook of Mathematics Education, identifies that the term numeracy tends to be more commonly used in the United Kingdom and Europe while the term mathematical literacy is more commonly used in North America. Based on articles and documents from countries such as Australia and South Africa the term mathematical literacy seems to be much more commonly used than the term numeracy with the term numeracy being used synonymously with mathematical literacy (Clarke, Cheeseman, Sullivan, & Clarke, 2000; Clarke & Clarke, 2002; Department of Education (South Africa), 2003; Wright, Martland, Stafford, & Stanger, 2002). Linda Ball and Kaye Stacey (Ball & Stacey, 2003) from the University of Melbourne describe mathematical literacy as being used to "describe students' capacity to use their mathematical knowledge for informed citizenship." In this way mathematical literacy is "an individual's capacity to use mathematics as a fully functioning member of a society." (Ball and Stacey)

If literacy refers to the use of language to support the performance of social activities then a literate person can not only use the language for social activities but knows the resources of the language and can use these resources for different social functions. Following this pathway would indicate that a mathematically literate person knows the resources of mathematics and can use these resources for a variety of purposes in the course of everyday 'social' and 'professional' activities.

The OECD Programme for International Student Assessment has incorporated these concepts into its definition which is stated as: "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen" (Organization for Economic Co-Operation and Development, 2003b). Before considering what or how students should learn it is essential that a clear understanding be established with respect to what it is that students should be striving for. If mathematical literacy is the ultimate goal, then consideration should be given to what students should be able to do. The OECD/PISA approach was to identify the realm of mathematical literacy as being concerned with the capacity of students to draw upon their mathematical knowledge and abilities to meet the challenges they might face both today and in the future. Thus, mathematical literacy should be concerned with "students' capacities to analyse, reason, and communicate ideas effectively by posing, formulating and solving mathematical problems in a variety of domains and situations" (Organization for Economic Co-Operation and Development, 2003a).

The National Curriculum Statement, Grades 10-12 (General), Department of Education, South Africa identifies mathematical literacy as providing learners with an awareness and understanding of the role of mathematics in the modern world. It goes on to indicate: "Mathematical Literacy, should enable the learner to become a self-managing person, a contributing worker and a participating citizen in a developing democracy. Mathematical Literacy will ensure a broadening of the education of the learner which is suited to the modern world"(Department of Education (South Africa), 2003).

Mathematical Literacy contributes to the attainment of the Critical and Developmental Outcomes in that it enables learners to:

- Use mathematical process skills to identify, pose and solve problems creatively and critically;
- Work collaboratively in teams and groups to enhance mathematical understanding;
- Organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal

concerns;

- Collect, analyse and organize quantitative data to evaluate and critique conclusions;
- Communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams;
- Use mathematical literacy in a critical and effective manner to ensure that science and technology are applied responsibly to the environment and to the health of others;
- Demonstrate that a knowledge of mathematics assists in understanding the interrelatedness of systems and how they affect each other;
- Be prepared to use a variety of individual and co-operative strategies in learning;
- Engage responsibly with quantitative arguments relating to local, national and global issues;
- Be sensitive to the aesthetic value of mathematics;
- Explore the importance of mathematical literacy for career opportunities;
- Realise that mathematical literacy contributes to entrepreneurial success.

Learners working toward Mathematical Literacy should be able to:

- Use numbers with understanding to solve real-life problems in different contexts including the social, personal and financial;
- Use mathematically-acquired skills to perform with understanding financially-related calculations involving personal, provincial and national budgets;
- Model relevant situations using suitable functions and graphical representation to solve related problems;
- Describe, represent and analyse shape and space in two dimensions and three dimensions using geometrical skills;
- Engage critically with the handling of data (statistics and chance), especially in the manner in which these are encountered in the media and in presenting arguments;
- Use computational tools competently (a scientific calculator is taken as the minimum).

The Mathematics Council of the Alberta Teachers' Association (MCATA) has identified several definitions that it feels address the issue of mathematical literacy. Along with the definition put forth by OECD/PISA it has also identified the definition of mathematical literacy as being: "An aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication abilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work" from the International Life Skills Survey"(Center for Educational Statistics of Statistics Canada, 2001; Mathematics Council of the Alberta Teachers' Association, 2004).

The MCATA has also identified several concepts that need to be considered in any definition of mathematical literacy. These ideas include:

- Connecting mathematics to the real world;
- Using mathematics appropriately in a variety of contexts;
- Communicating using the richness of the language of mathematics;
- Synthesizing, analyzing, and evaluating the mathematical thinking of others;
- Appreciating the utility and the elegance of mathematics; and,
- Understanding and being conscious of what has been learned mathematically.

In keeping with a wide range of jurisdictions across North America and around the world it would appear best to use the term mathematical literacy rather than numeracy, especially in light of the Alberta survey results, which found that most people defined numeracy as "…something about number and computation." This perception seems far too restrictive since most expectations for mathematical literacy incorporate many more dimensions than just numbers and computations.

Generating a concise but complete definition for mathematical literacy poses a monumental task. Within the definition consideration must be given to the grade level within the school system and the ultimate application of the definition. Steen (2000) identifies a range of goals for numeracy within five different dimensions. These dimensions are:

- Practical - for immediate use in the routine tasks of life;
- Civic - to understand major public policy issues;
- Professional - to provide skills necessary for employment;
- Recreational - to appreciate games, sports, lotteries;
- Cultural - as part of the tapestry of civilization.

Any definition of mathematical literacy probably needs to consider the above dimensions along with the following:

- The ability to think and express oneself effectively in quantitative terms;
- The capacity for quantitative thought and expression;
- The ability to use and apply mathematical skills efficiently and critically;
- The capacity to use their mathematical knowledge for informed citizenship and as a constructive, concerned and reflective member of society;
- The capacity to identify and understand the role that mathematics plays in the world;
- The ability to make well-founded mathematical judgements;
- The ability to engage in mathematics, in ways that meet the needs of that individual's

current and future life;

- The ability to analyse, reason, and communicate ideas effectively by posing, formulating and solving mathematical problems related to a variety of situations;
- The capacity to be able to interpret and communicate about mathematical information and reasoning processes;
- The aggregate of mathematical skills, knowledge, dispositions, habits of mind, and problem solving skills in combination with the communication abilities that people need in order to engage effectively in quantitative situations; and
- The recognition of the technological nature of our world.

Once agreement is reached among the jurisdictions that "Mathematical Literacy" will be the term used rather than "Numeracy" then agreement should be sought to arrive at a common definition for the term. The basis for a common definition for Mathematical Literacy might be the following: "Mathematical Literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to use, apply, analyse, process, interpret and communicate mathematical information, and solve problems involving that information in ways that meet the needs of the individual as a constructive, concerned, reflective and involved citizen."

What type of curriculum best supports the development of numeracy?

Once a definition has been established then consideration can be given to the type of curriculum that best supports this definition. There are a number of curriculum aspects that appear important regardless of the specific nature of the definition developed. Steen (2000) argues that "Logical thinking, analysis of evidence, and statistical reasoning are far more important for engaged citizenship in the twenty-first century, than traditional algebraic and mathematical skills. The new literacy, from this perspective, is really about reasoning more than arithmetic: assessing claims, detecting fallacies, evaluating risks, weighing evidence." It could also be about knowing and using the resources of mathematics as one would use the resources of language. The curriculum must look past the concept of arithmetic, which is more about facts and procedures.

The OECD/PISA tries to emphasize the importance of "… mathematical knowledge put to functional use in a multitude of different contexts and a variety of ways that call for reflection and insight. Of course, for such use to be possible and viable, a great deal of fundamental mathematical knowledge and skills (as often taught in schools) are needed. In the linguistic sense, reading literacy cannot be reduced to, but certainly presupposes, a wide vocabulary and a substantial knowledge of grammatical rules, phonetics, orthography, and so forth. In the same way, mathematical literacy cannot be reduced to, but presupposes, knowledge of mathematical terminology, facts, and procedures, as well as skills in performing certain operations, and carrying out certain methods" (Organization for Economic Co-Operation and Development, 2003b). This supports the notion that a mathematically literate person possesses a range of knowledge and skills but can use the resources of mathematics in order to further develop the knowledge and skills.

Mathematics is about "finding, making, and describing patterns. It is constructing mathematical models for both practical and theoretical situations, using technology when appropriate. It is representing and reasoning about quantities and shapes. It is devising and solving challenging problems and justifying and communicating about the solutions." (National Research Council, 1993). This description forms the basis for many curriculum documents from around the world.

Many sources have indicated that the application of mathematics and mathematics principles should form the basis for any curriculum adopted. Martin Hughes, Charles Desforges, Christine Mitchell and Clive Carre (Hughes et al., 2000) have indicated: "We believe that application is at the heart of numeracy. At all levels of learning mathematics pupils need to use and build on what they already know in order to progress further." Like others, they indicated that any curriculum should emphasize the process skills of decision making, communication and reasoning. These process skills all form part of the problem solving process that others have identified as a key component of any mathematics curriculum.

Public education serves many purposes but one purpose is likely "… to develop in young people the ability to acquire specialized knowledge and skills as and when they need them during the course of their working lives." (Devlin, 2000). This not only ties in with many of the proposals for the definition of mathematical literacy but also can form the basis for any curriculum that could be developed. A key is to identify the needs that young people are likely to have throughout the course of their lives. Any curriculum should support the capacity to identify and understand the role that mathematics plays in the world and the ability to engage in mathematics in ways that meet the needs of that individual's current and future life. The curriculum must extend beyond the realm of the school and the classroom to focus on the role of mathematics in employment making problem solving an important component. By solving a problem you are really transforming something you don't know into something you do know. This will foster the ability to make well-founded judgments in a variety of endeavours. To do this we must ask the question: "What kind of competencies are we looking for?"

English and Cudmore (English & Cudmore, 2000) proposed that the basic competencies or "reasoning processes" should take priority and be developed to support the general theme of mathematical literacy. These processes should include:

- Analyzing concepts and identifying relationships;
- Drawing distinctions and connections among ideas;
- Reasoning by analogy;
- Reasoning deductively and inductively;
- Thinking in diverse ways, including critically, creatively, and flexibly;

- Exploring alternatives and different possibilities;
- Taking all relevant considerations into account;
- Formulating and applying criteria;
- Constructing explanations and reasons, and distinguishing between effective and ineffective examples;
- Posing and critically evaluating mathematical problems and questions;
- Understanding and evaluating arguments;
- Anticipating, predicting, and exploring consequences;
- Constructing inferences;
- Generating and testing hypotheses;
- Acknowledging and respecting different perspectives and viewpoints; and
- Developing a commitment to the processes of inquiry and their improvement, including one's own thinking and reasoning processes.

Ball and Stacey propose that the new elements of curriculum to support mathematical literacy for solving equations numerically and graphically should include:

- Recognizing that there is a range of equation solving methods and that graphical and numerical methods are often a reasonable choice;
- Knowing techniques for efficient setting up and searching tables and lists;
- Being able to choose an appropriate viewing window for a graph;
- Appreciating that there may be multiples solutions to an equation and knowing approximately where they may be;
- Not being fooled by the pseudo-accuracy that technology sometimes gives (a solution of 2.5 exactly may be given mathematically as 2.499852751839 - twelve decimal places but all wrong.)

Given the technological nature of our society today they also propose that the aspects of mathematical literacy for using technology to solve equations should include:

- Recognising that numerical, graphical and algebraic methods are available.
- Knowing techniques for efficient setting up and searching tables and lists.
- Being able to select an appropriate viewing window for a graph.
- Appreciating that there may be multiple solutions to an equation and knowing approximately where they may be.
- Dealing with accuracy and pseudo accuracy of technology.
- Understanding the role of the most basic "equation solving" operations:

  - "Do the same to both sides " of an equation
  - Using inverse operations
  - Using the null factor law.
- Being sufficiently familiar with algebraic manipulation to be able to modify an equation before input or recognize calculator output that is in non-standard form.
- Being able to recognize the basic form of an equation (or set of equations);
- Knowing the nature of the solutions of equations of various forms.

As reported in *Early Math Strategy: The Report of the Expert Panel on Early Math in Ontario* (2003) any curriculum structure should be based on a variety of standards within two categories. Based on the work done by the NCTM the first five standards are in the mathematical content areas of number and operations, algebra, geometry, measurement, and data analysis and probability. The second five standards are in the area of process skills and involve the processes of problem solving, reasoning and proof, connections, communication, and representation. In combination, these standards describe the basic skills, knowledge, and processes that students will need to function effectively in the twenty-first century (National Council of Teachers of Mathematics, 2000).

Hughes and his colleagues (2000) proposed that any numeracy framework should be based around 5 strands: numbers and the number system, calculations, solving problems, measures, shape and space, and handling data. These strands appear to be common in many jurisdictions although the actual wording varies the overall intent seems to be there.

The current curriculum organization within the mathematics curriculum in British Columbia is fairly typical of the other jurisdictions in western and northern Canada. This curriculum organization includes:

- Problem Solving
- Number (Number Concepts and Number Operations)
- Patterns and Relations (Patterns, Variables and Equations, and Relations and Functions)
- Shape and Space (Measurement, 3-D Objects and 2-D Shapes, and Transformations)
- Statistics and Probability (Data Analysis and Chance and Uncertainty)

Ontario has built its curriculum around the following:

Number and Operations

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;

- compute fluently and make reasonable estimates.

Algebra

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Geometry

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specific locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Problem Solving

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;

- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Communication

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Denton and West (2002) looked at the curriculum from the standpoint of the first two years of school since there is strong evidence that what happens in these early years can affect the rest of a child's experience at school. They emphasized: "It is important to note that math is not simply about numbers and numerical operations. Mathematics knowledge and skills also include the ability to draw inferences, see relationships and reason logically (National Research Council 1989)." They referred to the work of Schunk and Zimmerman (1996) indicating that other skills are important at this age as well: "Skills such as paying attention and persisting at tasks are important skills in the learning process." All of this, they felt, was accomplished through the teaching style associated with the content indicated.

Their proposed content included the following:

> In mathematics, the proficiency levels are named as follows (their names reflect the most complex mathematical construct contained in the proficiency): (1) number and shape, (2) relative size, (3) ordinality and sequence, (4) add/subtract, and (5) multiply/divide. *Number and shape* refers to children's ability to recognize single-digit numbers and basic

> shapes. *Relative size* refers to children's ability to count beyond 10, recognize the sequence in basic patterns, and compare the relative size of objects. *Ordinality and sequence* means that children can recognize two-digit numbers, identify the next number in a sequence, and identify the ordinal position of an object. *Addition and subtraction* means children can perform simple addition and subtraction problems. *Multiplication and division* refers to children's ability to perform simple multiplication and division operations. The addition, subtraction, multiplication, and division items are presented in the form of word problems with picture support and in numerical statements.

The National Numeracy Framework in the United Kingdom has five strands. The first three have direct links to the National Curriculum programme of study for number. The fourth strand is linked to measures, shape and space, while the fifth incorporates handling data. Using and applying mathematics is integrated throughout. The strands, and the topics they cover, are:

Numbers and the number system

- counting
- properties of numbers and number sequences, including negative numbers;
- place value and ordering, including reading and writing numbers;
- estimating and rounding;
- fractions, decimals and percentages, and their equivalence; ratio and proportion;

Calculations

- understanding number operations and relationships
- rapid mental recall of number facts
- mental calculation, including strategies for deriving new facts from known facts
- pencil and paper methods
- using a calculator
- checking that results of calculations are reasonable

Solving problems

- making decisions: deciding which operation and method of calculation to use (mental, mental with jottings, pencil and paper, calculator...)
- reasoning about numbers or shapes and making general statements about them
- solving problems involving numbers in context: 'real life', money, measures

Measures. shape and space

- measures, including choosing units and reading scales
- properties of 2-0 and 3-0 shapes, position, direction and movement

Handling data

- collecting, presenting and interpreting numerical data

In their work "Future Basics: Developing Numerical Power" Charles and Lobato (1998) identify the basis for a curriculum for students by describing a numerically powerful child as one that:

1. Develops meaning for numbers and operations
    - connects numerals with situations from life experiences
    - knows that numbers have multiple interpretations
    - understands that number size is relative
    - connects addition, subtraction, multiplication, and division with actions arising in real-world situations
    - understands the effects of operating on numbers
    - creates appropriate representations for numbers
    - creates appropriate representations for operations
2. Looks for relationships among numbers and operations
    - decomposes or breaks apart numbers in different ways
    - knows how numbers are related to other numbers
    - understands how the operations are connected to each other
3. Understands computational strategies and uses them appropriately and efficiently
    - correctly performs the steps in an algorithm and discusses the underlying ideas and important relationships used
    - makes a conscious effort to complete calculations using prior knowledge and simpler calculations
    - often uses a variety of calculation strategies, even when completing calculations involving the same operation
    - chooses appropriate calculation techniques to obtain exact answers and to estimate
    - calculates with accuracy
    - calculates with relative efficiency

4. Makes sense of numerical and quantitative situations
    - expects numerical calculations to make sense
    - connects numbers to the quantities that the numbers are used to measure
    - relates the operations of addition, subtraction, multiplication, and division to a range of quantitative situations
    - seeks to understand relationships among quantities in real-world situations
    - relates computations to quantities in real-world situations
    - assesses whether the results of a calculation makes sense in the context of the numbers and the real-world quantities involved

The evidence would appear to support the use of the curriculum organization currently used within the CCF:

- Number (Number Concepts and Number Operations)
- Patterns and Relations (Patterns, Variables and Equations, and Relations and Functions)
- Shape and Space (Measurement, 3-D Objects and 2-D Shapes, and Transformations)
- Statistics and Probability (Data Analysis and Chance and Uncertainty)

Although the present CCF codes the learning outcomes with Mathematical Processes (Communication, Connections, Estimation and Mental Mathematics, Problem Solving, Reasoning, Technology, Visualization) that developers intend to be emphasized during instruction, it may be beneficial to make these processes more explicit by:

- Adding new curriculum strands (e.g., Problem Solving and Reasoning & Proof) to the curriculum organization. The expectation would be that teachers would integrate these learning outcomes as they teach. This has already been done in several of the WNCP jurisdictions (McAskill et al., 2004, p 11); or,
- Include explicit mathematical process learning outcomes into the four content strands as appropriate.

What is needed to promote the development of mathematical literacy?

There have been a number of suggestions identified by the various studies that can be used to help students succeed and become mathematically literate. The Ontario Report on Early Math indicates that students learn primarily through " … doing, talking, reflecting, discussing, observing, investigating, listening, and reasoning" (Copley, 2000, p.29). The study indicated that primary students learn by creating and using models that lead to being able to problem solve. Therefore students need to explore mathematics and communicate their explorations with peers and the teacher. In this way students can talk about and thus clarify their thinking and understanding of the concepts being studied.

"Meaningful mathematics instruction begins by engaging children's mathematical thinking, allowing children sufficient time to solve problems, and focusing on the use of incidental and integrated learning as well as programmed learning." (Expert Panel on Early Math in Ontario, 2003). This study also suggests that mathematics should be an integrated whole so that young students can more easily see the connections and relate the learning to real life situations. Like in almost all other areas of learning there needs to be a building process for students with new knowledge and understanding being built on prior knowledge and experience.

It is interesting to note that problem-solving played a major support role in the learning done by all students. Carpenter, Fennema, Peterson, Chiang, and Loef (1989) found that, when instruction focused on problem solving, children not only became better problem solvers but also had better mastery of computations than did children whose instruction focused solely on drill and the practice of basic facts. Children need time to practise and consolidate skills, balanced with opportunities to put those skills to use in a problem-solving context.

The National Numeracy Strategy from the Department for Education and Employment (United Kingdom) emphasizes that a teacher can help children acquire mathematical proficiency by giving a sharp focus to the relevant aspects of the programmes of study for mathematics. The outcome of this focus should be numerate pupils who are confident enough to tackle mathematical problems without going immediately to teachers or friends for help (Department for Education and Employment (United Kingdom), 1999).

The National Numeracy Strategy identified that where teaching is concerned, better numeracy standards occur when teachers:

- Structure their mathematics lessons and maintain a good pace;
- Provide daily oral and mental work to develop and secure pupils' calculation strategies and rapid recall skills;
- Devote a high proportion of lesson time to direct teaching of whole classes and groups, making judicious use of textbooks, worksheets and ICT resources to support teaching, not to replace it;
- Demonstrate, explain and illustrate mathematical ideas, making links between different topics in mathematics and between mathematics and other subjects;
- Use and give pupils access to number lines and other resources, including ICT, to model mathematical ideas and methods;
- Use and expect pupils to use correct mathematical vocabulary and notation;
- Question pupils effectively, including as many of them as possible, giving them time to think before answering, targeting individuals to take account of their attainment and needs, asking them to demonstrate and explain their methods and reasoning, and exploring reasons for any wrong answers;

- Involve pupils and maintain their interest through appropriately demanding work, including some non-routine problems that require them to think for themselves; and,
- Ensure that differentiation is manageable and centred around work common to all the pupils in a class, with targeted, positive support to help those who have difficulties with mathematics to keep up with their peers.

The approach to teaching recommended by the National Numeracy Strategy is based on four key principles:

- Dedicated mathematics lessons every day;
- Direct teaching and interactive oral work with the whole class and groups;
- Emphasis on mental calculation; and,
- Controlled differentiation, with all pupils engaged in mathematics relating to a common theme.

As part of the strategy teachers were told that the expectations for students should include:

- Having a sense of the size of a number and where it fits into the number system;
- Knowing by heart number facts such as number bonds, multiplication tables, doubles and halves;
- Using what they know by heart to figure out answers mentally;
- Calculating accurately and efficiently, both mentally and with pencil and paper, drawing on a range of calculation strategies;
- Recognising when it is appropriate to use a calculator, and be able to do so effectively;
- Making sense of number problems, including non-routine problems, and recognise the operations needed to solve them;
- explaining their methods and reasoning using correct mathematical terms;
- judging whether their answers are reasonable and have strategies for checking them where necessary;
- Suggesting suitable units for measuring, and make sensible estimates of measurements; and
- Explaining and make predictions from the numbers in graphs, diagrams, charts and tables.

The National Numeracy Strategy suggests that to ensure there is adequate time for developing numeracy skills, each class teacher should provide a daily lesson for mathematics, which should last about 45 to 60 minutes, depending on the grade level of the students.

The strategy also indicated that it is important to find time in other subjects for pupils to develop and apply their mathematical skills thus giving them the chance to practise the mathematics skills and to use them in a context. For example, a teacher could plan regular opportunities for measuring in science and design and technology, using properties of shapes and patterns in art, and collecting and presenting data in history, geography and ICT.

The strategy also states that during each lesson the teacher should aim to spend as much time as possible in direct teaching and questioning of the whole class, a group of pupils, or individuals. High-quality direct teaching is described as oral, interactive and lively. It is not achieved by adopting a simplistic formula of 'drill and practice' and lecturing the class, or by expecting pupils to teach themselves from books. It is a two-way process in which pupils are expected to play an active part by answering questions, contributing points to discussions, and explaining and demonstrating their methods to the class.

The National Strategy describes good direct teaching as achieving a balance of the following different elements:

- **Directing:** sharing your teaching objectives with the class, ensuring that pupils know what to do, and drawing attention to points over which they should take particular care, such as how a graph should be labelled, the degree of accuracy needed when making a measurement, or how work can be set out ...
- **Instructing:** giving information and structuring it well: for example, describing how to multiply a three-digit number by a two-digit number, how to interpret a graph, how to develop a mathematical argument...
- **Demonstrating:** showing, describing and modelling mathematics using appropriate resources and visual displays: for example, showing how to scribe numerals, showing how to measure using a metre stick or a protractor, demonstrating on a number line how to add on by bridging through 10, using a thermometer to demonstrate the use of negative numbers...
- **Explaining and illustrating:** giving accurate, well-paced explanations, and referring to previous work or methods: for example, explaining a method of calculation and discussing why it works, giving the meaning of a mathematical term, explaining the steps in the solution to a problem, giving examples that satisfy a general statement, illustrating how the statement 7 -3 = 4 can represent different situations...
- **Questioning and discussing:** questioning in ways which match the direction and pace of the lesson and ensure that all pupils take part (if needed, supported by

apparatus or a communication aid, or by an adult who translates, signs or uses symbols), listening carefully to pupils' responses and responding constructively in order to take forward their learning, using open and closed questions, skilfully framed, adjusted and targeted to make sure that pupils of all abilities are involved and contribute to discussions, allowing pupils time to think through answers before inviting a response...

- **Consolidating:** maximising opportunities to reinforce and develop what has been taught, through a variety of activities in class and well-focused tasks to do at home, asking pupils either with a partner or as a group to reflect on and talk through a process, inviting them to expand their ideas and reasoning, or to compare and then refine their methods and ways of recording their work, getting them to think of different ways of approaching a problem, asking them to generalise or to give examples that match a general statement...
- **Evaluating pupils' responses:** identifying mistakes, using them as positive teaching points by talking about them and any misconceptions that led to them, discussing pupils' justifications of the methods or resources they have chosen, evaluating pupils' presentations of their work to the class, giving them oral feedback on their written work...
- **Summarising:** reviewing during and towards the end of a lesson the mathematics that has been taught and what pupils have learned, identifying and correcting misunderstandings, inviting pupils to present their work and picking out key points and ideas, making links to other work in mathematics and other subjects, giving pupils an insight into the next stage of their learning...

Direct teaching and good interaction are as important in group work and paired work as they are in whole-class work but organising pupils as a 'whole class' for a significant proportion of the time helps to maximise their contact with the teacher so that every child benefits from the teaching and interaction for sustained periods.

A survey of results conducted by the Education Quality and Accountability Office (EQAO) found that the strongest positive indicators of success in mathematics were:

- Access to computers at home
- Access to scientific / graphing calculators at school
- Access to scientific / graphing calculators at home
- Dynamic geometry software
- Time spent on homework

On the other end of the scale the survey found that the strongest negative indicators of success:

- Student absence

- Student feeling unsafe at school

As part of the National Literacy Strategy developed in the United Kingdom, schools considered ways of promoting higher standards of literacy. The actions that have been taken in this area were felt to also be helpful when considering ways of promoting pupils' numeracy skills, since many of the same factors would apply. Where school management is concerned, better numeracy standards occur when:

- The administrator is well-informed, provides active leadership and sets high expectations for what can be achieved by staff and students;
- A co-ordinator for mathematics has the expertise, opportunity and support needed to influence practice;
- A desire to secure high standards through effective teaching and learning pervades the whole school;
- There are clear, realistic targets for raising standards, and a manageable plan for achieving them, with regular evaluation of the school's progress towards the targets- including effective arrangements, which take account of national standards, for assessing the progress of whole year groups and each class;
- There is systematic monitoring and self-review, under the head teacher's direction, of teachers' planning, teaching and assessment;
- There is a whole-school approach to the professional development of teachers and other staff involved in the teaching of mathematics, with emphasis on developing knowledge of the primary mathematics curriculum and appropriate teaching methods;
- Classroom assistants take part in planning and are used effectively to support teachers in mathematics lessons;
- Parents are kept well-informed and encouraged to be involved through discussions at school and sometimes in work with pupils at home; and
- Governors are involved actively in policy, monitoring and evaluation.

Taking the curriculum and assessment in the school as a whole, better numeracy standards occur when:

- Staff share a common understanding of numeracy and how best to promote it;
- There is a daily, dedicated mathematics lesson in every class, with lesson time extended through out-of-class activities and regular homework;
- The teaching programme is based on identified learning objectives, and is planned thoroughly, to ensure high expectations, consistent approaches and good progression throughout the school;
- The foundations of mental calculation and recall of number facts are established thoroughly before standard written methods are introduced;

- Assessments are used to identify pupils' strengths and difficulties, to set group and individual targets for them to achieve and to plan the next stage of work;
- Assessments include informal observations and oral questioning, regular mental tests, and half-termly planned activities designed to judge progress; and
- Recording systems give teachers the information that they need to plan and report successfully, but are not too time-consuming to maintain.

Implication of Mathematical Literacy Initiatives for the WNCP

Since it is not possible to direct the instructional strategies incorporated by teachers there should be a part of each curriculum document describing effective methods of helping students learn the necessary skills to become mathematically literate. Instructional strategies and teacher expertise play a major part in the development of mathematical literacy of students. Every possible effort should be made to help teachers develop both the knowledge and the instructional expertise to help students become effective learners of mathematics. Support needs to be given to teachers to help them develop ways of going deeper into the chosen areas of the curriculum rather than trying to cover more topics. In this way students can become more proficient in the areas with which they do work at any given grade level so that they can build on topics as they progress.

Mathematical Understanding of Children Entering School

Mathematical learning and understanding does not begin with formal school education. Children entering school already have certain mathematical skills. Many researchers have commented on this vast array of mathematical knowledge that children bring when then they begin school(Aubrey, 1993; Baroody, 2000; Bobis & Gould, 1999; Ginsburg, 2000; Hunting & Davis, 1991; Suggate, Aubrey, & Pettitt, 1997; Tang & Ginsburg, 1999). Some of these skills appear to be universal while others depend on the child's pre-school environment. These skills include strategies for carrying out arithmetical operations, basic shapes and their properties, knowledge that a ruler marked in units is used to measure lengths, patterning and tessellations, and notions of fairness and fractions (Perry & Dockett, 2002).

Although research based on the stages identified by Piaget have resulted in significant programs, materials, and approaches to early mathematics education, it also meant that children were considered incapable of using logical thought and abstraction till around age 8 (Perry & Dockett, 2002). More recent studies and evidence now suggests that children have the beginnings of such understandings in early childhood and that these understandings develop gradually over time (Schwitzgebel, 1999). Understanding what children already know and are capable of understanding is crucial both for identifying what still needs to be taught and for identifying strengths on which further instruction can be based.

The concept of cardinality of numbers appears to be universally present in all children from a very early age. Infants as young as four months of age have the ability to discriminate one object from two objects, or two objects from three objects (Starkey, Spelke, & Gelman, 1990; van Loosbrock & Smitsman, 1990). This sense of cardinality also enabled infants to intuitively understand the effect of adding or subtracting a small number of objects in a set. A study by Wynn (1992), later replicated by Simon, Hespos, and Rochat (1995) showed infants one or two objects and then placed a screen in front of them. A hand then placed another object behind the screen and the screen was removed. Sometimes the number of objects behind the screen was the expected result of adding the additional object; other times it was not. The infants looked longer when the number of objects was not the anticipated amount, suggesting they expected the correct number of objects to be present.

Even though children have this innate understanding of the cardinality of numbers up to four, they do not distinguish between larger numbers, even as low as five or six, until they are three or four years of age (Strauss & Curtis, 1984). Researchers have thus concluded that the infant abilities are due to subitizing. Subitizing is defined as the process of recognizing the cardinality of sets of objects up to three or four without consciously counting the objects. Sets larger than these induce uncertainty and often require individuals to count the objects. This subitizing of up to four objects is a skill that children entering school almost universally possess, and is a skill that remains at this ability level through to adulthood.

Proficiency in counting skills usually occurs at age three or four. Although rote counting is still common for many children at this age, most children understand the principles of rational counting by age five. These principles are:

1. The *one-one principle*: One and only one number word is assigned to each object.
2. The *stable order principle*: The numbers are always assigned in the same order.
3. The *cardinal principle*: The last count indicates the number of objects in the set.
4. The *order irrelevance principle*: The order in which objects are counted is irrelevant.
5. The *abstraction principle*: The other principles apply to any set of objects.

Even when children err in counting, they show knowledge of these principles. For instance, evidence of the *one-one principle* is shown when children assign exactly one number word to an object. Errors caused by omitting an object or counting one twice are errors of execution, rather than errors of understanding of the principle. Understanding of the *stable order principle* is evident when children use the same number words in the same order. This is usually the correct order, but even an idiosyncratic order the child uses tends to be stable. Children show understanding of the *cardinal principle* by placing special emphasis on the last number said in the counting sequence. Teachers can also check for understanding of this principle by asking the child to show them the number of objects just counted. For instance, if the child has just counted seven objects, the teacher might ask, “Can you show me 7 (objects)?” If the child only shows the last object counted, they do not yet understand the cardinal principle. If they show the entire set, they understand this principle. Understanding of the *abstraction principle* is evident when children readily count sets of different types of objects. The *order irrelevance principle* is understood when the child recognizes that counting can start with any object in the set. This appears to be the most difficult of the principles, but most 5-year olds demonstrate understanding even of this principle.

Ordinal number concepts also begin development early in childhood. The simplest concepts are more and less. Children as young as 12 to 18 months of age have a basic understanding of these concepts when comparing sets of up to four objects (Strauss & Curtis, 1984). As with cardinality, ordinal concepts beyond four objects take a few years to develop. Older children are usually asked to demonstrate understanding of ordinal relationships by answering questions such as “Which is more: eight or six?” Children entering school from middle-class or higher backgrounds can typically solve these ordinality problems correctly for numbers from 1 to 9. The greatest difficulty in choosing, which is more, occurs when two numbers are close together and relatively large (e.g. 7 and 8). Counting skills may be a factor here. The larger number is the one that comes later in the counting sequence and it is easier for children to remember which number comes later if the numbers are far apart in the counting sequence (Siegler, 2003).

The importance of counting to young children’s development of number sense and computation is well known. Many early number programs now focus on enhancement of children’s counting skills, including both forward and backwards counting, skip counting, and counting in realistic situations (Clarke et al., 2000; Clarke & Clarke, 2002; Wright, Martland, & Stafford, 2000; Wright et al., 2002). The need for facility in using composite units in base ten representation of numbers is seen as a critical aspect of this approach. This facility is within reach of children in their first years of school and has already been developed by many children entering school (Horne, Cheeseman, Clarke, Gronn, & McDonough, 2002; Jones, Langrall, Thornton, & Nisbet, 2002; Tang & Ginsburg, 1999).

Unfortunately, children from low-income backgrounds, at least in the United States, often have little or no sense of the relative magnitudes of the single-digit numbers, even on entry into first grade (Griffin, Case, & Siegler, 1994). The Early Numeracy Research Project in Australia also found wide ranges of ability in basic numeracy skills (Clarke et al., 2000; Horne et al., 2002). At the beginning of the study, childrens’ counting levels ranged from level 0 (not yet able to state the number names to 20) through level 5 (able to count by 2s, 5s, and 10s from any non-zero starting point). Overall, approximately 17% of the children in Prep through grade 2 were operating at level 0. The percentage of children entering school still at level 0 was 43.6%. This lack of understanding of relative magnitude of numbers causes difficulty for these children in understanding the basis of simple arithmetic operations and is likely related to their slow learning of the basic arithmetic facts (Jordan, Huttenlocher, & Levine, 1992).

Early intervention is particularly important to this group of children. Results from the Australian Early Numeracy Research Project showed that students who participated in the project retained gains made in the first year of school (prep), but that growth curves for all children studied were parallel after the first year (Horne et al., 2002). Thus, all groups progressed at the same growth rate after the first year. Children who enter school deficient in early numeracy skills who do not receive early intervention will continue to progress, but will likely always lag behind their peers who enter school with more mathematical skills. However, intervention studies indicate that teaching counting on in a conceptual way makes all single-digit additions accessible to first-grade children, including learning disabled students and limited-English-proficient students (Fuson, 2003; Fuson & Fuson, 1992).

Many preschool programs are designed as early intervention alternatives for low-income, minority children. Preschool programs are typically one of three types: child-initiated, academically directed, or a combination approach. A study by Marcon (1999) compared the three different approaches for their effect on children's development and mastery of basic skills at the end of preschool. Findings indicated that children whose preschool experiences had been child-initiated demonstrated greater mastery of basic skills at the end of preschool than did children in programs where academics were emphasized and skills were directly taught. At the end of preschool, children in the combination model did significantly poorer on all measures except self-help and development of social coping skills compared to children in either the child-initiated or academically directed models.

A follow-up study was conducted to examine the influence of three different preschool models on later school success (Marcon, 2002). The children were again studied in the transition from their fifth (grade 3) to sixth (grade 4) year in school. The study examined report card grades, retention rates, and special education placement of 160 children at the end of their fifth year in school and 183 children at the end of their sixth year in school. Results showed that children whose preschool experience was more academically directed had been retained less often (10%) than peers in the other two preschool models (24% and 26%). No significant effect for preschool model was found by the end of the fifth year in school (grade 3) in overall GPA or any specific subject area for either "retained" or "on-schedule" children. Also, no differences attributable to preschool model were found for special education placement. However, by the end of their sixth year in school, children whose preschool experiences had been academically directed earned significantly lower grades compared to children who had attended child-initiated preschool classes. One possible reason suggested by the study is that an academically directed approach typically emphasizes the act of reading over comprehension. Beginning in grade 4, comprehension becomes critical. Marcon concluded that "Children's later school success appears to be enhanced by more active, child-initiated learning experiences. Their long-term progress may be slowed by overly academic preschool experiences that introduce formalized learning experiences too early for most children's developmental status. Pushing children too soon may actually backfire when children move into the later elementary school … and are required to think more independently."(p. 17)

Aboriginal Students and Mathematics

If the performance of students in the public education system is a critical measure of the success of our education systems, all WNCP jurisdictions have a problem. Why, you might ask?

Consider the following:

- Numerous newspaper articles point out that issues relating to the education and health of young Aboriginals must be addressed. For example, Paula Simons (2002) wrote the following in the *Edmonton Journal*:

  > Pick any social indicator you like—teen pregnancy, alcohol and drug addiction, fetal alcohol syndrome, youth crime—and you'll find Native children top the list. Canada's Aboriginal infant mortality rate is twice the national average. The Native teen suicide rate has been estimated to be as much as 3 times higher. A study released this spring by the Manitoba College of Physicians found that Aboriginal children … up to 14 years were 7.2 times more likely to die than other children. A report on Native education, released by Alberta Learning, found that more than 30% of Aboriginals in Alberta suffer some kind of disability…

- The high school completion rate for Aboriginal students in Alberta is estimated to be a disturbing 8%.
- In October 2000, Human Resources Development Canada (HRDC) (Human Resources Development Canada, 2000) released *Dropping Out of School: Definitions and Costs* in which Quebec and the Atlantic provinces had the highest dropout rates. The report states:

  > Aboriginal youth, one of the fastest growing segments of the youth population, are burdened with virtually all the socio-economic risk factors listed in the School Learners Survey (SLS) and, as would be expected, have a high incidence of dropping out. Close to half of Aboriginal youth lived with a single parent or with neither parent in their last year of high school, and their parents are likely not to have completed high school. Four times as many Aboriginal youth have dependent children … The SLS shows that 40% of Aboriginal 18–20 year-olds dropped out compared to 16% of the Canadian population overall (p.25).

- In the January 12, 2004 edition of Maclean's, Sue Ferguson writes "… that a significant number of Alberta's First Nations students don't finish high school" (p.32).
- American students fare no better. The final report of the Indian Nations at Risk task force (INAR) (Charleston, 1994) stated that American Indians experience the poorest quality of life of all minority groups in the United States, the highest school

dropout rate, and the lowest rate of school attendance. In 1985, standardized-test scores for all schools funded by the Bureau of Indian Affairs showed a dramatic, steady decrease in grade-level performance for American Indian students as they progress through school. Although these students perform near grade level when they enter school, by the time they reach ninth grade they are two grade levels behind majority-cultural students, and by twelfth grade they are nearly four grade levels behind.

- Judith Hankes (1993), a graduate student working at the Wisconsin Center for Education Research writes, " I understand how poverty impacts on math performance. Today's children of poverty have parents, like mine, who are alienated from the system, and their alienation influences the level of math assistance and support they are capable of giving"(p.1).
- And, mathematics is one of the school subjects that Aboriginal children struggle with (Charleston, 1994; Davison, 2002; Hankes & Fast, 2002; Hilberg, Doherty, Dalton, Youpa, & Tharp, 2002; Perso, 2002; Trumbull, Nelson-Barber, & Mitchell, 2002).

Background

The past decade has seen widespread growth in research related to the learning of aboriginal students. Terms commonly used to identify these students include Aboriginal (Australia, New Zealand, and several WNCP jurisdictions) and Indian or Native American (United States). Pertinent information also uses the terms Native, First Nations, Inuit or Indigenous. For the purposes of our discussion, we will use the term Aboriginal to refer to all such students, unless referring to a study specific to a certain subgroup of aboriginal students.

Aboriginal students are participants in two cultures - the culture of the home and the culture of the school. Many of these students see little connection between these two cultures; consequently, many potentially rich situations from the native culture are lost to the school (Davison, 2002). This is particularly true in mathematics, where Aboriginal students feel alienated from the mathematics curriculum. For Aboriginal students to succeed in mathematics, they and their families must see mathematics as a valuable component of their life rather than a difficult subject. This is likely true for most students who struggle with mathematics learning

General information on Aboriginal mathematics is widespread. However, explicit mathematical examples and activities suitable for use with Aboriginal students are much more difficult to locate. Even those that do exist are often discussing a specific content topic as applied to a specific group of Aboriginal students. One useful resource synthesizing many different activities used with various North American Aboriginal students is the book Perspectives of Indigenous People of North America from the series Changing the Faces of Mathematics (Hankes & Fast, 2002). This book nicely combines information on general guidelines for teaching and interacting with Aboriginal students and examples of lessons and activities that have been used effectively with these students.

Cultural Frameworks

Mathematics education is embedded in cultural frameworks (Davison, 2002; Frigo, 2001; Hilberg et al., 2002). The predominant framework in Australia (and also in WNCP jurisdictions) is Western, and there are numerous differences between our framework and the framework in which Aboriginal mathematics is embedded. Many Aboriginal students can be considered as living in dual "mathematics worlds"—the Western mathematics of the classroom and the Aboriginal mathematics of their community or homeland. Aboriginal mathematics may include understanding of the kinship system; how to use money; local seasonal time; mental computation in the context of card games; order and pattern of events in ceremonies; traditional counting; measurements; sense of direction; and the language of relative position. It is important to recognize that intuitive ways of solving problems or pragmatic routines that Aboriginals devise to carry out computation or measurement are no less valuable than "school mathematics." A role clearly exists for the "principled" mathematics that we tend to think of as the most abstract but it should not displace the intuitive and ethnomathematical bases that Aboriginal children develop through their own experience (Trumbull et al., 2002).

Gillespie, Mercredi, and Robinson (2003) identify Aboriginal cultural concepts for First Nation, Métis and Inuit as the customs, achievements and behaviours of a people whose perspectives reflect a distinct world view. Thelma Perso (2001) expresses a belief that all educators need to know about Aboriginal cultures—aspects such as how questioning is used by Aboriginal parents, learning styles and cognitive differences in Aboriginal children, etc.—so that Aboriginal children feel comfortable, accepted and able to take risks in using Western mathematics.

Davison (2002) asserts that the use of cultural situations can improve the learning of mathematics by Aboriginal students in several ways. The use of familiar situations is one way of helping students attach meaning to the concepts of the mathematics curriculum. When mathematics ideas from the culture are used as a basis for developing academic mathematics, students value their cultural heritage more. The integration of the students' experiential mathematics with their school mathematics will help them make connections that they have not previously made.

Cultural examples work particularly well when used as part of an integrated presentation of the mathematics topic. Aboriginal learners respond well to a thematically integrated curriculum (Davison, 2002). Many examples from the native culture involve the use of games and cultural events that provide practice in simple arithmetic operations. Including such examples in the mathematics curriculum should be accompanied by investigation of the problem-solving implications. Otherwise they native culture is used only in a cosmetic way.

Implications for the WNCP Mathematics Curriculum

With this in mind, it is important for educators planning to include Aboriginal cultural concepts to seek the guidance of the local communities to fully understand and address the concepts unique to those communities. Examples can be used to bring cultural knowledge to the school to enhance and personalize the outcomes from WNCP mathematics. Where possible, when introducing a cultural topic to the classroom, educators should be encouraged to invite a member of the local community skilled in the cultural art to make the presentation. This step enhances the credibility of the cultural activity and forges a connection between the culture of the community and the culture of the school. It also helps avoid violating any cultural taboos - for example, the creation of objects that have religious significance.

Learning Styles

The preferred learning styles of Aboriginal children are heavily influenced by the cultural practices of their communities. Some of these cultural practices include (Trumbull et al., 2002):

1. Concepts are taught in the context in which they will be needed (e.g. numeration and quantification are taught in the natural setting for practical purposes).
2. Learning is by observation rather than by verbal rules or prescriptions.
3. Adults and older peers serves as models, guides, or facilitators rather than as direct instructors.
4. Children have considerable responsibility for their own learning, often working together in small groups to solve real-world problems or accomplish tasks.
5. Children have the latitude to choose when they will demonstrate their mastery of a particular task or skill, a situation that supports autonomy, self-evaluation, and perseverance until mastery is achieved.

Many of these practices appear in the preferred learning styles of Aboriginal children identified by researchers. For example, Robinson and Nichol (1998) identify six learning styles common for Aboriginal children. These include holistic learners, imaginal learners, kinesthetic learners, co-operative learners, contextual learners, and person-oriented learners. Characteristics of each category are as follows:

Holistic learners—prefer an integrated approach to learning, reflecting a view that everything is inter-related.

Imaginal learners—relay on unstructured situations involving visual images, symbols and diagrams. Such students learn best through observation and imitation rather than verbalization.

Kinesthetic learners—information is taken in through handling objects or by moving physically around them

Co-operative learners—function best in groups using a collaborative process. Co-operation is more important than competition or individual achievement for these learners.

Contextual learners—learning occurs in the specific context to which it relates. By contextualizing learning, all students discover that education is meaningful and relevant to them.

Person-oriented learners—Aboriginal cultures are more person-oriented than information-oriented. Students who feel a personal connection with the teacher will be more interested in learning, willing to take risks and attempt new tasks.

These learning styles concur with those identified by Frigo (2001) as preferred Aboriginal learning styles:

- Learning by observation;
- Learning by trial and error;
- Learning in real-life situations;
- Context-specific learning; and,
- Person-oriented instruction, where knowledge is valued because of who gives it.

Interestingly, some of the approaches of aboriginal peoples to teaching and learning coincide with some of the most highly touted elements of the research-based instruction called for in reform movements. For instance, aboriginal pedagogies that support Reform Math Pedagogy include:

- Concepts are taught in meaningful contexts and serve authentic purposes
- Adults serve as models and facilitators, guiding children to learn by observing and doing.
- Children are encouraged to take responsibility for their own learning.
- Children are encouraged to evaluate their own learning.
- Children are allowed choices about when and how to display learning (e.g. choices about being tested, portfolios, etc.).

Unfortunately, not all reform pedagogies may match the needs of aboriginal students. Even those that do match are not always implemented in our classrooms. Reform pedagogies that do not match well include (Trumbull et al., 2002):

1. The majority of learning and interaction is still based on verbal interchange. Children used to learning through observation may not be comfortable with this.
2. Aboriginal students may be less amenable to approaches calling for trial and error (discovery, inquiry approaches). They prefer to observe till they feel ready to try something themselves.

3. Children taught to respect elders' knowledge without questioning may not be comfortable questioning teachers as part of the process of developing critical thinking skills.
4. Asking students how and why they solved problems a certain way is counter to aboriginal students' culture. Questioning a person about the reasons for his or her actions is not a normal part of the culture. They may interpret it as calling for a personal defense rather than a mathematical rationale. Teachers need to develop more indirect ways of getting at students' thinking, such as eavesdropping on students' discussions or learning about the students' communities to get insight into their thinking.

Implications for the Mathematics Instruction of Aboriginal Students

The identified Aboriginal learning styles generally are not the ones rewarded in WNCP mathematics classrooms. The mismatch between our teaching and Aboriginal learning styles may inhibit effective learning. Teachers of aboriginal students must be encouraged to adapt their teaching to better match the styles of their students. The INISSS Project (Improving Numeracy for Indigenous Students in Secondary Schools) in Tasmania is an example of a project that successfully supported teachers in transforming their teaching of Aboriginal students (Callingham & Griffin, 2001). Meeting the needs of Aboriginal students requires teaching that incorporates approaches that provide students with interesting and motivating mathematics and understanding of local Aboriginal issues and culture. Many teachers of Aboriginal students are not members of that community. Therefore, extensive professional development opportunities were needed to help teachers gain the knowledge needed. The philosophy of the project was inclusive, assuming all students could learn in mainstream classes if they had the right instruction. It was based on the view that providing students with interesting and challenging problems would be more motivational and ultimately produce better results.

Professional development sessions were provided over a period of 18 months, mainly in two-day workshops, with special one-day meetings for particular purposes. Evaluation of the project took two forms. The first was videotaped data from professional development sessions and classrooms. The second was achievement of students' learning outcomes. A system of numeracy performance assessment was devised that mirrored the approach being taken to teaching and learning. Results showed:

- All tests, including the performance assessment tasks were highly reliable.
- No tests, including the performance assessment tasks, showed bias against any sub-groups.
- The performance assessments were measuring the same construct in the same way across all schools and all teachers who marked them.
- On the performance tasks indigenous students, and girls in particular, made gains that "closed the gap".

- On the conventional multiple-choice tests, comparisons with the control group showed overall statistically significant differences on both numeracy and literacy.
- On conventional multiple-choice tests, indigenous students made the same gains as the control group in numeracy but much greater gains in literacy.
- Structural equation modelling suggests a “method” effect – how the assessment was carried out makes a difference. (Callingham & Griffin, 2001, p.3)

Useful Teaching Strategies

The teaching strategies suggested by Robinson and Nichol (Robinson & Nichol, 1998) should equip teachers with a range of approaches reflecting the diverse learning needs and preferred ways of learning for Aboriginal students. However, it is important to note, that it is the teacher’s underlying beliefs that are essential in the effective teaching of mathematics.

Some successful strategies to help Aboriginal students gain mathematical understandings include:

1. Provide opportunities to learn by doing with as much community involvement and teaching by Aboriginals as possible;
2. Emphasize showing and modeling rather than explaining;
3. Use models and examples from the local environment to demonstrate concepts;
4. Use images, charts, diagrams and materials to convey mathematical information and concepts;
5. Use multimedia resources including video technology to demonstrate concepts.
6. Incorporate the manipulation of materials into mathematics lessons;
7. Use geometric shapes to encourage a concrete understanding before proceeding to written, symbolic work; and,
8. Demonstrate the meaning of mathematical vocabulary.

Frigo (2001) emphasized the reality of what happens in the classroom and what the Aboriginal student achieves. Interviews with parents, students and teachers led to the following suggestions:

- Use of activities incorporating the mathematics encountered outside the classroom;
- Displaying student work;
- Discussing beliefs about mathematics;
- Establishing problem-solving contexts;
- Working in small groups;
- Students generating questions;

- Listing mathematical vocabulary;
- Students identifying when mathematics is of use to them;
- Establishing a supportive environment where student error is not the focus of assessment;
- Supporting students self-esteem and belief in self; and,
- Recognizing cultural differences in communication and learning styles.

Researchers focusing specifically on the learning of geometry among Inuit students in Northern Quebec made the following recommendations (Pallascio, Allaire, Lafortune, Mongeau, & Laquerre, 2002):

1. Encourage the implementation of a project-based teaching approach.
2. Propose projects that require the use of geometric transpositions. This is valuable because:
   a) Spatial and geometric skills are required in numerous professional training programs;
   b) The cultural patterns of the Inuit tend to favor skills in determination and generation over skills in structuration and classification; and,
   c) Transposition skills are central to transferring skills between these two areas;
3. Let students choose their own partners. Inuit like to work in teams and have natural cultural aptitudes based on cooperation. Students whose metacognitive skills complemented one another formed teams on their own, without teacher intervention;
4. Intervene frequently for the purpose of making metacognitive activity explicit. The Inuit students in the study were receptive to the requests and encouragement of the researchers to discuss their metacognition. It is best done in an inductive fashion (e.g. through discussions of the learning objectives, the problem-solving process, the knowledge and skill involved, the corrective measures that need to be taken with respect to the objectives of a project.);
5. Propose projects that have a relationship to high school mathematics classes and training in certain professional programs; and,
6. Develop strategies of enculturation as a means to further the development of solid pre-professional training in spatial and geometric skills.

The Center for Research on Education, Diversity and Excellence (CREDE) developed five standards for maximizing the school achievement of students at risk of academic failure and two standards specific to effective education for Aboriginal students (Hilberg et al., 2002). The standards are not comprehensive. Additional data acquired over time will undoubtedly lead to additional or refined standards. However, these standards summarize many of the strategies identified by the researchers previously cited. The seven standards are:

Standard 1: Facilitate learning through joint productive activity among teachers and students.

Learning is more effective when teachers and students work jointly to solve practical, real-world problems. In such activity, academic concepts are connected to everyday concepts. This allows the creation of a common context of experience within the school itself, which is often necessary to learning when the teacher and the students are not of the same background.

Standard 2: Develop competence in the language of instruction across the curriculum.

Effective mathematics learning depends on the ability to speak "mathematically". Most teachers rely heavily on verbal analytic teaching methods. Therefore, cultural groups that do not emphasize verbal analytic problem solving are at a disadvantage in school. In addition, the way of asking and answering questions or challenging claims often used in mathematics classes are in contrast to the cultural conversation patterns of native students. Thus, many Aboriginal students develop patterns of short answers, interruptions, and silence. Children's speech often is brief, simple, infrequent, or unconventional for the classroom, leading to a school diagnosis of "low verbal ability" even for children who are highly verbal in other cultural settings.

Standard 3: Connect teaching and curriculum to students' experiences and to the skills of their home and community.

Instruction is more effective when contextualized in students' personal experiences and knowledge. This helps students make sense of instruction and to construct new knowledge accordingly.

Standard 4: Challenge students toward cognitive complexity.

Instruction must include thinking and analysis, not merely rote or repetitive drills to teach basic skills. This does not mean omitting mastery of basic facts, but it does mean that instruction must go beyond the instruction and practice of basic skills. Too often Aboriginal students are not held to the same standards as mainstream students from the majority culture. Although this is often due to benign motives, the effect is to deny many Aboriginal students the basic requirements for achievement: high academic standards and meaningful assessment that allows for necessary feedback and responsive assistance.

Standard 5: Engage students through dialogue, especially the Instructional Conversation (the means by which teachers relate formal, academic knowledge to the students' personal, family, and community knowledge.)

Standard 6: Include activities that are generated and directed by individual students or small groups.

Because of the high level of autonomy and decision-making granted to youth in Aboriginal cultures, native students are more comfortable and more motivated to participate in activities that they generate, organize, or direct themselves. Students even participate more in instructional conversations with teachers in classrooms organized into small student-directed groups.

Standard 7: Include some performance and demonstration.

This standard is based on the tradition of Aboriginal learning through observation and on the custom of allowing students to develop competence before requiring them to perform publicly. This observational learning style is closely tied to the visual learning patterns of native children and their holistic cognitive style. Inclusion of a modeling or demonstration activity even facilitates conversation, as it increases students' understanding of explanations, especially for students whose proficiency in the language of instruction is limited.

Mathematical Topics Used in Aboriginal Cultures

Different authors (Frigo, 2001; Moloney, 2003; Munro, 2003; Owens, 1999) identify somewhat different mathematical areas used in Aboriginal cultures. Most of the differences result from the background and languages of the diverse Aboriginal groups and societies. However, there are many commonalties in the mathematics used in Aboriginal civilizations and cultures.

Topics common to many Aboriginal cultures that could include mathematical knowledge are:

- Origin of counting systems
- Counting systems and their patterns
- Use for numbers
- Measurement
- Spatial thinking and symmetry
- Time
- Balance
- Seasons
- Maps
- Games
- Totems
- Food
- Head dress
- Designs
- Quilts
- Shields
- Symbols

Specific Activities Employing Aboriginal Culture

Glen Aikenhead, of the University of Saskatchewan (Aikenhead, 2000) provides a gentle warning to proceed cautiously as it is easy to misunderstand culturally embedded meanings when we do not fully share the other person's culture. Show respect for Aboriginal knowledge. Learn from the Aboriginal people and remember that gaining Aboriginal knowledge is a process of coming to knowing. Always emphasize that Aboriginal knowledge is inter-connected with many areas or fields of thought.

The following activities are intended to show different sources for Aboriginal content that might be incorporated into a WNCP mathematics revision.

1. Gillespie, Mercredi and Robinson, in work for Citizenship and Youth in Manitoba Education, (2003) have developed a theme-based curricular approach incorporating Aboriginal perspectives using selected content areas in grades 2, 5, and 10. Subject areas include ELA, mathematics, science and either social studies or physical education. The themes used are cooperation and respect (grade 2), knowing our places (grade 5) as well as water and collective decision making (grade 10).

The chart on the following page outlines the use of mathematics for the grade 10 thematic approach.

Incorporating Aboriginal Perspectives: A Theme-Based Curricular Approach Example S2 Applied Mathematics Water						
General Learning Outcome/s	Specific Learning Outcome/s	Cultural Concept/s	Instructional Strategies	Assessment	Resources	Family/ Community Involvement
Demonstrate an understanding of scale factors and their interrelationship with the dimensions of similar shapes and objects.	Prescribed Learning Outcomes E3. Determine the relationships among linear scale factors **areas, surface-areas,** and volumes o similar figures and objects. • Investigate scale factors, area, surface area, and volume of scale diagrams using sketches, and scale diagrams. E4. Interpret drawings and use the information to solve problems. • Solve problems using 2 dimensional drawings to create 3 dimensional objects	• Aboriginal people have an affinity with the circle. Their philosophy of life is represented in the circle known as the Medicine Wheel. (see *Integrating Aboriginal Perspectives Into Curricula 2003)* One of the most important stones in the Medicine Wheel is the water stone. The symbolism of the circle is an integral part of their culture and lifestyle. The belief of Mother Earth as provider and the Aboriginal Peoples close relationship with all things natural leads to circles of many forms including the sun the moon tree rings, life cycles, hydrological cycle, food chain, equality in terms of shared social responsibility and everything that is important in their lives. • Another important aspect connected to the circle is the number 4. Often the circle is divided into quadrants *to* reflect the 4 stages of life, 4 seasons, 4 directions etc.	Activating: • Have students view a BB display of Aboriginal art forms and memorials that have a polygon or circular base. Some possible suggestions are Dream Catchers, Totem poles, drums. rattles, tipis, and the Aboriginal Center on Higgins Ave and Medicine Wheels. In groups of 4 have students discuss list and record the Aboriginal symbols they have seen, what they have in common, or what they may mean. There should be a reporter in each group to report to the large group. Acquiring: • After the discussion show students the video on Medicine Wheels (permission pending) As they watch the video have students list where Medicine Wheels have been found and the symbolism and meaning of the placements of the rocks, using general directional information such as north, south, east and west. • Have students sketch their initial understanding *of* the Medicine Wheel. Applying: Provide students with a copy of. Medicine Wheel. Review the vocabulary and relationships of linear scale factors. Utilizing prior and existing knowledge on scale diagrams construct the rough layout of the wheel, placing stones to their designated positions, such as the four great stones, creator stone, seven foundation stones of life, and the spirit paths and all the other stones, The meaning of the rocks can be illustrated using a medium of their choice to create an individual work of art. Students should get the message that the Medicine Wheel is like a giant compass setting the direction for a good life. The Medicine Wheel is a place of great happiness where dancing, stories, friendships and the love of life Is celebrated Through the positioning of the rocks students will understand and incorporate the scalar qualities of Math in the Medicine Wheel. Students should see the quadrants in the wheel and calculate the placement of stones 13,14.15,16.17,18,19.20,21,22,23,24. In calculating the placement of stones 2,3,4,5,6,7.8, students should be encouraged to experiment with different calculations to reach the exact placement of the foundation stones.	• Tile diagram • Have students produced or completed a diagram using a scaled down technology? Is the scale provided in one of the following forms ration 1 cm : 100 cm, Words. 1 centimetre represents 1 meter, Fraction: scale = 1/100 • Have students calculated the number degrees of separation for the 7 foundation stones using the student worksheets to show their work. • Have students calculated the exact placements in degrees the stones on the Medicine Wheel?	• *The Smudgings and Blessings Book, Jane Alexander* • National geographic magazines to get Aboriginal artwork • Aboriginal greeting cards that reflect what the stones on the medicine wheel represent • Old Calendars that have an Aboriginal theme, which depict the symbols identified for the stones. • Senior 2 Applied Mathematics: A Foundation for Implementation (1998) • http://web.onramp.ca/rivemen/med_6.htm • Videos permission pending	• Elders and other individuals from the Aboriginal community can help students with the artwork for their stones. These same individuals would determine the difference between appropriate and inappropriate artwork. • The community must he involved in the discussion of the Medicine Wheel and it must be clear the activity is for Math purposes only. • Invite parents to assist on a field trip 10 the Aboriginal Center on Higgins Ave. or other Aboriginal buildings that Incorporate a circular designs

2. Manitoba Education, Citizenship and Youth is developing a series of mathematics activities for grades 2–8 focusing on Aboriginal perspectives. All information is found online in the Curriculum Navigator. Activities are keyed to Western mathematics outcomes as outlined in Manitoba documents. For more detailed information contact Sophia Munro, Technology Unit, at smunro@gov.mb.ca.

   Aboriginal Perspectives Support Files in the Curriculum Navigator Mathematics

   - 2-D Shapes
   - 3-D Objects
   - Beadwork/Quillwork
   - Birch Bark Biting
   - Bits of Bannock Beliefs, Trivia and Tips
   - Designs
   - Directions
   - Games
   - Moon Names
   - Northern Food Guide
   - Rosette/Medallion Beadwork
   - Star Blanket
   - Statistics
   - Stick Guessing-Counting Game: Grade 2
   - Stick Guessing-Counting Game: Grade 4
   - Sweatlodge
   - Symbols
   - Timekeeping
   - Tipi
   - Winter Counts

3. The University of Saskatchewan website provides access to Rekindling Traditions: Cross-Cultural Science and Technology Units (CCSTU, 2004, http://capes.usask.ca/ccstu/welcome.html). The project produces teaching materials for grades 6–12 that exemplify cross-cultural teaching and that engage students in both Aboriginal and Western sciences and technology.

   Lessons include:

   - Survival in our land
   - The night sky
   - Nature's hidden gifts
   - Snowshoes
   - Trapping
   - Wild rice

4. The University of Regina mathcentral database (Arnason, Maeers, McDonald, & Weston, http://mathcentral.uregina.ca/RR/database/RR.09.00/treptau1) contains a good source of Games from the Aboriginal People of North America compiled by Karen Arnason, Vi Maeers, Judith McDonald and Harley Weston. The educational value of many of these games extends beyond the skills they were intended to teach. They are significant examples of a natural learning environment with mathematical concepts coming from them. Games and resources included in the database are:

Games of Chance

- Kutepuchkunuputuk—Stick Guessing
- Ahkitaskoomnahmahtowinah—Counting Sticks
- Stick Dice
- Chekutnak—Stick Dice
- Hubbub
- "Wa'lade hama'gan"—Bowl and Dice
- Throw Sticks
- Ball & Triangle
- Stick Flipping

Games of Strategy

- Awithlaknannai
- Picaria

Lifestyle

- String Games

Other Information

- Pre-Contact Culture Areas
- Games Categorized by Mathematical Content
- Resources

5. *Nehiyaw Ma Two We Na*: *Games of the Plains Cree* is a digital collection of games produced under contract to the SchoolNet Digital Collections program, Industry Canada (Saskatchewan Indian Cultural Centre, http://collections.ic.gc.ca/games/).

6. *The Learning Circle: Classroom Activities on First Nations in Canada* is available from Indian and Northern Affairs Canada (Indian and Northern Affairs Canada, 2000,). http://www.ainc-inac.gc.ca/ks/12010_e.html. This print resource is available online and is for students ages 4 to 7, ages 8 to 11 and ages 12 to 14. Unit 5 for ages 4 to 7 is on Games.

7. *Aboriginal Resources in Mathematics* is available online from SUNTEP—Saskatchewan Urban Native Teacher Education Program, University of Regina (Arnason)http://education.uregina.ca/arnasonk/lessonplans.htm].

   Resources include:

   - Aboriginal Education;
   - Multicultural Resources;
   - Indigenous Knowledge: Annotated Bibliography & Links;
   - Children's Literature; and,
   - Lesson Plans

19. Shared Learnings: Integrating BC Aboriginal Content K-10 provides good information for teachers integrating Aboriginal topics in all subject areas (BC Ministry of Education, 1998,) http://www.bced.gov.bc.ca/abed/shared.htm. It is written to assist educators in creating greater sensitivity to the Aboriginal peoples of British Columbia.

Relationship to the WNCP Mathematics Curriculum

Many Aboriginal children struggle with WNCP mathematics. Contextualizing that mathematics and making it much more applicable to the needs of the Aboriginal population will help some students. A rethink in how mathematics is assessed and an explicit development of appropriate strategies for struggling learners to make them feel more confident in attempting mathematics test items appears to be warranted.

Assessment can be very confronting, particularly for Aboriginal children. Teachers should aim to use methods with which the students are comfortable. Suggestions are to:

- Include assessment tasks that allow students to demonstrate their knowledge visually rather than verbally;
- Use assessment that rewards teamwork;
- Provide choice for students both within and among assessment tasks and,
- Introduce self-assessment in an attempt to avoid alienating struggling students through criticism.

Finally, if the WNCP member jurisdictions wish to improve the learning situation for Aboriginal students, they should provide information related to learning styles and appropriate teaching strategies for aboriginal students. They should also be strongly encouraged to provide professional development to assist teachers in making appropriate adjustments to their teaching and assessment practices.

The WNCP mathematics revision should consider:

- Including, sometime after grade 6, pertinent workplace examples illustrating the use of mathematics outside school. This may help young Aboriginal students to stay in school thereby increasing their chances of employment. Workplace examples are likely available in all WNCP jurisdictions. Two useful resources are:
  - *Numeracy at Work*, a binder by the BC Construction Industry Skills improvement Council (BC Construction Industry Skills Improvement Council, 2002). A sample chapter is available online at http://www.skillplan.ca/numeracy_at_work.htm
  - *Essentials of Mathematics*, student textbooks for grades 10–12 include information on careers for students not intending to pursue post-secondary study in mathematics;
- Cutting the number of mathematics outcomes to allow development to proceed more in depth and at a slower pace. Teachers then might have more time for using manipulatives, alternate assessment techniques, multicultural practices, etc. which could assist Aboriginal students to be more successful in their mathematics;
- Re-balancing the skill-based numeracy outcomes arranged in strands and the function-based ones that focus on the social purpose and use of mathematics within meaningful contexts. This suggestion is made to emphasize the dichotomy between Aboriginal mathematics and Western mathematics. Aboriginal mathematics is focused on relationships rather than on quantities. Much of this could be accomplished through tighter connections within the mathematics and through a deeper embedding of problem solving within the specific outcomes; and,
- The inclusion in revised WNCP documents of appropriate activities, games, sources, etc., to depict Aboriginal culture and the use of mathematics within Aboriginal societies. If possible, references should come from Aboriginal communities within WNCP jurisdictions.

ANALYSIS OF INTERNATIONAL MATHEMATICS ASSESSMENTS

Assessment is a critical component of any educational program. Effectiveness of curricula and instruction in promoting student learning can only be determined when some type of evaluative process is undertaken. Knowing how students perform relative to peers in other jurisdictions can help determine student needs and inform decision making. This section summarizes some of the findings of several international assessments (TIMSS, TIMSS-R, PISA) and national assessments (NAEP, SAIP, NWEA) and provides an analysis to help us better understand Canadian successes. It concludes with a discussion of factors contributing to differential Mathematics achievement and a content analysis of curricular topics in top achieving countries.

Summary of results for TIMSS, TIMSS-R, and PISA

Canada has participated in a number of international assessments since 1995. These include TIMSS 1995 (Robitaille, Taylor, Brigden, & Marshall, 1998; Robitaille, Taylor, & Orpwood, 1996, 1997; Robitaille, Taylor, Orpwood, & Donn, 1998); the 1999 TIMSS-R (Robitaille & Taylor, 2000); and, the OECD's PISA 2000 (Organization for Economic Co-Operation and Development, 2003a) study. Over all, Canadian students performed quite well compared with students from other countries.

The following are general results of these assessments:

1. TIMSS 1995
    - Grade 4 (9 year old students): Canada performed at about the middle range ranking 8th among the 15 countries that met the sampling requirements. Among the five participating provinces only two were from Western Canada. Both Alberta and British Columbia statistically had results as good as those of Canada.
    - Grade 8 (13 year old students): Canada performed quite well with overall performance as good as or better than that of 30 countries and only 10 countries with results that were statistically better. Five provinces (British Columbia, Alberta, Ontario, New Brunswick and Newfoundland) over-sampled so that their results can be compared with other participating countries. British Columbia results were statistically higher than those of Canada while Alberta's were as good as those of Canada.
    - Grade 12 (students enrolled in their final year of secondary school): Of the countries participating, five countries had results significantly higher than Canada's, six had the same and nine countries had results below. Three of the four Canadian provinces (Ontario, Alberta and British Columbia) with large enough samples had results the same as Canada.

2. TIMSS-R 1999
    - Only six of the 38 countries in the study had mathematics scores significantly higher than Canada. Results for Canada and each province were significantly higher than the international average. Japan, Korea, Singapore, and Taiwan had scores significantly higher than Canada.
    - Scores for Canadian students increased significantly in both mathematics and science from 1995 to 1999. Canada was one of only two countries that showed significant improvement in both subjects. Within Canada, Ontario showed significant improvement in mathematics from 1995 to 1999.
    - Quebec students' mathematics scores placed them within the group of the top six countries and significantly higher than the Canadian average. Alberta, British Columbia and Ontario did not have scores significantly different than the Canadian average. Newfoundland scored significantly lower than the Canadian average.
3. PISA 2000
    - Thirty-two countries participated in PISA 2000. PISA 2000 sampled 15-year-old students through reading, mathematics and science assessments.
    - Overall, Canadian students performed well compared with students in most other countries, ranking sixth in mathematics. Only Japan and Korea performed significantly better than Canada in mathematics. The performance of students in Alberta and Quebec was significantly above the Canadian average. British Columbia, Manitoba, Saskatchewan and Ontario students performed at about the Canadian average in mathematics.

It is important to be cautious when making definitive statements based on the results of international studies. TIMSS was the world's largest and most complex educational research study. In some cases the TIMSS results appear to challenge conventional wisdom. For example, many Western nations with strong and well-resourced educational traditions ranked relatively poorly. Sweden, Germany, New Zealand, England, Norway, Denmark, USA and Scotland scored in the lower half for TIMSS mathematics. Of these, only New Zealand and the United Kingdom scored at Canada's level on PISA 2000. However, there were some relevant and interesting findings. Some of these include:

- Canada experienced an increase in mathematical scores between TIMSS 1995 and TIMSS 1999 and achievement for Grade 8 mathematics improved in all content areas except measurement. Canada's achievement ranking has improved over time. This may simply indicate that Canada has advanced relative to other countries (Organization for Economic Co-Operation and Development, 2003a).
- American textbooks are way too big compared to those of top-performing countries. TIMSS data shows the typical US mathematics textbook at about 700 pages whereas textbooks are no more than 200 pages in Singapore and other top performing countries. Is WNCP closer to the US or Singapore?

- Curricula are different in top-performing countries. It appears that Asian countries believe that less is more. Both American and Canadian data from TIMSS show students studying many more topics in mathematics at every grade level than their Asian counterparts. This causes some North American teachers to rush, leaving many students confused and somewhat baffled. WNCP survey data also expresses concern at the length and over-abundance of outcomes at virtually every grade level.
- Expectations in North America often neither require nor expect all students to do well in mathematics. In Asia, on the other hand, parents and teachers expect all students to do well in mathematics.
- Students in North America make much more use of calculators than students in most top-performing nations. For example, in six of the top seven scoring countries in TIMSS, 85 percent of the students didn't use calculators.
- American mathematics lessons seem to be characterized by an emphasis on procedure and skills over understanding. Is there not a similar pattern in WNCP jurisdictions?

A Comparison of NAEP, TIMSS-R, and PISA

Nohara (2001) compared the eighth-grade mathematics portions of the National Assessment of Educational Progress (NAEP 2000) with TIMSS-R (the report of the Third International Mathematics and Science Study) and with PISA (the OECD's Programme for International Student Assessment). *NAEP and TIMSS-R* are both designed to assess students' mastery of basic knowledge, concepts, and thinking skills in mathematics while *PISA* is designed to assess students' abilities to handle mathematical skills in real-world contexts. Comparisons were based on the work of an expert panel that examined items on each of the three assessments in terms of content, response type, context, multi-step reasoning, and other characteristics. The findings for each of these categories is presented below:

Content

Table 3 (next page) shows the percent and number of items that address specific mathematics strands. The most commonly addressed strand for NAEP and TIMSS-R was "Number sense, properties, and operations." The most commonly addressed strand for PISA was "Data analysis, statistics, and probability."

Table 3: Percent and number of items that address specific mathematics strands (p. 24)

Mathematics Strand	NAEP (165 items)		TIMSS-R (164 items)		PISA (32 items)	
	Percent	Number of Items	Percent	Number of Items	Percent	Number of Items
Number sense, properties, and operations	32	52	46	76	9	3
Measurement	15	24	15	24	25	8
Geometry and spatial sense	20	33	12	20	22	7
Data analysis, statistics, and probability	14	23	11	18	31	10
Algebra and functions	20	33	19	31	19	6

Context

Table 4: Percent and number of items that present students with real-life situations or scenarios as settings for the problem (p.26)

Assessment	Percent	Number of Items
NAEP	48	79
TIMSS-R	44	72
PISA	97	31

All but one PISA item was judged to present students with real-life situations. PISA items used challenging contexts that were either highly unique—not typically encountered in mathematics instruction or textbooks or required significant thought regarding how the nature of the context affects the mathematics in the problem.

Computation

Table 5: Percent and number of items that require computation (p.26)

Assessment	Percent	Number of items
NAEP	27	44
TIMSS-R	34	55
PISA	25	8

Panel members found a similar percentage of items that required non-trivial computations on all three assessments.

Multi-step Reasoning

Table 6: Percent and number of items that require multi-step reasoning (p.27)

Assessment	Percent	Number of items
NAEP	41	68
TIMSS-R	31	51
PISA	44	14

NAEP and PISA contained similar proportions of items requiring multi-step reasoning.

Interpret Figures and Charts

Table 7: Percent and number of items that require interpretation of figures (p.28)

Assessment	Percent	Number of items
NAEP	56	92
TIMSS-R	45	73
PISA	91	29

All three assessments included a large proportion of items that required the use or interpretation of figures or visual data. Panel members did find several items on PISA whose figures they judged to be significantly more complex.

Overall Difficulty

Four factors thought to make items more difficult were context, multi-step reasoning, computation, and extended response requiring students to justify their answer. Looking only at these factors, PISA was judged to be the most difficult.

A Comparison of Proficiency Levels of US Students Across States

The US federal No Child Left Behind (NCLB) Act requires each state to set proficiency levels in mathematics and reading to categorize students as proficient or not. The legislation requires each state to set its own proficiency levels but does not specify how these levels should be set or what the definition of "proficiency" should be. This raises questions concerning the consistency and comparability of results from state to state.

The Northwest Evaluation Association (NWEA) conducted a research study investigating the student proficiency standards established by 14 states (Arizona, California, Colorado, Idaho, Illinois, Indiana, Iowa, Minnesota, Montana, Oregon, South Carolina, Texas, Washington, and Wyoming) to answer the following questions (Kingsway, Olson, Cronin, Hauser, & Houser, 2003):

1. Are proficiency levels consistent and comparable among states?
2. Within each state, are proficiency levels consistent and comparable across grade levels in each subject?
3. Are proficiency levels consistent and comparable between subject areas within a state?

Students in the study completed their mandated state test and a second NWEA test. In each state, the NWEA test instruments were designed to align with the content standards of that state. Results showed significant differences in proficiency levels of students across the 14 states. While it is not surprising that the proficiency levels differ from state to state, the degree to which they differ and the potential for misinterpretation is surprising. Three general conclusions drawn from the study are:

1. Proficiency standards among states differ enough to cause dramatic differences in the percentages of students categorized as proficient even if the students have exactly the same skills. For example, the eighth grade math proficiency level varies from the 36th percentile in Montana to the 89th percentile in Wyoming using the NWEA percentiles. This means that for these neighbouring states that one could expect over twice as many students in Wyoming to be identified as below proficient as in Montana.
2. Proficiency standards within individual states differ across grades enough so that they may provide teachers with inconsistent proficiency indications for a large percentage of students. For example, the Arizona math proficiency level is set at the 46th percentile in grade 3 and at the 75th percentile in grade 8. As a result, a large percentage of students in grade 3 will be identified as proficient and those same students may eventually be categorized as below proficient in grade 8.
3. Proficiency standards between subject areas within and across states differ enough that they may provide schools with inconsistent information when comparing proficiency of students in reading to proficiency in mathematics. For example, the Washington fourth-grade proficiency level in reading is at the 53rd percentile, while the level in mathematics is at the 76th percentile. This will cause more students to be identified as proficient in reading than in mathematics.

Analysis of National and International Mathematics Assessment Data to Better Understand Canadian Successes

It is evident from interprovincial comparisons that some jurisdictions have been better than others at securing and maintaining improvements in student achievement—at least on national and international mathematics assessments. Students from two jurisdictions, notably Quebec and Alberta, have consistently outperformed those from other jurisdictions. This analysis will look at the results and possible reasons for the success of top-achieving Canadian jurisdictions.

Let's begin the discussion by reviewing some of the data (Tables 8 & 9 starting on the next page) from *Exploring Quebec's and British Columbia's Student Achievement on Large-Scale Mathematics Assessments* by Raptis and McAskill (2003).

Table 8: SAIP Mathematics 1997 and 2001

13 year olds		**16 year olds**	
Content 1997			
Above BC	Alberta, Quebec, Yukon	Above BC	Alberta, Canada, Quebec
In BC's range	BC, Canada, Manitoba, PEI, New Brunswick, Newfoundland and Labrador, Nova Scotia	In BC's range	BC, Manitoba, Nova Scotia, Ontario, PEI, Saskatchewan, Yukon
Below BC	NWT, Ontario, Saskatchewan	Below BC	New Brunswick, Newfoundland and Labrador, NWT
Problem Solving 1997			
Above BC	Alberta, Canada, Quebec	Above BC	Alberta, Canada, Manitoba, Quebec, Saskatchewan
In BC's range	BC, Manitoba, New Brunswick, Newfoundland and Labrador, Nova Scotia, Ontario, PEI, Saskatchewan	In BC's range	BC, New Brunswick, Newfoundland and Labrador, Nova Scotia, Ontario, PEI, Yukon
Below BC	NWT, Yukon	Below BC	NWT
Content 2001			
Above BC	Alberta, Quebec	Above BC	Alberta
In BC's range	BC, Canada, Manitoba, Ontario, Newfoundland and Labrador, Yukon	In BC's range	BC, Canada, Manitoba, New Brunswick, Nova Scotia, Ontario, PEI, Saskatchewan, Yukon
Below BC	New Brunswick, Nova Scotia, Nunavut, NWT, PEI, Saskatchewan	Below BC	Newfoundland and Labrador, Nunavut, NWT

13 year olds		16 year olds	
Problem Solving 2001			
Above BC	Alberta, Canada, Quebec	Above BC	Alberta
In BC's range	BC, Manitoba, New Brunswick, Newfoundland and Labrador, Ontario, Saskatchewan, Yukon	In BC's range	BC, Canada, Manitoba, New Brunswick, Nova Scotia, Ontario, PEI, Saskatchewan
Below BC	Nova Scotia, Nunavut, NWT, PEI	Below BC	Newfoundland and Labrador, Nunavut, NWT, Yukon

Table 9: International Mathematics Assessments Results Comparison With BC

	Jurisdictions Above/Below/Same as BC
Third International Mathematics and Science Study (TIMSS) 1999 **Grade 8 Students**	
Above BC	Belgium, Hong Kong, Japan, Korea, Quebec, Singapore, Taiwan
In BC's range	Alberta, Australia, BC, Bulgaria, Canada, Czech Republic, Finland, Hungary, Latvia, Malaysia, Netherlands, Newfoundland, Ontario, Russia, Slovak Republic, Slovenia
Below BC	Chile, Cyprus, England, Indonesia, Iran, Israel, Italy, Jordan, Lithuania, Macedonia, Moldova, Morocco, New Zealand, Philippines, Romania, South Africa, Thailand, Tunisia, Turkey, United States
Programme for International Student Assessment (PISA) 2000 **15-yr-old Students**	
Above BC	Alberta, Japan, Korea, Quebec
In BC's range	Australia, BC, Canada—as a whole, Finland, Manitoba, New Zealand, Ontario, Saskatchewan, Switzerland, United Kingdom
Below BC	Austria, Belgium, Brazil, Czech Republic, Denmark, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Liechtenstein, Luxembourg, Mexico, New Brunswick, Newfoundland, Norway, Nova Scotia, Poland, Portugal, Prince Edward Island, Russian Federation, Spain, Sweden, United States

Raptis and McAskill (2003), after considerable analysis, identify four systemic features that appear to be significant in beginning to explain Quebec's student achievement compared to British Columbia's. These are:

1. Teacher qualifications

The 1997 BC Task Force on Mathematics reported that 19% of grade 4 teachers and 21% of grade 7 teachers had no post-secondary mathematics courses. Similar data from Quebec showed few educators teaching mathematics with no post-secondary mathematics credits.

2. Teaching load

Teachers in Quebec fall at the lower end of the workload scale, averaging 18 hours per week at the elementary level and 15 hours per week at the secondary level. Teachers in BC average between 23 and 25 contact hours per week.

3. Streaming and tracking

Teachers in Quebec report no streaming of students prior to grade 10; whereas in BC streaming occurs as early as grade 4 even though this is not Ministry supported. In grade 10, students in both Quebec and BC are tracked into different programs. However, in Quebec students in different tracks cover essentially the same topics to varying depths.

4. Curricula rooted in different social, historical and philosophical foundations

In Quebec, curriculum design resides in the hands of a few highly skilled authorities who oversee the curriculum in its entirety. Quebec's curricula are narrower and deeper in scope than BC's. Quebec moves constantly between concrete and abstract mathematics while BC outcomes tend to emphasize either purely concrete operations or those that are decidedly abstract.

In examining the previous data it is important to note that another Canadian jurisdiction; namely, Alberta scored statistically better than British Columbia in all but one case. One would expect to find greater similarity between BC and Alberta teacher data than between BC and Quebec. What was found for each of the systemic features identified by Raptis and McAskill?

1. Teacher qualifications

Requirements for teacher certification in Alberta and British Columbia are very similar.

2. Teaching load

According to the Alberta Teachers' Association workload study the average instructional time in Alberta is 1220 minutes or 20.3 hours per week. This places Alberta teacher workloads midway between those of BC and Quebec. However, it is clear that Alberta teachers work well beyond the instructional time at lesson planning, marking, report card writing, supervision, extracurricular activities, etc.

3. Streaming and teaching

Alberta is acknowledged in Canada as a leader in school autonomy. Schools in Alberta must deliver the curriculum. Otherwise, they have a great deal of freedom within the block funding. Over the years, it has not been unusual to find Alberta schools exploring school-year organization as well as tracking/ streaming.

4. Curricula

Mathematics curricula in both Alberta and British Columbia are based on WNCP outcomes so are relatively similar.

The four features identified by Raptis and McAskill (2003) don't appear to explain the significant differences in student mathematics achievement between Alberta and British Columbia. This in no way diminishes the conclusions reached by Raptis and McAskill but rather demonstrates the complexity among factors contributing to student achievement results.

Factors Contributing to Differential Mathematics Achievement

Raptis and Fleming (2003) note eight characteristics widely identified in the research as factors that positively influence student achievement. They include: a focus on student achievement, effective classroom instruction, a shared vision about educational purpose among school staffs, an orderly and secure climate for learning, strong leadership, a linkage between assessment and curricular practices, high standards and expectations for students, and supportive home-school links. Other researchers identify similar characteristics affecting student achievement.

1. Focus on Student Achievement

The most frequently cited characteristic of effective education in current research is a focus on student achievement of both school and classroom levels. Barth et al (1999) report that increasing instructional time in reading and mathematics helps students meet standards that, in turn, raise achievement. Henchey et al (2001) compared effective schools in British Columbia, Alberta, and Quebec and found 14 elements of success including a focus on academic achievement. Several authors have noted that those schools that are most effective, place academic learning at their centre.

2. Effective Classroom Instruction

Research shows that variability between classes in a school is far greater than variability among schools. Australian researchers estimate that school-level factors account for 5 to 10 percent of variance in student outcomes, while classroom-level effects account for 40 to 55 percent (Raptis & Fleming, 2003, p. 9). Both Barth et al (1999) and Henchey et al (2001) report that top-performing high-poverty schools direct a larger proportion of funds toward teachers' professional development.

3. A Shared Vision About Educational Purpose

Canadian researchers Hajnel, Walker, and Sackney (1998) report the importance of a shared vision and a caring climate as two significant school-level factors. Reviews in 1995 by both Cotton and Sammons et al indicate "it is unproductive and confusing for students to study the same things over two different years with two different teachers. Schools that adhere to well-delineated curricular programs avoid repetition and allow successive teachers to deal with new subject content in ways that motivate and challenge learning" (Raptis & Fleming, 2003, p.10)

4. Linkage Between Assessment and Curricular Practices

School effectiveness researchers reported in the 1990s that effective schools assessed student learning and, then, used the results to inform school planning and administration. Although most education systems collect data on student achievement at various grade levels, these data are rarely analyzed to ensure alignment of curriculum, instruction, and assessment (Raptis & Fleming, 2003).

5. High Standards and Expectations for Students

Corbett, Wilson and Williams (2002) clearly state that "what distinguishes classrooms and schools where the achievement gap has been significantly reduced from those where it persists are educators who assert that all children can succeed in school, and it is our job to make sure they do" (p.156).

6. Supportive Home-School Links

Research has shown, in the case of minority learners, or uninterested parents, that attempts by schools to build home-school links are not always useful. In fact, it may be more productive to focus on home-school communication. PISA 2000 findings indicate that parental involvement correlates negatively with student achievement in every Canadian jurisdiction and all countries except Japan (OECD, 2000, table 3.7).

Relationship to WNCP

Of the eight factors contributing to differential mathematics achievement (Raptis & Fleming, 2003) there appears to be significance in some of them when trying to explain Alberta's success on national and international mathematics assessments over British Columbia and, for that matter, other WNCP jurisdictions. These are: focus on student achievement, high standards and expectations for students, and strong linkage between assessment and curricular practices.

Scholastic achievement is important in Alberta as evidenced by parental concern, educators focused on instructional issues, maximizing time for learning (Alberta Learning identifies 125+ hours of instruction for secondary mathematics, whereas BC identifies 100 hours per course), etc.

Alberta tests students in grades 3, 6, and 9 mathematics regularly. Henchey et al (2001) identified the best schools as those preparing their students well to write tests and examinations. It is likely significant that a unifying force in Alberta classrooms is imposed by external testing and the use of those events to strengthen collaboration between teachers and students in preparing for a common enemy—external testing from the Province.

Alberta, provincially and throughout the system, appears to use large-scale provincial, national, and international assessments in mathematics better than other WNCP jurisdictions. Results are used in instructional planning and the preparation of extensive support resources to promote student achievement.

Implications of National and International Assessments for the WNCP

It is important for WNCP jurisdictions to continue to do well on national and international assessments and, when jurisdictions do well, it is equally important to give credit to teachers and students for the success. This encourages teachers to use items from other countries, provinces or territories.

Ministry personnel in a number of WNCP jurisdictions do not have many opportunities to work with teachers. Therefore, a variety of worthwhile sample items from international assessments and from other countries should be included in revised WNCP mathematics documents.

In addition, WNCP mathematics revision should consider the following:

- Include some examples from Alberta Learning's testing for grades 3, 6, and 9 mathematics. Let's all learn from Alberta's success.
- Attempt to determine whether results for at-risk students, particularly those of Aboriginal ancestry, are better or worse in mathematics in Alberta as compared to other WNCP jurisdictions.

Analysis of National and International Assessments: Content and Results

Based on the *Trends in International Mathematics and Science Study* (TIMSS) data for grades 4 and 8, Canadian students performed at about the middle range in mathematics achievement in grade 4 [Canada ranked 13th out of 26 countries (Robitaille et al., 1997)] and somewhat better in grade 8 [Canada ranked 16th among 41 countries (Robitaille et al., 1996)]. At both the grade 4 and grade 8 levels a few countries had statistically higher overall performances than Canada. These countries include Singapore, Korea, Japan, Hong Kong, Austria, Slovenia, and the Czech Republic.

TIMSS was guided by 4 questions:

1. What are students expected to learn?
2. Who delivers the instruction?
3. How is instruction organized?
4. What have students learned?

The "intended curriculum" was determined from curriculum guides, textbooks, and questionnaires administered to subject experts.

Role of Mathematics Curriculum Guides

Many differences existed among various countries' curriculum guides. In virtually all cases, curriculum guides seem to have been carefully targeted documents by which curriculum designers communicated their intentions to those shaping mathematics curriculum and to those providing instruction. These guides all carried some official status, although the authority varied greatly. If we consider those countries (Singapore, Korea, Japan, Hong Kong, Austria, Slovenia, and Czech Republic) with significantly higher overall performance than Canada at both the grade 4 and the grade 8 levels, several points are worth noting:

1. Singapore's documents stressed pedagogy.
2. Documents from Austria, Czech Republic, Japan, and Hong Kong took a prescriptive approach emphasizing policy or content.
3. Korea seemed to balance objectives, content, and pedagogy.

Overall, mathematics curriculum guides specified mathematical content with great frequency. More often this information identified mathematical topics for study rather than detailed statements outlining student performance expectations. Slovenia, Korea, and Hong Kong identified content and performance expectations most specifically. Japan identified content most explicitly with only general statements on performance.

The Intended Flow of Mathematics Curricula

The flow of mathematical topics including time of introduction, duration, and departure varies widely among countries. TIMSS data for this analysis came primarily from panels of national experts. The following chart generally shows mathematics topics introduced within specific grade ranges.

Table 10: Mathematics Topics Introduced Within Specific Grade Ranges

Grade Group	Topics That Half the Countries Intended for Introduction in Grade Group	
1 to 3	• Whole Number: Meaning • Whole Number: Operations • Whole Number: Properties of Operation • Common Fractions • Estimating Quantity & Size • Estimating Computations	• Measurement: Units • Measurement: Perimeter, Area & Volume • 2-D Geometry: Basics • 2-D Geometry: Polygons & Circles • Data Representation & Analysis
4 to 6	• Decimal Fractions • Relationships of Common & Decimal Fractions • Percentages • Properties of Common & Decimal • Fractions • Negative Numbers, Integers & Their Properties • Number Theory • Rounding & Significant Figures	• 2-D Geometry: Coordinate Geometry • 3-D Geometry • Geometry: Transformations • Constructions using Straightedge & Compass • Proportionality Concepts • Proportionality Problems • Equations & Formulas
7 to 8	• Rational Numbers & Their Properties • Real Numbers, Their Subsets & Their Properties • Exponents, Roots & Radicals • Exponents & Orders of Magnitude	• Measurement: Estimation & Errors • Geometry: Congruence & Similarity • Proportionality: Slope & Trigonometry • Patterns, Relations & Functions
9 to 12	• Complex Numbers & Their Properties • Counting • Vectors • Uncertainty & Probability	• Infinite Processes • Change • Validation & Justification • Structuring & Abstracting

The patterns for when topics are introduced by countries vary greatly. In the analysis, the median grade was determined for each topic area and each country was compared to that median. Some countries introduced topics up to eight years before the median and others up to 10 years after the median grade. For example, Canada introduces "uncertainty and probability" eight years before the median of grade 9, while Singapore introduces "equations and formulas" three years after the median of grade 4.

The following table shows the number of topics introduced at least three years earlier or three years later than the median grade for top performing countries.

Table 11: Number of Topics Introduced ±3 Years from the Median Grade

Country	Number 3 Years Earlier	Number 3 Years Later
Austria	0	0
Czech Republic	4	0
Hong Kong	2	9
Japan	5	0
Korea	5	0
Singapore	2	3
Slovenia	7	2
Canada	8	1

Duration of Topic Coverage

Next, we look at variations in how many grades different mathematics topics were intended to be included in the curriculum. Duration data were derived by determining the number of grades that curricular attention was given to each topic within each country. The mean duration for each topic (across countries) and the mean duration for each country (across topics) were then calculated. After the mean country durations were determined, the median country duration was found for all the countries. Then, the difference from that median was calculated for each country. Sample data is found in the following chart:

Table 12: Difference in Duration of Topics in the Curriculum from the Median International Duration

Country	Number of Grades Average Duration Is From the Median
China	–3.2
Hong Kong	–1.8
Japan	–1.2
Korea	–1.1
Czech Republic	–0.8
Singapore	0.8
Slovenia	1.0
Canada	2.7
Switzerland	3.3

Interpreting the data - China had an average duration far less than the median. This means they devoted fewer years than the median to covering a topic. Canada, on the other hand, with the second largest average duration spends longer than the median covering a topic.

Covering Multiple Topics

Generally, curricular attention proceeds from a few topics in early grades, to an increasing diversity in the middle grades and returns to a more focused approach toward the end of the secondary grades. The following chart identifies the number of topics to be covered for each grade in selected countries.

Table 13: Number of Topics to be Covered for Each Grade in Selected Countries

Grade	Canada	Czech Republic	Hong Kong	Japan	Korea	Singapore	Slovenia
1	17	3	5	8	8	5	7
2	17	12	5	10	11	6	10
3	18	13	5	18	15	6	15
4	22	14	9	20	17	14	21
5	24	21	11	26	22	19	23
6	28	21	11	21	22	20	28
7	33	17	18	14	26	30	36
8	37	19	17	19	18	22	37

Generally, the smaller the number in the chart the more focused the mathematics curriculum. Singapore is a country that gradually increases its diversity of math topics. Canada, on the other hand, is an example of a country that begins and continues with significant diversity or breadth. New Zealand is similar to Canada. Japan decreases its diversity after grade 5.

Although not evident from the chart, several countries introduced and dropped topics frequently. The overall number of topics did not increase as much as in other countries. These countries did not simply have a focused mathematics curriculum, they continually changed focus. Examples of such countries were Korea, Japan, and the Czech Republic.

Sequencing of Curricular Topics

William Schmidt, in an article entitled “A Vision for Mathematics” in the February 2004 edition of *Educational Leadership* writes: “The most striking feature of the school experiences of students in most …countries whose test performance is very high, is that of a common, coherent, and challenging curriculum through 8th grade (Schmidt, 2004). He further indicates that differences in mathematics achievement among countries are clearly related to those countries’ different curriculum (Schmidt et al., 2001).

A mathematics curriculum should identify a progression of topics and outcomes that build on those covered in previous years. And, it follows that those topics and outcomes should not be addressed over and over and over. One version of such a progression of mathematics topics for grades 1–8 is found in Table 14: The Sequence of Mathematics Topics in Top-Achieving Countries (Schmidt, 2004).

Legend

X represents the grade-level content expectations in a majority of the top-achieving countries in TIMSS.

√ represents the placement of similar content in the June 1995 WNCP document - "The Common Curriculum Framework for K–12 Mathematics."

Table 14 The Sequence of Mathematics Content in Top-Achieving Countries

	Grade Level								
Topic	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Whole Number Meaning	X √	X √	X √	X √	X √	√			
Whole Number Operations	X √	X √	X √	X √	X √	√			
Measurement Units	X √	X √	X √	X √	X √	X √	X √	√	
Common Fractions	√	√	X √	X √	X √	X √			
Equations and Formulas			X	X	X	X √	X √	X √	√
Data Representation & Analysis	√	√	X √	X √	X √	X √	√	X √	√
2-D Geometry: Basics	√	√	X √	X √	X √	X √	X √	X √	
Polygons & Circles	√	√	√	X √	X √	X √	X √	X √	
Perimeter, Area, & Volume	√	√	√	X √	X √	X √	X √	X √	√
Rounding & Significant Figures		√	√	X √	X	Rounding √			
Estimating Computations				X	X √	X √	√	√	
Properties of Whole Number Operations				X	X				
Estimating Quantity & Size	√	√	√	X √	X √	√	In example	√	
Relationship of Common & Decimal Fractions				X √	X √	X	√		
Properties of Common & Decimal Fractions				√	X √	X √	√	√	

	Grade Level								
Topic	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Percentages					X	X √	√	√	
Proportionality Concepts					X	X	X √	X √	
Proportionality Problems					X	X	X √	X √	√
2-D Coordinate Geometry				√	X √	X √	X √	X	√
Geometry: Transformations					√	X √	X √	X	√
Negative Numbers, Integers, & Their Properties						X √	X √		
Number Theory					√	√	X	X	
Exponents, Roots, & Radicals							X	X √	√
Exponents & Orders of Magnitude							X √	X √	√
Measurement Estimation & Errors	Estimate √	Estimate √	Estimate √	Estimate √	Estimate √	Estimate √	X Angles √	Estimate √	
Constructions w/Straightedge & Compass							X	X	
3-D Geometry	√	√	√	√	√	√	X	X √	√
Congruence & Similarity							Intro Congrue nce √	X	√
Rational Numbers & Their Properties								X √	√
Patterns, Relations, & Functions	√	√	√	√	√	√	X √	√	√
Slope & Trigonometry								X	√

Relationship to WNCP

Studies such as TIMSS show that delineating a clear curriculum does make a difference in terms of what students learn (Schmidt et al., 2001) and that national differences in student mathematics achievement correlate with differences in curricular coverage.

Current Western and Northern Canadian curricula are based on the outcomes outlined in the 1995 edition *of The Common Curriculum Framework for K–12 Mathematics*. From an examination of Table 14 it is evident that WNCP mathematics content often spans grades on either side of those from TIMSS top-achieving countries. In other words, WNCP content has greater breadth, and consequently less depth at a selected grade level than that of TIMSS top-achieving countries. Also, WNCP mathematics deals with more content areas at any given grade level.

Implications for the WNCP

It is critical to remember that international results must be interpreted with caution. TIMSS can be used to get a broad sense of the kinds of relationships that exist but the WNCP needs to move purposefully and cautiously in its use of data from other regions.

It is likely that a curriculum with many topics devotes less time in a given year to each. Multiple topics imply shifts in attention among topics. This lessens the intensity of instruction and may confuse some students, particularly if they do not make explicit connections from one topic to the next. It is worrisome that teachers may devote little sustained attention to critical aspects of the mathematics they are teaching simply because there is so much to cover.

The number of topics introduced in mathematics classrooms has implications for how teachers present the material. In an attempt to cover a wide range of topics, teachers tend to lecture rather than to allow time for student groups to engage in the mathematics. Such a practice, leads to the results previously described in Boaler's study (Boaler, 1998, 2002) where students develop a procedural knowledge that they find difficult to use in anything other than typical textbook exercises.

Some suggestions for future WNCP revisions in mathematics include:

1. Our curriculum lacks focus. The breadth of topics prevents them from being treated with real depth at any particular grade level. There is a need to focus upon fewer topics and to deal with them in greater depth thereby decreasing the duration of topic coverage. What is learned in grade 4, for example, must be applied using worthwhile problems and contexts in future grades in lieu of explicit lengthy review. This is consistent with the findings of the BC provincial survey on high school mathematics curriculum. Post-secondary institutions in the province were surveyed to determine whether topics in the BC mathematics curriculum required mastery, exposure only, or were not applicable to various post-secondary programs. Findings suggest that up to 30% of the secondary curriculum is not needed for entry into any post-secondary program, including those that are calculus-based (McAskill, 2004). It appears that topics

could be eliminated with no detrimental impact on student entry into post-secondary programs.

2. Our curriculum needs more coherence among topics. The way in which topics are included and developed in the curriculum should reflect essential mathematics and not a list of unrelated ideas borrowed from member jurisdictions.

SUMMARY OF WNCP MEMBER SURVEY RESULTS

As part of this project Hold Fast surveyed the WNCP member jurisdictions to determine the degree of implementation of the present version of the K to 12 Mathematics Framework.

Each member jurisdiction was asked to complete the survey in order to gather information to assist in the formation of an overall picture of the adoption, use, and perceived success of *The Common Curriculum Framework for K-12 Mathematics*. The survey included questions related to the following topics:

- Use of the CCF in Jurisdictional Development of Curriculum Documents;
- Implementation of Jurisdictional Curriculum Documents;
- Instructional Time;
- Jurisdictional Large-scale Assessment of Student Achievement;
- Secondary Course Structure & Enrolment;
- High School Completion Rates;
- Kindergarten;
- Estimates of Stakeholder Satisfaction;
- Sources of Evidence Used in Responding to This Survey; and,
- Jurisdictional Recommendations

The WNCP member survey was distributed to jurisdictional representatives on October 14, 2003. The response deadline was initially December 19, 2003. A preliminary report was submitted to the WNCP Project Steering Committee on January 14, 2004 and members of the Committee were asked to review the preliminary draft and report any discrepancies to Hold Fast. The response deadline was extended to March 19, 2004 to allow jurisdictions time to review the preliminary results and provide additional information as required.

This section of the report includes the specific survey questions along with a summary of jurisdictional responses.

Context

In order to establish each jurisdiction's context when responding to the survey, member representatives were asked to identify what grade level(s) of the mathematics curriculum their jurisdictions' were presently focusing on, or planning to focus on, over the next three years. Four out of six responding WNCP member jurisdictions (representing approximately 66% of the student population) are focusing their efforts on elementary mathematics education over the next three years. Although the same proportion of jurisdictions (4/6) are also focusing their efforts at the junior secondary level, these jurisdictions represent 59% of the student population. The area that the fewest number of jurisdictions (2/6), representing only 25% of the student population of Western Canada, reported focusing on is senior secondary. Table 15 summarizes the WNCP jurisdictional responses as a percentage of jurisdictions and as a percentage of total student population affected.

Table 15: Jurisdictional Focus on Mathematics

Grade Grouping	Percentage of Jurisdictions	Percentage of Population
Primary (K to 3)	67	66
Intermediate (4 – 6)	67	66
Junior Secondary (7 – 9)	67	59
Senior Secondary (10 – 12)	33	25

Use of the CCF in Jurisdictional Development of Curriculum Documents

1. How much of the CCF (General & Specific Outcomes, Illustrative Examples) has been incorporated into the documents created for use in your jurisdiction?

Grades 8 and 9 were the only levels where not all jurisdictions indicated that they had incorporated at least 81% of the General and Specific Outcomes of the CCF into their curriculum documents. Contrasting this was the lack of use of the corresponding Illustrative Examples (IEs) by a significant proportion of jurisdictions. On average, WNCP jurisdictions used 50% to 80% of the IEs as part of their curriculum documents. Figures 1 and 2 summarize the use of the three components of the CCF

Figure 1: Average
Jurisdictional Implementation of WNCP Mathematics Framework

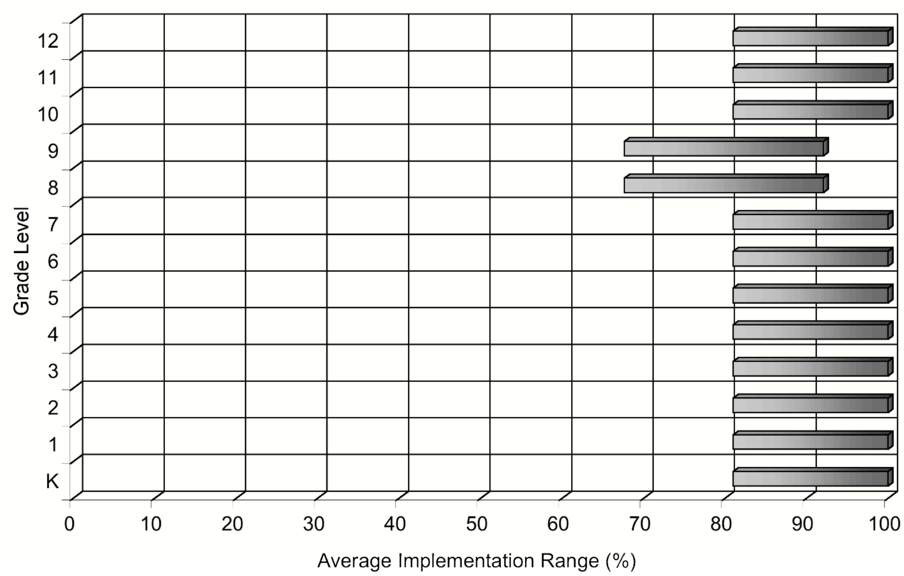
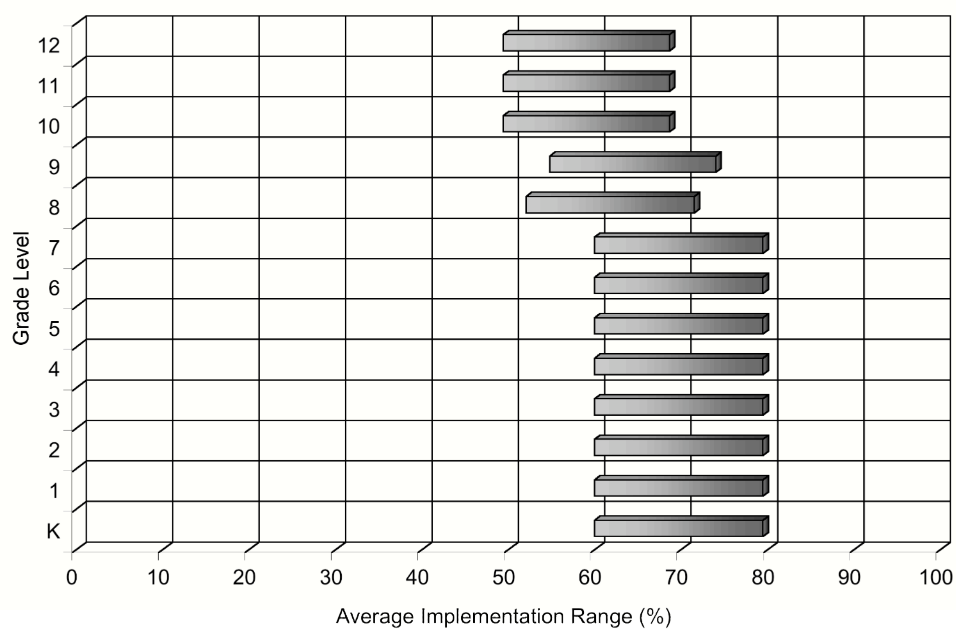


Figure 2: Average Jurisdictional Use of WNCP Illustrative Examples



2. Describe, in general terms, how the CCF has been modified for use within your jurisdiction's documents.

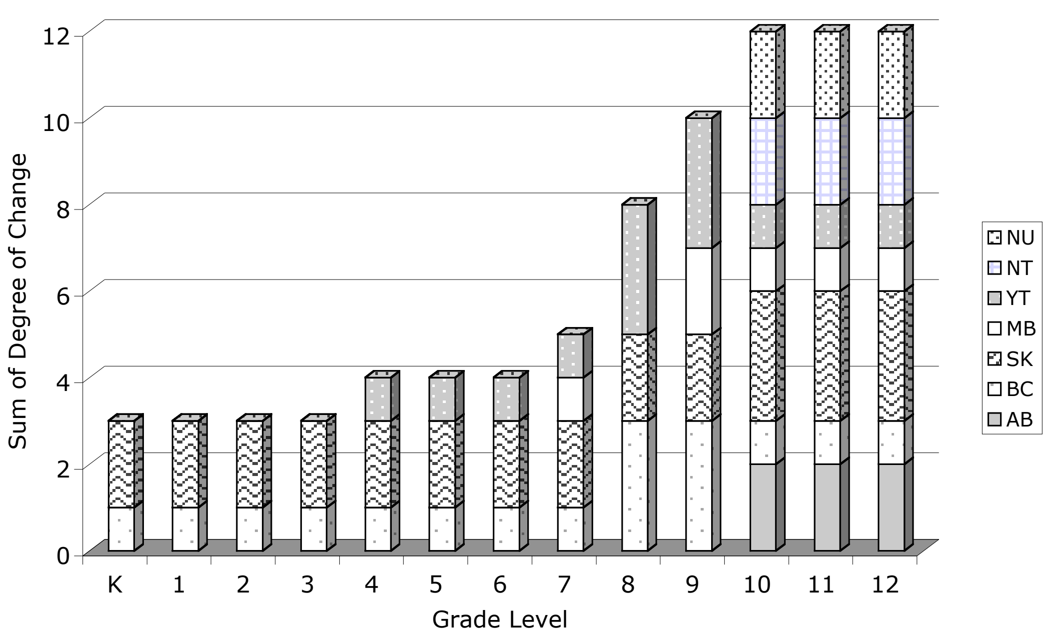
3. Identify any CCF Strands or Substrands that have been omitted from, or added to, your jurisdiction's documents.

4. Identify any CCF Strands or Substrands that have been modified for use in your jurisdiction's documents. Indicate why and how they were modified.

Figure 3 summarizes the cumulative degree of change (based on the sum of each jurisdictions assigned modification score for each grade level) that the CCF has undergone since 1995.

Analysis of jurisdictional responses indicate that, with the exception of Saskatchewan, no CCF Strands or Substrands have been completely deleted from provincial or territorial curriculum documents. At the secondary level all jurisdictions except British Columbia and Yukon have renamed the Strands and substrands in some way, usually to make them more topical in nature. British Columbia, Yukon, and Saskatchewan have all added a Problem Solving strand to their curricula, although in the case of British Columbia and Yukon this is at the draft stage for Grades K to 7. Finally, all jurisdictions except Saskatchewan have developed a third program or pathway at the Grade 10 and 11 levels. British Columbia, Manitoba, Yukon, Northwest Territory, and Nunavut have extended this third program through to Grade 12.

Figure 3: Summary of Degree of Jurisdictional Modifications of the WNCP Mathematics Framework



5. Identify any CCF General/Specific Outcomes that have been omitted from, or added to, your jurisdiction's documents.

6. Identify any CCF General/Specific Outcomes that have been modified for use in your jurisdiction's documents.

Saskatchewan was not included in the following analysis as the differences between its mathematics curriculum and the CCF would result in almost all outcomes being identified as *added*, *deleted*, or *modified*. At Grades K to 6 all jurisdictions, with the exception of Saskatchewan as just noted, made no substantive changes to the CCF outcomes (Specific or General). British Columbia's minor differences at this level can be attributed to the fact that they finalized their provincial curriculum using an older version of the CCF. Starting at Grade 7 Manitoba began making more direct changes to their curriculum documents. These included adding specific outcomes with the intent to better prepare students for Grade 8. Manitoba also made significant changes to their Grade 9 curriculum document by deleting a number of outcomes in an attempt to reduce the content overload teachers reported their students experiencing at this level.

British Columbia deleted a significant number of outcomes at the Grade 8 & 9 level as well. This was done to focus the curriculum more on developing students' numeracy skills and abilities and to reduce the content overload that teachers had reported their students were also experiencing.

It should be noted that Alberta is presently reviewing its junior high document and thus it appears that the majority of jurisdictions have identified Grades 8 and 9 as problem areas where teachers have reported that there is too much content in the curriculum to effectively cover in the given class time.

At the Grades 10 to 12 levels all jurisdictions have made changes to their curriculum documents. These range from specific outcome deletions or modifications that had been agreed to by the jurisdictions involved (Alberta, British Columbia, Nunavut, Manitoba, Yukon) to the more extensive modifications made by Alberta (and subsequently Nunavut and Northwest Territories) to its curriculum documents. At the same time Manitoba has reworded specific outcomes in their curriculum documents to make the intent clearer to teachers.

Implementation of Jurisdictional Curriculum Documents

7. Estimate to what extent the different regions of your jurisdiction have implemented your curriculum documents.

Figures 4, 5, and 6 (beginning next page) summarize the average extent that jurisdictions estimate their different regions have implemented their curriculum documents. Weighted levels of implementation for all jurisdictions differed primarily by grade level and not by jurisdiction. Implementation levels in Kindergarten were lowest, ranging from 60% to 77%. Grades 2 through 9 had similar levels of implementation, ranging from 68% to 89%. Grades

10 through 12 reported implementation ranges that averaged from 79% to 100%. Grade 12 was the only grade that reported at least 81% implementation of the curriculum documents in all jurisdictions. Most jurisdictions attributed this high level of implementation to the provincial exams the students had to complete in that grade. The figures below summarize these implementation ranges.

Figure 4: Extent of Implementation of Jurisdictional Curriculum Documents
Large Urban

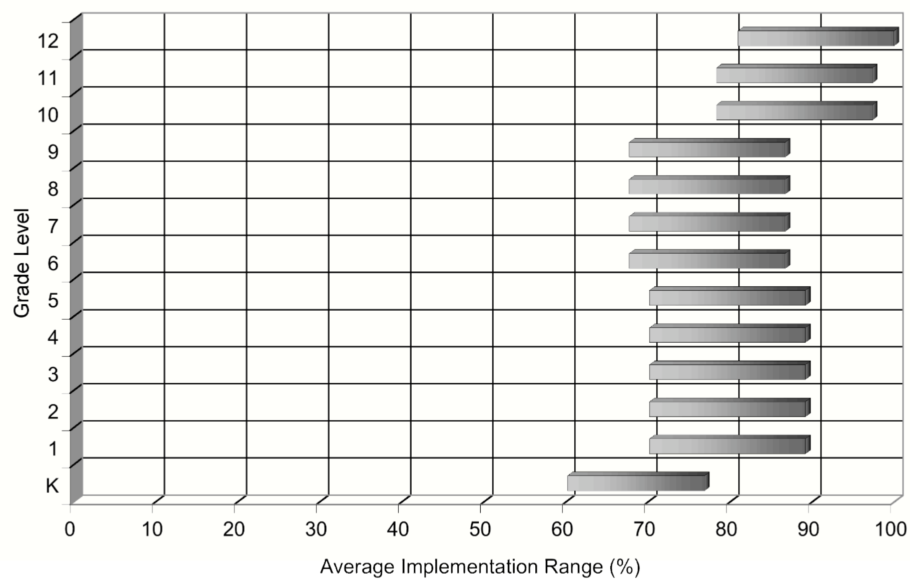


Figure 5: Extent of Implementation of Jurisdictional Curriculum Docements
Small Urban

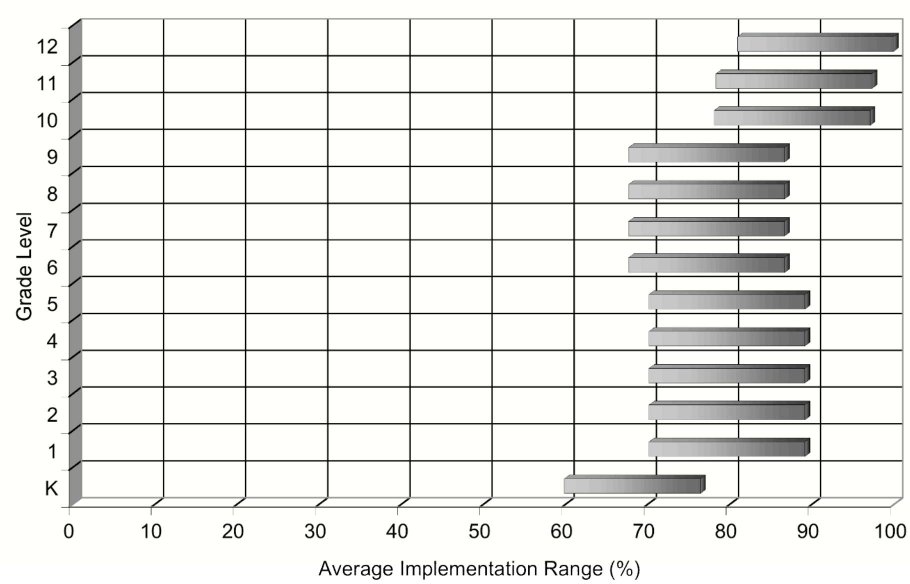
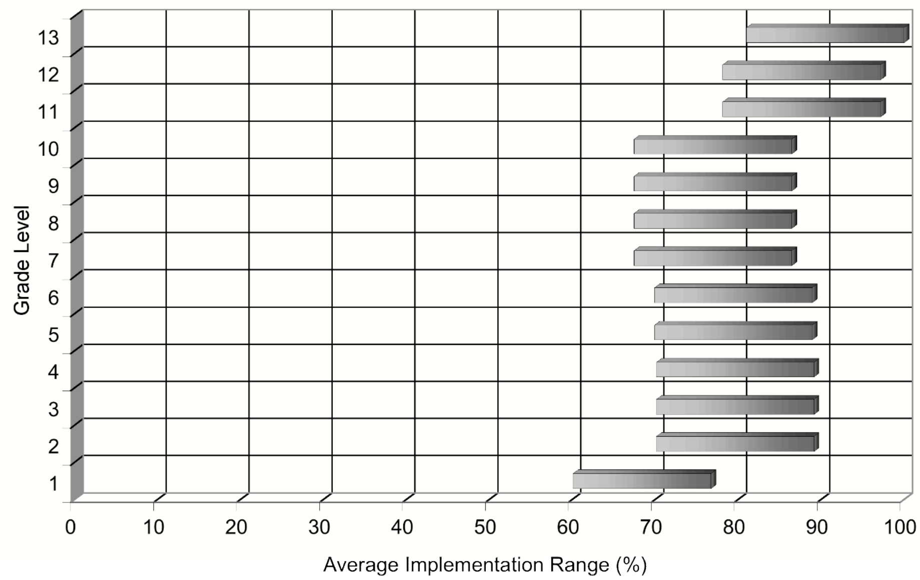


Figure 6: Extent of Implementation of Curriculum Documents
Rural



8. What specific evidence (e.g., surveys, anecdotal evidence, etc.) did you use to respond to the previous question (#7)?

The most commonly reported source of evidence used to determine levels of curriculum implementation was personal communication, either with curriculum consultants, math committees, or anecdotal feedback from different stakeholders. Other forms of informal evidence included feedback from inservice sessions, workshops, test development committees and curriculum writers. Four jurisdictions reported use of more formal data sources. Alberta, Saskatchewan, and Northwest Territories reported use of large-scale assessments as a source of evidence, while Manitoba used results from a recent survey to guide their estimates.

9. Explain any significant differences in levels of implementation.

A variety of reasons were provided to explain differences in levels of implementation of the curriculum in the various jurisdictions. By weighted percent, the four most frequently cited explanations dealt with resources, leadership issues, teacher factors, and course factors. Whether or not regions in a jurisdiction used authorized or non-authorized resources was a clear factor in level of implementation. Leadership factors included presence or absence of math specialists or coordinators, as well as level of support from administrators and level of inservice provided to teachers. Teachers themselves were a factor, with some jurisdictions reporting teachers' comfort with the math content and technology, levels of anxiety, and knowledge of appropriate methodologies as factors. Nunavut also reported teachers' ability to incorporate cultural relevance into the curriculum as a significant factor in their jurisdiction. Course factors dealt with issues of scheduling, time available to teach the curriculum, and whether or not courses were streamed.

Other factors impacting levels of implementation were in the categories of assessment, parent factors and student factors. Grade levels where students were completing large-scale assessments were viewed as having higher levels of implementation of the curriculum. Although only two jurisdictions reported this as a reason for differences in implementation levels, this impacted 54.6% of the students (weighted percent) of all jurisdictions. Only Saskatchewan reported parental pressure to return to more "traditional" mathematics as a factor. In contrast to the provinces, the three territories were the only jurisdictions reporting student factors in levels of implementation of the curriculum. Social (extensive FASD), cultural (native language usage, cultural relevance of curriculum), and educational factors (below grade level literacy levels of students) were all significant factors that severely limited the ability of teachers in the jurisdiction to implement the prescribed curriculum.

10. Identify any comprehensive learning resources (including any created by your jurisdiction) that are commonly used in your schools at each grade level.

Although jurisdictions identified the currently WNCP approved learning resources (Quest 200 and Interactions) as being commonly used in elementary schools, it is important to note that a significant percentage of schools/teachers appear to be using other learning resources that have either not been evaluated and approved or have been de-listed from jurisdictional lists because they no longer correspond to the jurisdictional curricula. This suggests a number of possibilities including:

- Teachers/schools/boards have purchased the approved learning resources, but feel they are not meeting the needs of students and/or the curriculum and therefore use what they feel is appropriate;
- Teachers/schools/boards have purchased the approved learning resources, but continue to use more familiar/traditional resources rather than change instructional practices;
- Teachers/schools/boards have (for whatever reason) not purchased the approved learning resources.

At the secondary level the use of comprehensive resources approved by the WNCP is more prevalent than at the elementary level. This difference is not unexpected as elementary teachers typically use a wider range of resources to deliver the curriculum.

11. What <u>steps</u> were taken to implement the CCF in your jurisdiction?

12. What <u>support</u> was given to teachers during the implementation process?

13. What <u>on-going support</u> is available for teachers with respect to the Mathematics curriculum in your jurisdiction?

Jurisdictions consistently identified two primary activities they undertake or support (i.e., funded) to implement the curriculum. These activities are:

- Develop/authorize learning resources (supporting materials); and,
- Provide or support professional development.

Jurisdictional ongoing support in the form of local initiatives (e.g., advisory groups, teacher associations, etc.) was reported by five out of the seven responding jurisdictions.

Instructional Time

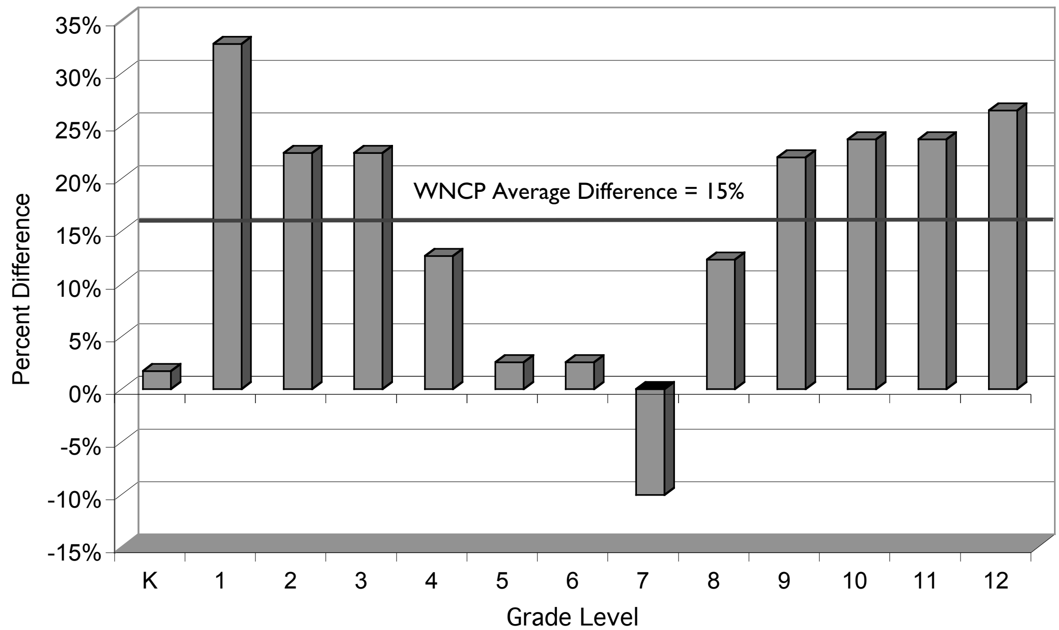
14. What, if any, is the legislated or mandated number of hours of instruction per week or total instructional time set for each grade level? Please specify hrs/week or total instructional time.

15. What is the typical number of hours of practical instruction per week or total instructional time for each grade level? Please specify hrs/week or total instructional time.

This question identified the variability that exists between jurisdictions in terms of legislated instructional time and more noticeably, in terms of the practical amount of instructional time available to teachers and students. On average, the CCF and resulting jurisdictional curriculum documents, are being taught in classrooms with 19 fewer hours than is needed. In some cases (e.g., Grade 1) teachers and students have one third less time to cover the curriculum than is needed. Paradoxically Grade 7 is the only level where it appears teachers and students have too much time (10% extra) to complete the CCF.

Figure 7 (next page) graphically summarizes the percent difference between legislated instructional time and required instructional time for each grade level as a WNCP weighted average. On average, teachers and students have 15% less time to complete the curriculum at each grade level than the CCF is designed for. The greatest discrepancy in terms of lost instructional time is at Grades 1 – 4 and 8 – 12.

Figure 7: Difference Between Average Legislated Instructional Time & Typical Instructional Time



16. What are the reasons for any discrepancies between legislated and practical instructional time?

All jurisdictions identified similar issues to explain the differences between legislated instructional time and practical instructional time. These include:

- Subject specific time allotments are not mandated at the elementary level – where there is legislation it deals with total instructional time. Teachers therefore have discretion as to how much time they teach mathematics, which results in a great deal of variability from teacher to teacher;
- School activities (e.g., assemblies, fire drills, field trips, etc.) reduce instructional time;
- Weather;
- Provincial/territorial testing requirements;
- School time tables (e.g., 65 minute block is no different in terms of practical instructional time compared to a 55 or 60 minute block, but the 5 or 10 minute difference over a semester adds up to at least 8 hours of "lost" instructional time)
- Non-mathematics trained substitute teachers (particularly at secondary) are often unable to adequately teach the subject. The returning teachers typically end up re-teaching the concepts – resulting in further loss of time.

17. What evidence have you used to identify any discrepancies identified between legislated and practical instructional times?

Almost all jurisdictions reported that the evidence used to answer question 16 was anecdotal in nature. Three jurisdictions were able to cite documented evidence including:

- Curriculum revision reports;
- Recent survey to the field; and,
- Reports from school boards.

Jurisdictional Large-scale Assessment of Student Achievement

18. Identify all large-scale assessment tools that have been used to measure student achievement relative to your jurisdictional curriculum?

Alberta, Manitoba, Northwest Territories and Yukon all begin assessing students at Grade 3, while British Columbia starts in Grade 4, Saskatchewan in Grade 5 and Nunavut in Grade 6. Jurisdictions have a large array of assessment tools at their disposal to measure student achievement in a variety of contexts. Although none of the jurisdictions indicated that they used information from international assessments (i.e., TIMSS & PISA) to measure student achievement, they have been included for those jurisdictions known to have participated in them. This was done, as these assessment instruments are known to have sufficient curriculum validity to be used in this manner. Additionally, a number of jurisdictions did not include SAIP on their list of large-scale assessments – these were also included, as they have been developed specifically to allow (in part) cross-jurisdictional comparisons. Table 15 (next page) summarizes the number and type of large-scale assessment instruments used across Western Canadian jurisdictions.

The following is a list of the large-scale assessments identified through the survey:

- AFL = Assessment for Learning (2001)
- AIN = Assessment in Numeracy (Classroom based assessment, used from 2000 on)
- FSA = Foundation Skills Assessment (* Graduation Program Exams will replace FSA in Grade 10 beginning in 2004/05 school year)
- MAT = Mathematics Achievement Test
- MST = Mathematics Standard Test (1997-99 Gr. 3, 1999 on Gr. 9, 2002 on Gr. 12)
- PLAP = Provincial Learning Assessment Program
- SAIP = Student Achievement Indicators Program
- Prov. Exam = Provincial (department) exams
- PISA = Program for International Student Assessment
- TIMSS = Third International Mathematics & Science Study

Table 15: Number of Jurisdictions Reporting Use of Various Large-scale Assessment Tools

Grade	Large-scale Assessment Tools at Each Grade Level										TOTAL
	AFL	AIN	FSA	MAT	MST	PLAP	SAIP	Prov. Exams	PISA	TIMSS	
K - 2											0
3		1		3	1						5
4			1								1
5	1					1					2
6				3							3
7			1				7				8
8	1					1	7			2	11
9				3	1				4		8
10			1				7				8
11						1	7				8
12					1			6			7

19. Based on the assessment tools used in your jurisdiction, what trends have been identified over the last five to eight years during implementation of the WNCP CCF?

Although the overall trend appears to be one of continuous improvement, a number of problematic areas have been identified that need to be addressed. These include:

- Grade 3: Number sense, recall of basic facts
- Grade 6: Pre-algebra skills
- Grade 9: Measurement, number (rate, ratio, proportion), algebra, problem solving
- Grade 12: No commonalities found

Secondary Course Structure & Enrolment

20. What stratification / pathways / course options are formally (i.e., supported by legislation) available at the secondary level (grades 9-12)?

All jurisdictions, with the exception of Nunavut, have one authorized course at the grade 9 level. Only Saskatchewan offers a single course at the grade 10 level with all other jurisdictions offering at least 3 courses at grade 10. At the grade 11 and 12 levels all jurisdictions offer a variety of courses with three jurisdictions offering students some level of Calculus. The names of courses vary but the indications are that the content is quite similar.

21. What, if any, informal stratification / pathways / course options are commonly used by schools and districts at the secondary level.

Several, but not all jurisdictions, indicate that schools are offering a number of informally identified courses at Grades 8 and 9. The question to consider is: "Why is it necessary to offer informal courses?" Since course names and descriptions were not given it is difficult to determine how similar any of the indicated courses might be to the learning outcomes in the authorized courses.

22. How many students are enrolled in each of the secondary level pathways / courses?

Although not all jurisdictions were able to provide detailed student enrolment figures for each of the three years requested, a number of trends have emerged. The responses indicate that from 2000 to 2003:

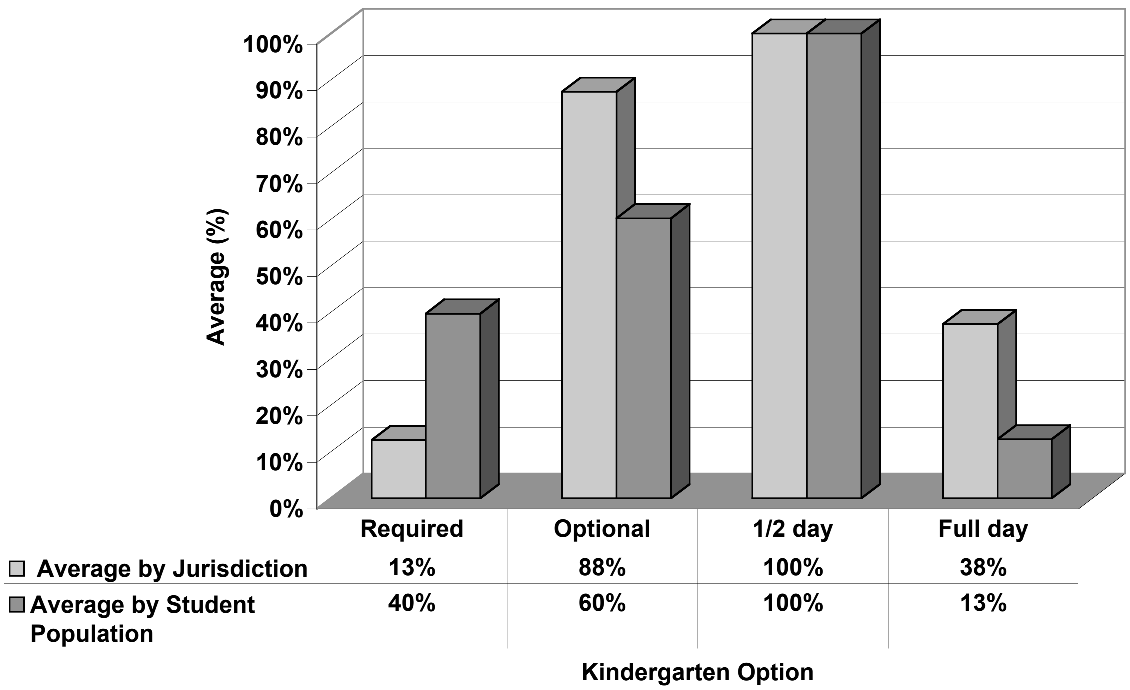
- Grade 10 enrolments in Applied Mathematics (AM), Pure Mathematics (PM), and Other Mathematics (OM) courses for both English and French populations remained constant as a percentage of students taking mathematics at each grade level (English: AM = 21%, PM = 53%, OM = 26%; French: AM = 15%, PM = 75%, OM = 10%);
- A greater percentage of students in the French sub-population consistently take PM 10 to 12 then in the English sub-population;
- Enrolment in Pure Mathematics 11 and 12 is declining as a percentage of total students taking mathematics (PM 11: from 57% to 49%; PM 12: from 67% to 60%) while at the same time enrolment in Applied Mathematics has been increasing as a percentage of students taking Grade 11 and 12 mathematics (AM 11: from 9% to 14%; AM 12: from 14% to 25%);
- A significant percentage of students in Grade 11 take a mathematics course other than Applied or Pure (~34% of students taking Other Mathematics); and,
- Course enrolments of French students for Grade 11 and 12 remains relatively constant.

Kindergarten

23. What is the status of kindergarten in your jurisdiction?

British Columbia is the only jurisdiction that requires students to attend Kindergarten. This accounts for 40% of the student population in Western Canada. For students in the rest of the WNCP jurisdictions Kindergarten is optional. All jurisdictions indicated that 1/2 day Kindergarten is available while 38% of the jurisdictions (including Manitoba (French), Saskatchewan, Yukon) reported that Full Day Kindergarten was available. Figure 8 (below) summarizes the results of this question as the proportion of jurisdictions and the proportion of students.

Figure 8: WNCP Kindergarten Options



24. Identify the basic mathematical skill levels students bring to kindergarten.

Three of the seven jurisdictions were able to provide responses to this question. Table 16 outlines the different skills and the expected proportion of students entering Kindergarten who should have mastered each skill (if at all).

Table 16: Basic Mathematical Skills Expected of Kindergarten Students Upon Entry

Skill	Level of Mastery	Proportion of Students Having the Skill on Entry
Number skills (counting 1 to 10, one-to-one correspondence, etc)	Average	50% - 80%
Number recognition 11 to 20	Below average	30%
Number concepts (conservation, more than, less than, counting on, etc)	Emerging	15% - 40 %
Spatial sense (puzzle problem solving, spatial relationships, etc)	Emerging	20% - 50 %

Skill	Level of Mastery	Proportion of Students Having the Skill on Entry
Patterning, simple routines, seriate	Average	50% - 70%
Counting backwards	Emerging	20%
Sorting to one attribute	Average	40%
Awareness of subtraction and addition through to addition/subtraction with one-digit numbers	Average	20% - 50%
Know basic shapes	Emerging	60% - 80%
Describe basic shapes / matching / sorting, greater / less	Emerging	60% -70%
Make choices (either or)	Emerging	60%

Estimates of Stakeholder Satisfaction

25. Estimate how satisfied the following stakeholder groups are with your jurisdictional mathematics curriculum. Be sure to identify any other stakeholder groups you include.

Responses indicate that jurisdictions believe that post-secondary institutions (related to Applied/Consumer courses) are the most dissatisfied stakeholder group with an average rating of *Somewhat Unsatisfied*. The stakeholder groups that are believed to be the most satisfied (i.e., *Somewhat Satisfied*) are Business and Post-secondary Institutions (Calculus based programs). All other groups were rated between *Somewhat Unsatisfied* and *Somewhat Satisfied*.

26. Summarize any consistent comments/complaints you have received about your jurisdictional curriculum documents OR the WNCP CCF, as expressed by the various stakeholder groups. Be sure to identify any other stakeholder groups you include.

The most common comments (in descending order of frequency) along with the stakeholder group making the majority of the comments (noted in parentheses) are:

- Students unprepared for further math courses for work (Business);
- Discontent with topics (Parents);
- Too much content (Teachers);
- Discontent with approaches/tools used (Teachers and Parents);
- Format/sequencing of topics is poor (Parents);

- Does not prepare for post-secondary (Post-secondary);
- Not enough resources (Teachers); and,
- Poor quality of resources (Teachers).

27. Indicate any other comments/complaints you have received about your jurisdictional curriculum documents OR the CCF.

The most common complaints that jurisdictions received were related to the curriculum documents themselves. Jurisdictions reported that teachers want their curriculum documents to be user-friendlier. Characteristics that teachers are looking for in their curriculum documents include a format that:

- Emphasizes important outcomes in clear language;
- Clarifies the level of understanding expected of students;
- Does not include unnecessary material.

Jurisdictional Recommendations

28. What changes do you think are needed in the CCF in order to better meet the needs of students in your jurisdiction?

The only (and perhaps most significant) recommendation that jurisdictions appear to have agreement on is that the breadth of the CCF needs to be decreased while increasing its depth.

29. Indicate any additional recommendations from within your jurisdiction for the next steps to be taken by the WNCP Mathematics group.

Jurisdictions provided a wide array of final recommendations. A number of common themes are evident in these recommendations. The commonalities include the following:

- Review the fundamental design of the CCF. To what degree should the curriculum be spiral in nature? How does this impact grade placement and sequencing of topics?
- Decrease the amount of content in the CCF to allow more time for in-depth exploration of topics;
- Review and revise/delete the Illustrative Examples – they need to be more effective in supporting the learning outcomes;
- Formally develop a third program of studies for Grades 10 to 12 (e.g., Essentials of Mathematics, Consumer Mathematics, Mathematics 14/24); and,
- Work more closely with post-secondary institutions to ensure that Applied and Pure meet their program needs.

SUMMARY OF STAKEHOLDER SURVEY RESULTS

From December 2003 to March 2004 stakeholder groups were invited to respond to a survey that asked them to identify what they believe are the most important mathematics concepts that our students should understand and use as they proceed from Kindergarten through to Grade 12.

In total 172 responses were received – 161 in English and 11 in French. The distribution of respondents by jurisdiction is shown in the table below.

Table 17: Number of Stakeholder Responses by Jurisdiction

	Jurisdiction									
	AB		BC	MB		SK	NT	NU	YT	Not Known
	E	F		E	F					
Number of Responses	44	2	45	34	9	26	9	1	1	1
Percentage of Responses	27		26	25		15	5	0.6	0.6	0.6

Respondents were asked to indicate which group best described them. As shown in Table 18 the majority of responses were from teachers (61%) with the next largest group being Mathematics post-secondary instructors (16%). No responses were received from "Students" or from "Business".

Table 18: Number of Stakeholder Responses by Stakeholder Group

	Stakeholder Group						
	Parent	Educator	Post-sec (Education)	Post-sec (Math)	Post-sec (Other)	Trades	Other
Number of Responses	10	105	8	27	7	1	14
Percentage of Responses	6	61	5	16	4	0.6	8

The following are the survey questions along with a summary of responses. There were no significant jurisdictional differences observed in the responses although there were differences noted between the responses of post-secondary Mathematics instructors and the rest of the respondents.

1. Describe what mathematics you believe students should be able to do by the end of each grade.

   a. By **Grade 3** students should be able to do the following mathematics:

High Priority	Other Considerations	Also Mentioned
Number operations	Relate numbers to money	Estimation
Addition and subtraction (up to 4 digits)	Start to do mental math	Use manipulatives
Multiply and divide (2 digits)	Simple fractions - 1/2, 1/3, 1/4	Understand function relationships e.g. addition to subtraction and multiplication to division
Geometric shapes	Create algorithms for addition & subtraction	Math in everyday life - buying things, walking home from school, games, time
Solve simple problems / puzzles	Patterns and relations	Simple probability
Number sense / know numbers / understand number relationships	Graphs - read & construct	
Measurement - distance, mass, time, volume/capacity, temperature		

   b. By **Grade 6** students should be able to do the following mathematics:

High Priority	Other Considerations	Also Mentioned
Emphasis on mental math	Gather, analyze and present data	Measurement - conversions between metric units (e.g. Metres - centimetres)
Problem solving (word problems)	Fractions, decimals, percents, ratios, proportions	Probability - basic understanding, ability to work with
Knowledge of numbers and their relationships	Geometric shapes - applied to daily lives (2d, 3d, parallel lines)	Make predictions
Number operations and patterns (e.g. Bedmas)	Graphs - reading, interpreting, constructing	Introduction to algebra
Measurements - perimeter, area, surface area, volume, angles		Fractals, tessellations

Many respondents indicated that calculators should not be used while some indicated that students should be introduced to the calculator and ensure that students are aware of the necessity of checking for errors.

c. By **Grade 9** students should be able to do the following mathematics:

High Priority	Other Considerations	Also Mentioned
Problem solving	Proportion	Trigonometry - basic introduction
High level of competence and confidence in basic number sense, facts and operations	Measurement	Scientific notation
Mental math	Calculators - effective and accurate use	Cartesian coordinate system
Algebra - linear & quadratic equations, factoring, polynomials	Rate, ratio	
Manipulation & use of formulas & equations		
Geometry - strong foundation - concepts & relations		
Consumer math - strong & confident		

2. For each group of students please describe what mathematics they should be able to do by the end of grade 12.

   a. Students who are **entering directly into the workforce** need to be able to do the following mathematics:

High Priority	Other Considerations
Problem solving	Probability
Mental math - basic operations	Fractions, decimals, percents, proportions
Basic spreadsheet operation	Rates and ratios
Consumer math	Interpretation of graphs
Measurement	Geometry
Use of technology to work with numbers	Communication of mathematical ideas through a variety of methods
Basic number operations	
Calculator use	
Money	
Financial math (e.g. income taxes, making change, budget, wages based on given rates)	

b. Students who are **entering post-secondary programs NOT requiring calculus** need to be able to do the following mathematics:

High Priority	Other Considerations
Problem solving	Fractions, decimals, percents
Logical thinking / reasoning	Ability to pick up a math resource to review a skill they have forgotten - read and understand the resource
Basic number operations	Introduction to Trigonometric functions
Number sense	
Mental math	
Estimating	
Money math / money sense	
Measurement	
Basic geometry	
Data analysis - graphs, tables, etc	
Probability and statistics	
Algebraic functions	
Work with formulas	
Use of technology (e.g. calculator) and knowing what technology to use and when to use it	

c. Students who are **entering post-secondary programs requiring calculus** need to be able to do the following mathematics:

High Priority	
Problem understanding	Trigonometry
Problem solving	Calculus
Writing mathematical proofs	Measurement
Logical thinking / reasoning	High level of geometry (e.g. related to design)
Basic number operations	Data analysis - graphs, tables, etc
Number sense	Probability and statistics
Mental math - less reliance on calculators	Deep understanding of formulas and how to manipulate them
Algebra	Use of technology (e.g. calculator) and knowing what technology to use and when to use it

3.

a. What mathematics is necessary for every day life?

High Priority	
Number sense	Measurement (unit conversions)
Number operations at a high level (without calculator)	Reasoning
Mental math	Problem solving
Probability	Geometry - space/shape/size
Estimation	Fractions, decimals, percents, rates, ratios, proportions
Presentation and interpretation of data	Algebraic manipulations
Consumer math (e.g. financing, credit, budget)	

b. What do you believe are the strengths of the current mathematics program?

High Priority	
Cross section of skills	Broken down into specific and manageable outcomes
Variety of concepts	Good breadth/range of topics
Focus on real world applications	3 pathways at 10-12 level
Strong focus on mental math	Emphasis on data management
Emphasis on estimation	Use of appropriate technology
Based on problem solving	Focus on hands-on experiences
Building process from one grade level to the next (sequential)	Requires an understanding of concepts - not just memorization

c. What changes would you like to see made to the current mathematics program?

High Priority	
More real life applications	Post secondary needs to respect all courses
More emphasis on processes of developing answers/solutions. Emphasize the communication of the knowledge and ideas Greater emphasis on why learn math	Fewer concepts and more emphasis of fully understanding these concepts ... more teaching to master
Methods of teaching should change - more inservice for teachers. Need to convince teachers that there are better ways to help students learn	More background information for teachers (e.g. why a concept is important, how it fits into the overall curriculum, why it is important). Teachers need more math training and more training in how to teach math
Harmonize the program across the country	Earlier options for students who are not academically inclined

High Priority	
More concentration on the basic skills, especially at lower grade levels	Allow more time to teach the basic skills more thoroughly, especially at the early grade levels
Concentrate on critical math skills such as numbers, number operations and problem solving	Put geometry back into the senior level curriculum
Reduce the emphasis on calculus	Ban or reduce the use of calculators at lower grade levels

d. Additional comments you wish to make concerning the Kindergarten to Grade 12 Mathematics program?

High Priority	
Post-secondary schools place too much emphasis on mathematics as the gatekeeper for admission	Teachers need to understand the mathematics before they can teach it
Teachers must be well trained to teach mathematics ... stop the approach that anyone can teach math	Students would be better served by having mathematics as a year long course rather than as a semestered course so they don't loose touch with their learning before going on
Reduce the use of calculators, especially at the lower grade levels where the use could be stopped altogether in favour of developing mental math	Curriculum must be followed at all levels to ensure that students at higher grade levels have the knowledge, skills and experience that they should have to succeed at the higher grades

ANALYSIS OF BC POST-SECONDARY INSTITUTION RESPONSES TO A GRADES 10 TO 12 MATHEMATICS CURRICULUM SURVEY

Over a nine-month period BC Ministry of Education staff met with a wide range of post-secondary institutions throughout British Columbia. The main reasons for the meetings were:

- To provide an overview of the present BC Grades 10 to 12 Mathematics curriculum; and,
- To request that instructors from as many post-secondary institutions as possible fill out a survey asking them to rate each of the learning outcomes found in the Grades 10 to 12 Mathematics curricula.

A total of 253 instructors representing 60 distinct departments or programs from four universities, three university-colleges, seven colleges, and one technical institute responded to the survey. The various programs or departments were further divided according to whether they required students to complete a first-year calculus course.

The survey results, although inconclusive in some aspects, did provide sufficient information to support a number of recommendations. It was recommended that the BC Ministry of Education:

- Should support the WNCP revision of the Principles of Mathematics 10 to 12 pathway (Pure) to ensure that the number of topics in this curriculum is reduced by at least 30%. The remaining topics (see page 137) should be sequenced and revised to provide students with more opportunities to study them in depth. This change will better ensure that students completing PM 12 are prepared for programs of study that include first-year calculus.
- Should support the WNCP revision of the Applications of Mathematics 10 to 12 pathway (Applied) to ensure that it meets the needs of programs of study that do not require students complete first-year calculus. To accomplish this secondary analysis of the survey data is recommended. Of paramount importance is the need to consult further with institutions offering non-calculus based programs of study in order to obtain assurances that the revised curriculum will be acceptable for general admission purposes as well as program specific admission.
- Should support the WNCP revision of the Essentials of Mathematics 10 to 12 pathway to ensure that it continues to meet the needs of students planning to enter the workforce directly upon graduation.
- Upon completion of the curriculum revision process, clearly communicate to parents, students, teachers, counselors, and school administrators who the intended student audience is for all thee pathways as well as the status of post-secondary admissions for each of the pathways.

Implications for the WNCP

As the BC Principles of Mathematics pathway (Pure Mathematics) was originally designed to meet the needs of students going on to further studies in mathematics (i.e., programs of study that include first-year calculus), it is not surprising that 71% of the present curriculum (overall) was rated as requiring student exposure or mastery. The fact that the Principles of Mathematics 10 to 12 pathway includes 29% more content than appears is necessary is supported by comments submitted by survey respondents. This is further supported by the anecdotal responses reported in the WNCP Stakeholder Survey.

Overall respondents from calculus based programs favour a reduction in the number of topics presented in the curriculum. Responses and comments also indicate that they would like to retain or see a greater emphasis in the Principles of Mathematics curriculum on the following:

- **Problem Solving**;
- **Algebra**;
- **Functions** (and associated transformations) including linear, polynomial, trigonometric, exponential and logarithmic;
- **Function notation**;
- **Conics**;
- **Geometry**;
- **Systems of Equations**;
- **Vectors**.

At the same time there is general agreement that the following topics are unnecessary in the Principles of Mathematics curriculum:

- **Finance**;
- **Probability (Grade 12)**;
- **Technological Applications (Grade 10)**

The following list of topics consists of those that respondents did not agree should or should not be included in the Principles of Mathematics curriculum:

- **Statistics**;
- **Probability (Grade 11 & parts of Grade 12)**;
- **Technological Applications (Grades 11 & 12);**
- **Measurement;**

Contrasting this are the responses from instructors of programs of study that do not require calculus. Virtually all of the mathematical content in the grade 10 to 12 curriculum received low ratings. The only topics that respondents consistently rated high were:

- **Problem Solving**;
- **Measurement in both SI and Imperial units**;
- **Creating, reading and interpreting graphs**.

The topics that non-calculus based program respondents clearly felt were unnecessary to have in the grade 10 to 12 mathematics curriculum (as preparation for their respective programs) include:

- **Algebra**;
- **Finance**;
- **Technological Applications**;
- **Functions** (and associated transformations) including linear, polynomial, trigonometric, exponential and logarithmic;
- **Statistics**;
- **Probability** (Grade 12);
- **Systems of Equations**;
- **Geometry**;
- **Conics**.

A number of topics received indeterminate ratings from respondents. These include:

- **Coordinate Geometry**;
- **Logic**.

SUMMARY & DISCUSSION

The following discussion section focuses back on the original questions and tasks of this research project.

1. What does the current academic literature pertaining to the teaching and learning of mathematics tell us?

Learning Theories:

Eighteen "theories of learning" were reviewed over the course of this project. The most important concept that needs to be adhered to is that not only do the theories of learning fall on a spectrum; they overlap in a variety of ways. The perceived flaws of one learning theory may be addressed by another and the combined effect of using a broad range of approaches is much more powerful than relying on any single model. There is no doubt that educators need to be kept apprised of new and evolving theories of learning, but policy makers must be very cognisant of the ongoing support teachers require as they are asked to incorporate this new information into their teaching.

The ten common "needs" identified by Sfard (2003) should be kept mind as the CCF is revised. Regardless of the theoretical underpinnings of the revised CCF, it must address our students' needs:

1. The need for meaning
2. The need for structure
3. The need for repetitive action
4. The need for difficulty
5. The need for significance and relevance
6. The need for social interaction
7. The need for verbal-symbolic interaction
8. The need for a well-defined discourse
9. The need for belonging
10. The need for balance

The most important of these needs is "balance". The reality is that there must be a bit of everything in the classroom: problem solving as well as skills practice, teamwork as well as individual learning and teacher exposition, real-life problems as well as abstract problems, learning by talking as well a silent learning (Sfard, 2003).

Conceptual vs. Procedural Knowledge

Results of the WNCP Member Survey, the Stakeholder Survey as well as the BC Post-secondary Mathematics Curriculum survey underscore that fact that there are competing expectations of our students. This is a case of wanting the best of both worlds: students that have developed computational skills without compromising the conceptual understanding needed to help them learn new, more sophisticated mathematics. To accomplish this mathematics curricula and classroom instruction must include learning outcomes that focus on both of these aspects of knowledge. It is not enough to include procedural and conceptual outcomes in a curriculum unless both types of knowledge of knowledge are assessed (by both classroom-based and large-scale assessment instruments). Correct answers are not always a safe indicator of understanding. Teachers and policy makers must ensure that curriculum and assessment are properly aligned (BC Ministry of Education, 2002).

Two other factors may also have an impact on conceptual learning – choice and connections. The problem-based instruction and open, project-based approaches to learning both provide opportunities for students to make their own choices and to form connections between their mathematical learning and real-world applications. These factors have a far greater impact on students than most educators realize.

Technology in the Mathematics Curriculum

Calculators, particularly graphing calculators, can have a positive impact in the learning of mathematics for students. It appears that benefits will be maximized if calculators are used in a pedagogical role and not just for performing calculations. This has curricular implications, as it requires adaptation of both content (i.e. representations and procedures) and instructional approaches. Support in terms of both resources and inservice is essential to ensure that the technology is appropriately integrated into the classroom.

The Impact of the Teacher in Educational Change

Although this topic may be considered outside the mandate of this project, there were simply too many studies that indicated that substantive change could be achieved with appropriate professional development (Arcavi & Schoenfeld, 2003; Callingham & Griffin, 2001; Schoenfeld, 2003; Sykes, 1996; Wilcox & Jones, 2003). This calls for both a commitment on the part of the teachers and the inservice providers. Professional development must be seen as a continuing enterprise for teachers and an integral component of any curriculum change. Thus, it is strongly felt that the WNCP member jurisdictions need to consider ways to encourage and support this important step in any recommended curriculum changes.

Aboriginal Students and Mathematics

Many Aboriginal children struggle with mathematics. Contextualizing that mathematics and making it much more applicable to the needs of the Aboriginal population will help some students. A rethink in how mathematics is assessed and an explicit development of appropriate strategies for struggling learners to make them feel more confident in attempting mathematics test items appears to be warranted.

Assessment can be very confronting, particularly for Aboriginal children. Teachers should aim to use methods with which the students are comfortable. Suggestions are to:

- Include assessment tasks that allow students to demonstrate their knowledge visually rather than verbally;
- Use assessment that rewards teamwork;

- Provide choice for students both within and among assessment tasks and,
- Introduce self-assessment in an attempt to avoid alienating struggling students through criticism.

Finally, if the WNCP member jurisdictions wish to improve the learning situation for Aboriginal students, they should provide information related to learning styles and appropriate teaching strategies for aboriginal students. They should also be strongly encouraged to provide professional development to assist teachers in making appropriate adjustments to their teaching and assessment practices.

2. Mathematical Literacy and the WNCP Curriculum

The evidence would appear to indicate that the use of the present CCF organization is consistent with many of the definitions of numeracy, or more accurately, mathematical literacy found in the literature. Regardless of the term used, the present structure includes those elements recognized throughout the world as supporting the development of mathematical literacy in our students. In order to better support the teaching of conceptual understanding it may be advisable to either add new curriculum strands (e.g., Problem Solving and Reasoning & Proof) to the curriculum organization with the expectation that teachers would integrate these learning outcomes as they teach, or to include explicit mathematical process learning outcomes into the four content strands as appropriate.

To support teachers in ensuring that these mathematical processes are truly part of the delivered curriculum the CCF should include: additional information describing effective teaching methods; and, examples of student performance related to these (and all) learning outcomes.

Common Curriculum Framework Design & Content

Results from the various surveys, as well as research into national and international assessment results all support a reduction in the number of topics at all grade levels. Teachers have reported for several years concerns about the number of topics – resulting in a superficial treatment of a large number of topics. Post-secondary instructors (particularly those from calculus-based programs of studies) have indicated that incoming students need to have in depth knowledge of fewer topics to have a reasonable chance of being successful. This is consistent with an analysis of the results of international assessment of student mathematics achievement. Those countries that have surpassed Canada and/or the WNCP jurisdictions on these assessments are ones that generally have a narrower, but deeper mathematics curriculum. A more in-depth treatment of the mathematics curriculum depth will also provide teachers and students greater opportunity to attain the levels of procedural and conceptual understanding that society is demanding. Once again, this type of change requires a significant commitment to teacher inservice on the part of the WNCP jurisdictions. This may be problematic as the WNCP Member Survey clearly indicates that, in more and more jurisdictions, teacher inservice responsibilities are being relegated to the district/board level.

In order to maintain the required curriculum depth at the secondary level it is vital that three distinct programs of study be available to students. The BC post-secondary study (McAskill, 2004) clearly shows that the present Pure Mathematics curriculum can, with relatively minor revisions, meet the needs of the 20% of our students that go on to programs that include higher mathematics. As all jurisdictions have programs of study designed to meet the needs of the 20% - 30% of students not going on to post-secondary studies, it seems logical to pool WNCP resources to accomplish this as part of a revised CCF. Unfortunately this leaves us with a significant gap as the needs of the approximately 50% of our students not going on to post-secondary studies are not being addressed by the Applied Mathematics curriculum. This suggests that further consultation with these programs is necessary to determine what it is they do want these students to know mathematically. A recent study done for Kwantlen University College in Surrey, BC (Dean, 2001) shows that many programs of study that require Pure Mathematics 11 for entry, in fact only require a Grade 8 or 9 mathematics level! It is essential the WNCP jurisdictions establish or expand communication channels with their respective post-secondary institutions. The post-secondaries may need as much education about the K – 12 system as we do about the post-secondary system.

A number of different sources of information (e.g., analysis TIMSS, PISA results, BC Post-secondary Math Curriculum Survey, WNCP Member Survey, Stakeholder Survey) provide enough preliminary information to suggest a few of the topics the CCF should focus on (and grade ranges). This is by no means all that should comprise a revised CCF, but it does provide a starting point. Table 19 (next page) provides a summary of suggested topics and appropriate grade ranges. Not all of the topics, specifically at the secondary level (Grades 10 to 12), were consistently identified as desirable by all data sources. These in particular are the issues that need to be clarified with post-secondary institutions before any curriculum revision is undertaken.

Table 19: Suggested Topics & Grade Grouping for the Common Curriculum Framework

	Grade Groupings					
				Applied	Pure	Other
Topic	K – 3	4 – 6	7 – 9	10 – 12		
Whole Number Meaning	X	X				
Whole Number Operations	X	X				
Measurement Units	X	X	X	X		X
Common Fractions		X				
Equations and Formulas		X	X	X		X
Data Representation & Analysis		X	X	X		X
2-D Geometry: Basics	X	X	X	X		X
Polygons & Circles		X	X			
Perimeter, Area, & Volume		X	X			X
Rounding & Significant Figures		X				
Estimating Computations		X				
Properties of Whole Number Operations		X				
Estimating Quantity & Size		X				
Relationship of Common & Decimal Fractions		X				
Properties of Common & Decimal Fractions		X				
Percentages		X				
Proportionality Concepts		X	X			
Proportionality Problems		X	X			
2-D Coordinate Geometry		X	X			
Geometry: Transformations			X			
Negative Numbers, Integers, & Their Properties			X			
Number Theory			X			
Exponents, Roots, & Radicals					X	
Exponents & Orders of Magnitude					X	
Measurement Estimation & Errors				X	X	X
Constructions w/Straightedge & Compass					X	
3-D Geometry (including vectors)					X	
Congruence & Similarity					X	
Rational Numbers & Their Properties					X	
Algebra					X	
Functions & Notation					X	
Systems of Equations					X	
Finance/Consumer Mathematics						X
Trigonometry				X	X	
Probability				X		
Statistics				X		

RECOMMENDATIONS

The following recommendations are put forward for consideration by the member jurisdictions as they prepare to revise of the WNCP *Common Curriculum Framework for K – 12 Mathematics*. It is recommended that:

1. The CCF not be developed using a single learning theory as its basis, but rather be developed using the common characteristics of learning theories identified in this report (see page 34).
2. The CCF consist of a single program of studies from Kindergarten to Grade 9.
3. In order for the CCF to effectively meet the needs of our secondary students, it include three distinct programs for Grades 10 to 12. Specifically, it is recommended that the secondary programs be developed to meet the needs of students who are:
   - 3.1. Entering post-secondary programs that require calculus (e.g., Mathematics, Sciences, Engineering, Commerce, etc.);
   - 3.2. Entering post-secondary programs that do not require calculus (e.g., Humanities, Fine Arts, somes Trades and Technical programs, etc.); and,
   - 3.3. Entering the workforce, Trades or Technical programs that do not require advanced mathematics.
4. Additional consultation with post-secondary representatives of programs of study that do not include calculus be undertaken to identify what mathematics is required of students entering these types of programs.
5. The WNCP reduce the breadth of the CCF so as to increase its depth. In any revision of the CCF, consideration should be given to decreasing the breadth or spread of specific outcomes across multiple grades. Specifically, it is recommended that:
   - 5.1. The curriculum for Kindergarten to Grade 3 be focused on Number and Measurement only;
   - 5.2. The K – 3 curriculum be based upon conceptual understanding with algorithms introduced only when conceptual understanding supports them;
   - 5.3. When developing the curriculum for K – 12 the WNCP take into consideration the Suggested Topics & Grade Groupings described in Table 19 (page 145); and,
   - 5.4. The number of topics in each secondary program (Grades 10 – 12) be reduced by as much as one third. This is needed to allow more time for in-depth instructional experiences consistent with the identified common learning theory characteristics.

   This does not mean that all content from such areas should be removed. One possibility would be to separate outcomes from applications. For example, the outcomes in grade 2 could emphasize "number" but some applications could be from "3-D geometry." However, the application would not be part of what students are expected to master. Rather, the application would be a vehicle to develop an understanding of number.

6. The CCF includes both procedural and conceptual learning outcomes. The intent of this recommendation is to better support the teaching of conceptual understanding by either:
    6.1. Adding new curriculum strands (e.g., Problem Solving and Reasoning & Proof) to the curriculum organization with the expectation that teachers would integrate these learning outcomes in their teaching; or,
    6.2. Including explicit mathematical process learning outcomes into the four content strands as appropriate.
7. The WNCP member jurisdictions develop large-scale assessment instruments that are consistent with the learning theory characteristics used to develop the CCF and thus assess students' procedural **and** conceptual understanding of mathematics. It is evident that large-scale assessments send a clear message to teachers concerning what is valued in the mathematics curriculum. The implication is that the procedural and conceptual mathematics teachers are being asked to teach in their classrooms must be reflected in any large-scale assessments used for accountability purposes.
8. For each grade level and topic, an Effective Instruction and Assessment Practices description be developed. The description should outline the classroom environment and instructional and assessment approaches that research (consistent with the identified learning theory characteristics) has identified as being most effective. Additionally, it is recommended that:
    8.1. The CCF support teachers in ensuring that mathematical processes are truly part of the delivered curriculum by including:
        8.1.1. Additional information describing effective instructional and assessment methods (including methods designed specifically for Aboriginal students); and,
        8.1.2. Examples and/or descriptions of the level of performance that is expected of students.
9. The CCF incorporate the use of technology such as computers, calculators and graphing calculators in a phased manner. Specifically, it is recommended that:
    9.1. The CCF should not require the use of technology in the Grades K to 3 learning outcomes;
    9.2. The CCF should introduce technology into the Grade 4 to 7 learning outcomes as appropriate as part of the mathematical learning and problem solving processes;

9.3. The CCF should require technology in the Grade 8 to 12 learning outcomes as appropriate as part of the mathematical learning and problem solving processes; and,

9.4. The Effective Instruction and Assessment Practices include instructional and assessment methods related specifically to the appropriate use of such technology along with examples and/or descriptions of the level of performance that is expected of students.

10. The WNCP jurisdictions pilot the Draft CCF for one full year prior to implementation and use the information collected from the pilot sites to revise both the learning outcomes and the Effective Instruction and Assessment Practices as needed.

11. The WNCP work with publishers to ensure that learning resources consistent with the identified learning theory characteristics and the mathematical content of the CCF be developed and piloted before implementation.

12. The WNCP jurisdictions provide a significant level of targeted support for teachers during the initial implementation of the CCF. Specifically, it is recommended that:

12.1. Jurisdictions provide ongoing inservice for teachers and school administrators for a minimum of two years from the date of first implementation. This is particularly important for elementary teachers (i.e., non-mathematics specialists), who would benefit the most from ongoing inservice that integrated mathematics content <u>and</u> mathematics pedagogy; and,

12.2. Jurisdictions communicate and work with post-secondary institutions to ensure that accurate information concerning course acceptance is available to students, parents, teachers, counsellors, administrators and the public in general.

13. The WNCP jurisdictions, prior to revising the CCF, develop and use a program evaluation model that allows them to assess the effectiveness of the revised curriculum. Throughout this project it was evident that the WNCP did not have a plan in place for the systematic collection of information that could be used for program evaluation purposes. Although this was not necessarily true of individual jurisdictions, the information that was collected by jurisdications was not done so in a consistent manner. This made data analysis and synthesis unnecessarily complex and less reliable than it otherwise could be. Good evaluation studies include four important criteria identified by the Joint Committee on Standards for Educational Evaluation (Gall, et al, 1996):

- Utility – the study is informative, timely and useful to the affected persons;
- Feasibility – the evaluation design the study is appropriate to the setting in which it is to be conducted (i.e., schools) and the design is cost effective;

- Propriety – the study is conducted legally and ethically; and,
- Accuracy – the study produces valid, reliable, and comprehensive information for making judgements of a program's worth.

Some Final Thoughts

In a paper recently prepared by the Canadian Mathematics Education Study Group (CMESG) (Whitely & Davis, 2003) the following statement was made:

> The CMS endorses the general aims of the current K – 12 mathematics curriculum documents across Canada. However, we believe that the structure of these curricula is an obstacle to student learning of mathematics. Over-specified and fragmented lists of expectations misrepresent what mathematics is and militate against deep and authentic engagement with the subject.

As qualified as this endorsement is, it should be viewed in a positive manner as the WNCP has already taken the first critical steps towards addressing some of these concerns. Even though there is considerable work ahead for curriculum developers, teachers and the myriad of educators involved in any reform effort, we can take heart in the words of Alan Schoenfeld (1992):

> In sum, the imminent implementation of curricula with ambitious pedagogical and philosophical goals will raise a host of fundamentally difficult theoretical and practical issues. It is clear that we have our work cut out for us, but it is also clear that progress over the past decade gives us at least a fighting chance for success. (p. 366)

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