The Common Curriculum Framework

for

# K–12 MATHEMATICS

## Grade 10 to Grade 12

**Western Canadian Protocol for Collaboration in Basic Education**

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# I. BACKGROUND

*Partners for Collaboration in Basic Education:*

*Manitoba*
*Saskatchewan*
*Alberta*
*British Columbia*
*Yukon Territory*
*Northwest Territories*

The Western Canadian Protocol for Collaboration in Basic Education Kindergarten to Grade 12 was signed in December 1993 by the ministers of education from Manitoba, Saskatchewan, Alberta, British Columbia, Yukon Territory and the Northwest Territories. The protocol states that the four western provinces and the two territories agree to collaborate in basic education because of the importance they place on:

- common educational goals
- high standards in education
- removing obstacles for student access to educational opportunities, which includes improving the ease of transfer from jurisdiction to jurisdiction
- optimum use of educational resources.

*The Common Curriculum Framework for K–12 Mathematics* (The Common Framework) is the first in a series of joint development projects in basic education. It has been developed by the six ministries of education in collaboration with teachers, administrators, parents, business representatives, post-secondary instructors and others.

The Common Framework identifies beliefs about mathematics, general and specific student outcomes and illustrative examples agreed upon by the six jurisdictions. Each of the provinces and territories will determine when and how The Common Framework is to be implemented within its own jurisdiction.

In June 1995, the first phase of *The Common Curriculum Framework for K–12 Mathematics* was published. The 1995 document had a focus on Kindergarten to Grade 9 mathematics. This second phase of the project has a focus on Grade 10 to Grade 12 mathematics.

The third section of each document—Conceptual Framework for K–12 Mathematics—is identical. Here, the philosophical view toward mathematics and mathematics education is presented.

*This second phase focuses on Grade 10 to Grade 12 mathematics.*

# II. INTRODUCTION

## PURPOSE OF THE DOCUMENT

*The Common Framework communicates high expectations for students.*

The Common Framework addresses the major goals of the protocol. This document provides a common base for the curriculum expectations mandated by each province and territory. This common base will result in consistent student outcomes in mathematics across jurisdictions and will enable easier transfer for students moving from one jurisdiction to another. Its intent is to **communicate clearly high expectations for students in mathematics education to all educational partners across the jurisdictions** and facilitate the development of common learning resources.

### Document Design

This document presents mathematics expectations for high school students. These expectations are presented in three ways:

- general outcomes
- specific outcomes and
- illustrative examples.

The *Common Curriculum Framework for K–12 Mathematics (Grades 10–12)* is built upon the same design principles as the Kindergarten to Grade 9 materials that were published in June 1995. The 10–12 framework provides:

- an overall view of all student expectations, through the presentation of K–12 General Outcomes and 10–12 General Outcomes and Specific Outcomes that include Grade 9 from the June 1995 document
- the identification of 24 clusters of outcomes (specific outcomes) that are intended to be used as a menu from which provinces and territories can create courses and programs.

All students engaged in a 10–12 program will be expected to realize the outcomes in the common clusters. Further information on clusters occurs on pages 18–19 and pages 61–190.

## BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

Students are curious, active learners who have individual interests, abilities and needs. They come to classrooms with different knowledge, life experiences and backgrounds that generate a range of attitudes about mathematics and life.

*Students must construct their own meaning of mathematics.*

Students learn by attaching meaning to what they do; and they must be able to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives can address the diversity of learning styles and developmental stages of students and can enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with appropriate materials, tools and contexts when constructing personal meaning about new mathematical ideas. The learning environment should value and respect each student's way of thinking, so that the learner feels comfortable in taking intellectual risks, asking questions and posing conjectures.

Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions and actively create new knowledge in striving for self-fulfillment.

## GOALS FOR STUDENTS

*Mathematics education must prepare students to use mathematics to solve problems.*

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

At the completion of a program, students should have developed a positive attitude toward mathematics and have a base of knowledge and skills related to Number, Patterns and Relations, Shape and Space, and Statistics and Probability.

*Positive attitudes toward mathematics are important.*

It is important for students to develop a positive attitude toward mathematics so that they can become confident in their ability to undertake the problems of a changing world, thereby experiencing the power and usefulness of mathematics. Students also should gain an understanding and appreciation of the contributions of mathematics, as a science and as an art, to civilization and to culture.

Students should:

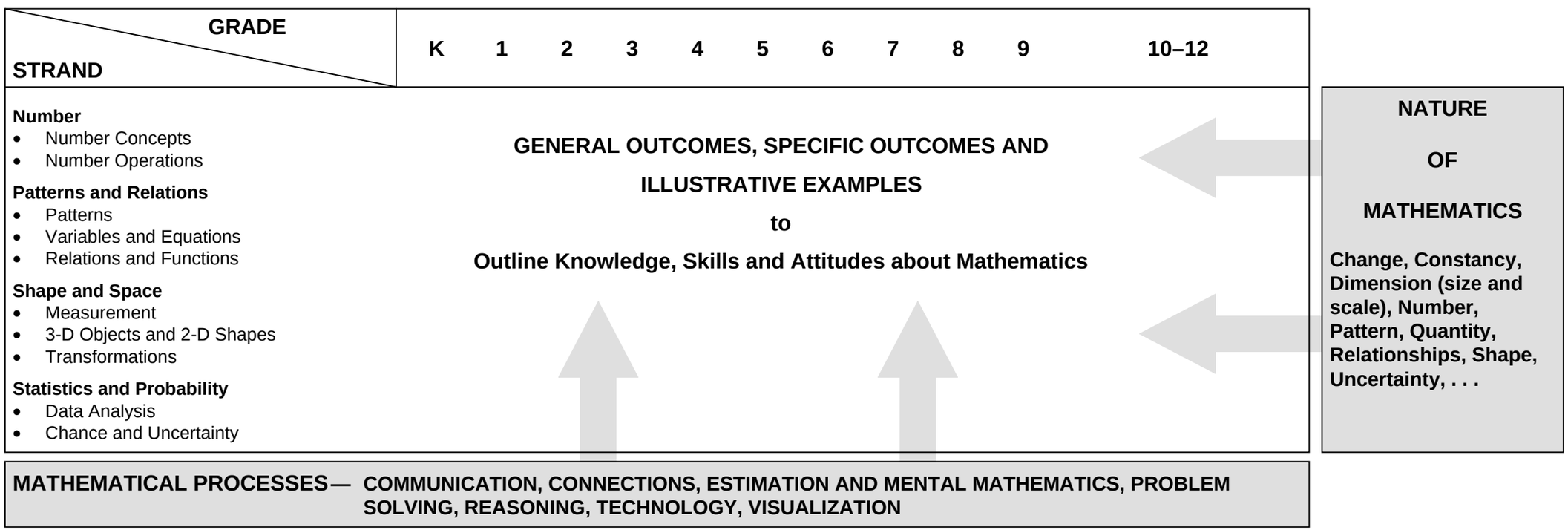
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity
- show some enjoyment of mathematical experiences.

All students should receive a level of mathematics education appropriate to their needs and abilities.

## III. CONCEPTUAL FRAMEWORK FOR K–12 MATHEMATICS

Students of mathematics, regardless of age or experience, struggle to do mathematics in settings that are new to them. The conceptual framework outlined in this section presents a multifaceted view of mathematics and presents the discipline as skills, procedures and concepts woven together.

The framework chart below shows how student outcomes, organized by grade and strand, are designed to be influenced by Mathematical Processes and the Nature of Mathematics. These components are described more fully in this section.



## MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. Students are expected to:

| | |
|---|---|
| • *Communication* [C] | • communicate mathematically |
| • *Connections* [CN] | • connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines |
| • *Estimation and Mental Mathematics* [E] | • use estimation and mental mathematics where appropriate |
| • *Problem Solving* [PS] | • relate and apply new mathematical knowledge through problem solving |
| • *Reasoning* [R] | • reason and justify their thinking |
| • *Technology* [T] | • select and use appropriate technologies as tools to solve problems |
| • *Visualization* [V] | • use visualization to assist in processing information, making connections and solving problems. |

The Common Framework incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

## Communication

Students need to communicate mathematical ideas clearly and effectively, orally and in writing.

Communication will help students make connections among different representations of mathematical ideas; namely, "physical, pictorial, graphic, symbolic, verbal and mental representations." (NCTM, p. 26)

*Students must be able to communicate effectively how an answer was obtained.*

It is not enough to arrive at an answer. Students must be able to communicate effectively how the answer was obtained. In other words, students need opportunities to read, to explore, to investigate, to write, to listen to, to discuss and to explain ideas in their own language of mathematics. Thus, students can create their own links "between their informal, intuitive notions and the abstract language and symbolism of mathematics." (NCTM, p. 26)

## NCTM COMMUNICATION STANDARDS

| K–4 | 5–8 | 9–12 |
|---|---|---|
| *The study of mathematics should include numerous opportunities for communication so that students can:* | *The study of mathematics should include opportunities to communicate so that students can:* | *The mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can:* |
| • relate physical materials, pictures, and diagrams to mathematical ideas<br>• reflect on and clarify their thinking about mathematical ideas and situations<br>• relate their everyday language to mathematical language and symbols<br>• realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics. | • model situations using oral, written, concrete, pictorial, graphical, and algebraic methods<br>• reflect on and clarify their own thinking about mathematical ideas and situations<br>• develop common understandings of mathematical ideas, including the role of definitions<br>• use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas<br>• discuss mathematical ideas and make conjectures and convincing arguments<br>• appreciate the value of mathematical notation and its role in the development of mathematical ideas. | • reflect upon and clarify their thinking about mathematical ideas and relationships<br>• formulate mathematical definitions and express generalizations discovered through investigations<br>• express mathematical ideas orally and in writing<br>• read written presentations of mathematics with understanding<br>• ask clarifying and extending questions related to mathematics they have read or heard about<br>• appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas. |
| (NCTM, p. 26) | (NCTM, p. 78) | (NCTM, p. 140) |

## Connections

*Through connections students should begin to view mathematics as an integrated whole.*

Students need numerous and varied experiences in order to appreciate the usefulness of mathematics and, at the same time, to explore connections within mathematics, from mathematics to other disciplines, and from mathematics to their daily experiences. When mathematical ideas are connected to each other through concrete, pictorial and symbolic representations, students begin to view mathematics as an integrated whole.

This integration "allows students to see how one mathematical idea can help them understand others, and it illustrates the subject's usefulness in solving problems, describing and modeling real-world phenomena, and communicating complex thoughts and information in a concise and precise manner." (NCTM, p. 94)

## NCTM CONNECTIONS STANDARDS

| K–4 | 5–8 | 9–12 |
|---|---|---|
| *The study of mathematics should include opportunities to make connections so that students can:* | *The mathematics curriculum should include the investigation of mathematical connections so that students can:* | *The mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can:* |
| • link conceptual and procedural knowledge<br>• relate various representations of concepts or procedures to one another<br>• recognize relationships among different topics in mathematics<br>• use mathematics in other curriculum areas<br>• use mathematics in their daily lives. | • see mathematics as an integrated whole<br>• explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations<br>• use a mathematical idea to further their understanding of other mathematical ideas<br>• apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business<br>• value the role of mathematics in our culture and society. | • recognize equivalent representations of the same concept<br>• relate procedures in one representation to procedures in an equivalent representation<br>• use and value the connections among mathematical topics<br>• use and value the connections between mathematics and other disciplines. |
| (NCTM, p. 32) | (NCTM, p. 84) | (NCTM, p. 146) |

## Estimation and Mental Mathematics

*Mental mathematics is the cornerstone for estimation.*

Students need to know when and how to estimate. The context of a problem helps to determine when it is necessary or desirable to have an exact answer or an estimate of that answer. Problem contexts include number, patterns and relations, shape and space, and statistics and probability. The use of technology increases the emphasis on estimation skills to enable students to determine the reasonableness of computed answers.

A variety of estimation strategies assists students in arriving at quick approximations for exact answers.

Facility with mental mathematics is an important outcome for students. A focus on mental mathematics forces students to think and improve their efficiency and accuracy in calculating, including pencil and paper calculations. Mental mathematics is the cornerstone for estimation and leads to better understanding of number concepts and number operations. (Hope, pp. 161–173)

## Problem Solving

"*Problem solving—which includes the ways in which problems are represented, the meanings of the language of mathematics, and the ways in which one conjectures and reasons—must be central to schooling so that students can explore, create, accommodate to changed conditions, and actively create new knowledge over the course of their lives."* (NCTM, p. 4)

*Problem solving is the focus of mathematics at all grade levels.*

Problem solving is the <u>focus</u> of mathematics at all grade levels. The development of each student's ability to solve problems is essential. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all of the strands.

Problem solving provides an opportunity for students to be active in constructing mathematical meaning, to learn problem-solving strategies, to practise a variety of concepts and skills in a meaningful context, and to communicate mathematical ideas. Most problem-solving situations in the elementary years come from the everyday experiences of the student. Students are able to attach mathematical meaning to familiar activities. As they progress through school, the problems become more complex. The problems will arise from an exploration of mathematics itself, as well as from the world around them. Gradually, students become more confident in their ability to use and communicate mathematics, using correct terminology.

As students develop mathematically, they are able to solve more challenging problems on an increasing variety of topics. Students need the opportunity "to solve problems that require them to work cooperatively (and individually), to use technology, to address relevant and interesting mathematical ideas, and to experience the power and usefulness of mathematics." (NCTM, pp. 75–76) By the time students reach the secondary level, many problem-solving strategies should be internalized and problem solving should be a process for constructing and reinforcing mathematical concepts.

Students should be confident and flexible problem solvers, using a wide range of strategies in their work, and accept that some problems have different solutions.

## NCTM PROBLEM-SOLVING STANDARDS

| K–4 | 5–8 | 9–12 |
|---|---|---|
| *The study of mathematics should emphasize problem solving so that students can:* | *The mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can:* | *The mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can:* |
| • use problem-solving approaches to investigate and understand mathematical content<br>• formulate problems from everyday and mathematical situations<br>• develop and apply strategies to solve a wide variety of problems<br>• verify and interpret results with respect to the original problem<br>• acquire confidence in using mathematics meaningfully. | • use problem-solving approaches to investigate and understand mathematical content<br>• formulate problems from situations within and outside mathematics<br>• develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems<br>• verify and interpret results with respect to the original problem situation<br>• generalize solutions and strategies to new problem situations<br>• acquire confidence in using mathematics meaningfully. | • use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content<br>• apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics<br>• recognize and formulate problems from situations within and outside mathematics<br>• apply the process of mathematical modeling to real-world problem situations. |
| (NCTM, p. 23) | (NCTM, p. 75) | (NCTM, p. 137) |

## Reasoning

*Reasoning helps students to make sense of mathematics and to be logical in their thinking.*

Students need to develop confidence in their ability to reason and to justify their thinking within and outside of mathematics. The power of reasoning helps students to make sense of mathematics, to be logical in their thinking, and to convince others.

Inductive reasoning helps students explore and make conjectures from activities that allow generalizations from a pattern of observations.

Deductive reasoning helps students test conjectures and build arguments that serve to validate thinking. Deductive reasoning builds a structured body of knowledge.

### NCTM REASONING STANDARDS

| K–4 | 5–8 | 9–12 |
|---|---|---|
| *The study of mathematics should emphasize reasoning so that students can:* | *Reasoning shall permeate the mathematics curriculum so that students can:* | *The mathematics curriculum should include numerous and varied experiences that reinforce and extend logical reasoning skills so that all students can:* |
| • draw logical conclusions about mathematics<br>• use models, known facts, properties, and relationships to explain their thinking<br>• justify their answers and solution processes<br>• use patterns and relationships to analyze mathematical situations<br>• believe that mathematics makes sense. | • recognize and apply deductive and inductive reasoning<br>• understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs<br>• make and evaluate mathematical conjectures and arguments<br>• validate their own thinking<br>• appreciate the pervasive use and power of reasoning as a part of mathematics. | • make and test conjectures<br>• formulate counterexamples<br>• follow logical arguments<br>• judge the validity of arguments<br>• construct simple valid arguments. |
| (NCTM, p. 29) | (NCTM, p. 81) | (NCTM, p. 143) |

## Technology

*Technology will aid students in solving complex problems.*

Improvements in technology, and its increased availability in schools, have changed the focus of mathematics education. The time saved by using calculators or computers to perform complex calculations can be used to help students better understand mathematical concepts. Students can then understand the relationships among concepts and use these relationships to solve problems.

Calculators and computers can be used as tools to:

- develop concepts
- explore and demonstrate mathematical relationships and patterns
- organize and display data
- assist with solving problems and thus promote independence
- encourage students to be inquisitive and creative
- decrease the time spent on tedious computations
- reinforce the learning of basic number facts and properties
- develop an understanding of computational algorithms
- create geometric displays
- simulate situations.

In some cases, technology will allow teachers to ask questions requiring a high level of thinking and will allow students to solve complex, multifaceted problems. Technology can foster environments in which the growing curiosity of students can lead to rich mathematical discoveries. In these environments, the control of exploring mathematical ideas can be turned over to students.

## Visualization

*Images are useful in describing the physical and mathematical environment.*

Visualization "involves thinking in *pictures* and *images* and the ability to perceive, transform and re-create different aspects of the visual–spatial world." (Armstrong, p. 10, italics in original) The use of images in the study of mathematics provides students with the opportunity to understand mathematical concepts and to make connections among them.

The physical environment is full of images. The images are of 3-D objects, 2-D shapes, 1-D lines and pictures. In geometry, the study of a 3-D object is assisted by visualizing either the net of 2-D shapes or the skeleton of 1-D lines required to construct the object.

The mathematical environment is full of images. These images are used to communicate mathematical concepts and multiple solutions to problems. At an elementary level, four piles, each containing three coins, can be used to represent $3 + 3 + 3 + 3 = 12$. Rearranging the piles into four rows of 3 can then be used to represent $4 \times 3 = 12$. Connecting the two images links the process of multiplication with that of repeated addition. At a more advanced level, analytic geometry describes figures algebraically and provides a tool for the visualization of algebraic relations. The analysis and interpretation of data, using a visual summary, aids in understanding the data and making predictions from it.

## NATURE OF MATHEMATICS

- *Change*
- *Constancy*
- *Dimension*
- *Number*
- *Pattern*
- *Quantity*
- *Relationships*
- *Shape*
- *Uncertainty*

By enriching our view of mathematics and the learning environment, the outcomes of The Common Framework can be accomplished.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding. . . . Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching." (Caine, p. 5)

There are additional critical components that must be addressed in a mathematics program beyond those listed as mathematical processes. The components discussed are: Pattern, Number, Shape, Change, Constancy, Dimension (size and scale), Relationships, Quantity and Uncertainty. They are used to describe mathematics in a broad way in order to establish the wide variety of connections that can be made among the various strands used to organize the outcomes central to The Common Framework.

### Change

*Change is a very broad concept. Students must become sensitive to patterns, such as linear, exponential, logarithmic and periodic.*

Change can be discussed from Kindergarten to Grade 12 across many aspects of mathematics. The study of change is often discussed in the context of calculus, but is often limited to this context. However, change is a much broader concept than that used in calculus. In order to make predictions, students need to describe and quantify their observations, attempt to build patterns, and identify those quantities that remain fixed and those quantities that change. For example, look at the pattern 4, 6, 8, 10, 12, . . . An elementary school student can describe this as skip counting by 2s, starting from 4. A senior high school student may describe this pattern as an arithmetic sequence, with first term 4, and a common difference of 2. Another student may describe it as a linear function with a discrete domain. All three interpretations are focusing on the changing size of the numbers within the sequence. To be able to understand change, students must become sensitive to patterns, such as linear, exponential, logarithmic and periodic. (Steen, p. 184)

### Constancy

*Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.*

Students are expected to communicate ideas visually, using diagrams and oral and written words, when describing constancy or invariance. Different aspects of constancy "are described by the terms stability, conservation, equilibrium, steady state, and symmetry." (AAAS–Benchmarks, p. 270) The most important properties in mathematics and science relate to those properties that do not change when outside conditions change. Elementary school students deal with constancy in situations where different methods are used to solve a single multiplication problem, such as finding the area of a 3-tile by 4-tile tabletop. Secondary students need to deal with constancy when they solve the more complicated multiplication problems that appear in determining the number of elements present in the sample spaces of probability problems. Many of these situations will involve permutations and combinations.

In geometry, a circle can be transformed into an ellipse by a simple stretch, and into a square by a more complex series of transformations; but there is no way that the circle can be transformed into a parabola. The closed figures, such as circles and squares, remain closed and cannot be transformed into open figures, such as parabolas. Triangles can be distorted in many ways, but all will have an angle sum of 180°. The straight line is characterized as having all its parts with the same slope. In solving many of the most important problems in mathematics, students need to concentrate on the properties that remain constant. This idea enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

### Dimension (size and scale)

*The concept of dimension needs to be developed within an environment of physical objects.*

The concept of dimension, most usually associated with 3-D objects, 2-D shapes or 1-D lines, needs to be developed within an environment of physical objects for all grades from Kindergarten to Grade 12. The prediction of the change in dimension of objects can be done using numbers attached to appropriate units. For example, with no knowledge of a formula, students in upper elementary grades can predict that doubling the side of a square generates four times the area. Junior and senior high school students need to be able to use algebraic structures to formalize this relationship.

Physical objects can all be described using measurement concepts. The development of perimeter, area and volume concepts relies on pattern recognition, not on memorization of formulas. Descriptions of geometric patterns (number of vertices, sides and edges of various 3-D objects, 2-D shapes and 1-D lines); and the angle sum of various 2-D shapes is also encouraged. This type of data should be placed in charts and/or graphs to help students visualize their findings and predict patterns.

### Number

*The use of number must include number sense.*

Number, number systems and the operations on numbers are vital to all mathematics learning. The use of number must go beyond procedure and accuracy to include what is called number sense. Number sense includes:

- an intuitive feeling about numbers and their multiple relationships
- construction of the meaning of number through a variety of experiences, and development of an appreciation of the need for numbers beyond whole numbers (NCTM, p. 38)
- an appreciation and ability to make quick order of magnitude approximations (Steen, p. 79) with emphasis on establishing quick and accurate estimations for computation and measurement
- the ability to detect arithmetic errors
- knowledge of place value and the effects of arithmetic operations.

Many numerical calculations are performed with calculators and computers, and students must be able to determine if the desired calculations have been done correctly. Students must plan for the efficient use of technological tools.

Number patterns should be recognized and used to count, to make predictions, to describe shapes and to compare.

## Pattern

*Mathematics is an exploratory science that seeks to understand every kind of pattern.*

"What humans do with the language of mathematics is to describe patterns. Mathematics is an exploratory science that seeks to understand every kind of pattern. . . ." (Steen, p. 8) Patterns exist in number, geometry, algebra and data. By helping students recognize, extend, create and use patterns as a routine aspect of their lives, mathematics will become a useful tool to assist them in their systematic and intellectual understanding of their environment.

## Quantity

*Quantitatively literate people use numbers to describe phenomena in all aspects of mathematics.*

"Quantitatively literate young need a flexible ability to identify critical relations in novel situations, to express these relations in effective symbolic form, to use computing tools to process information, and to interpret the results of those calculations." (Steen, p. 65)

Students have a strong desire to measure, code and order things. To this end, some of the outcomes are about single numbers, numbers attached to units of measure, and ordered sets of numbers. Other outcomes are about the interpretation of numbers and of number systems. The use of single numbers and of ordered pairs to describe phenomena in all aspects of mathematics, the natural sciences and the social sciences is very important.

With the growing use of technology to process numerical information, it is becoming essential for students to have a wide range of estimation skills so that they can evaluate whether or not the numerical output provided by a calculator or a computer is a reasonable solution to a given problem.

## Relationships

*The study of mathematics is the development of relationships between and among things.*

The study of mathematics is the development of relationships between and among things. Part of mathematics should help students develop a sense of discovery that mathematicians over the years have felt and should prepare the way for students to make their own discoveries. Students should look for relationships among physical things, as well as the data used to describe those things. Descriptions of the attributes of objects are used to analyze symmetry and congruence and to classify things, using increasingly sophisticated language. Relationships will be described visually, symbolically, orally and in written form.

## Shape

*Shape in mathematics includes geometric representations of algebraic relations, the geometry of maps and the creation of networks of figures.*

Shape in mathematics is central to geometry but also includes geometric representations of algebraic relations, the geometry of maps and the creation of networks of plane figures that can be used to construct 3-D objects. It is very important for students to look for and use similarities, congruences, patterns, transformations, dilatations and tessellations in the solution of a range of problems.

The use of language to describe shapes is an important aspect of mathematics. This description allows for the classification of objects according to various attributes, the naming of objects, and the analysis of objects. The study of shape can be used to build a deductive system, which can assist in further, more detailed analysis. Shape is used in the development of visual models in other disciplines, such as the use of molecular models in chemistry and biology.

The use of technology to analyze and depict shape will increase in importance for students of mathematics as more and better software and hardware become available in classrooms.

## Uncertainty

*Uncertainty involves data, chance, measurements and errors.*

Uncertainty involves data, chance, measurements and errors. Problems dealing with data, together with numbers in context found in the mass media, can be solved within the school mathematics program so long as the data provided and the problems posed have some meaning and relevance to students.

Chance deals with the predictable and the unpredictable outcomes of events. Students from an early age are expected to deal with the concept of chance. As they mature, the language they use to describe chance becomes more sophisticated and involves the vocabulary of probability theory.

When dealing with random events and complex experiments, students can generate large quantities of data requiring analysis. The use of various technologies enables the student to summarize data easily and to create a visualization of the data to help identify patterns in the information. In some instances the functions describing patterns are linear, periodic, logarithmic or exponential, and senior high school students are expected to use the appropriate algebraic structures to model the information contained within the pattern.

The quality of the output information is directly related to the quality of the input data. The study of uncertainty allows students to assess the reliability of input data, and to learn the processes whereby input data is converted to output information.

## STRANDS

- *Number*
- *Patterns and Relations*
- *Shape and Space*
- *Statistics and Probability*

The student outcomes are organized within four strands. The strands are the formal aspects of the discipline of mathematics that form the foundation of The Common Framework and act as connections across the grades. Four strands have been identified for the entire Kindergarten to Grade 12 mathematics framework to reinforce the interrelationship of mathematical concepts and skills. These strands are split into substrands. However, any such grouping into strands and substrands is for organizational purposes only, and does not reflect the connections among the strands and the underlying themes running throughout all of mathematics.

## Number

Number Concepts

*Students will:*
- use numbers to describe quantities
- represent numbers in multiple ways.

Number Operations

*Students will:*
- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

## Patterns and Relations

Patterns

*Students will:*
- use patterns to describe the world and to solve problems.

Variables and Equations

*Students will:*
- represent algebraic expressions in multiple ways.

Relations and Functions

*Students will:*
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

## Shape and Space

Measurement

*Students will:*
- describe and compare everyday phenomena, using either direct or indirect measurement.

3-D Objects and 2-D Shapes

*Students will:*
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations

*Students will:*
- perform, analyze and create transformations.

### Statistics and Probability

Data Analysis

*Students will:*
- collect, display and analyze data to make predictions about a population.

Chance and Uncertainty

*Students will:*
- use experimental or theoretical probability to represent and solve problems involving uncertainty.

## STUDENT EXPECTATIONS

The content of The Common Framework is stated in terms of outcomes. These outcomes are measurable and identify what students are required to know and do.

The outcomes are developed and are based on the expectation that they are appropriate to a large majority of the students. They are stated at the time where they are expected to be "mastered". There may be some time delays between where students first encounter the learning and where they are expected to demonstrate knowledge of, or mastery in, that learning.

### General Outcomes

General outcomes are general statements that identify what students are expected to know and to be able to do upon completion of a grade.

### Specific Outcomes

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome.

### Illustrative Examples

Illustrative examples are sample tasks that demonstrate and elaborate on the general and specific outcomes. They are important in conveying the richness, breadth and depth intended in the outcomes.

## SUMMARY

The components of the Conceptual Framework for K–12 mathematics, as described, dictate what should be happening in mathematics education. The components are not meant to stand alone, but are to be interrelated to enhance one another. Activities that take place in the classroom should stem from a problem-solving approach built on the mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes related to each of the strands.

*Student expectations are described in terms of:*

- *general outcomes*
- *specific outcomes*
- *illustrative examples.*

# IV. INSTRUCTIONAL FOCUS

## SUGGESTED TIME ALLOTMENTS

The Common Framework is arranged into four strands, each of significance. Therefore, considerable time should be spent on the concepts and processes identified in each strand.

Several additional considerations are important:

- Integration of the mathematical processes, within each strand, is encouraged and expected.

- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.

- Problem solving, reasoning and connections are vital to increasing mathematical power and must be integrated throughout the program. A minimum of half the available time within all strands needs to be dedicated to activities related to these processes.

- There is to be a balance between estimation and mental mathematics, paper and pencil exercises and the appropriate use of technology, including calculators and computers. Concepts should be introduced, using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.

- There is an assumption made that all students have regular access to appropriate technology. For most of the work in the patterns and relations strand, the most appropriate technology is the graphing calculator. For the work in number and in statistics and probability, standard spreadsheet programs are appropriate.

## COMMON, APPLIED, PURE FOCUS FOR GRADES 10–12

Each specific outcome, starting on page 30, has a code of (C), (A) or (P) as it has been judged to provide a **C**ommon focus for *all* students, or an **A**pplied or a **P**ure focus for *some* students.

These outcomes are grouped into clusters starting on page 62.

Common clusters, numbered C1 to C6, include the mathematics expected of all students completing a K to 12 mathematics program.

Applied clusters, numbered A1 to A9, emphasize applications of mathematics rather than precise mathematical theory. The approaches used are primarily numerical and geometrical.

Pure clusters, numbered P1 to P9, place more emphasis on precise mathematical theory. The approaches used are primarily algebraic and graphical.

The order of the clusters is intended to indicate a sequence that might be used to construct courses and programs of study.

Any Grade 10 courses identified would be made up of clusters early in the sequence, while any Grade 12 courses would be made up of clusters later in the sequence.

After the clusters for a course have been selected, the outcomes can be reordered by strand. This reordering may help establish connections among various mathematical and problem-solving contexts included in clusters.

# V. STUDENT OUTCOMES

This section of the document is divided into three parts, each of which serves a different but cumulative purpose.

**General Outcomes** (pages 22–29)

This section presents the general outcomes, of The Common Framework, for each strand, Kindergarten through Grade 12, to show the direction and scope of the total curriculum.

**General Outcomes and Specific Outcomes**
(pages 30–59)

This section presents the general and specific outcomes, organized by strand, for Grade 9 through Grade 12. This grouping shows the relationships between the general outcomes and the specific outcomes and the coding of (C) for Common, (P) for Pure and (A) for Applied.

The Grade 9 outcomes are included to provide continuity from the June 1995 document (Kindergarten through Grade 9) to this document (Grade 10 through Grade 12).

Each specific outcome is coded for mathematical processes, using the codes listed on the top of each page from page 30 to page 59.

**General Outcomes, and Specific Outcomes with Illustrative Examples, Organized by Cluster**
(pages 61–190)

This section adds sample tasks to the general and specific outcomes, and is organized by strand, within a cluster. Most of these examples add clarity about the intended depth and breadth of the specific outcomes. A few illustrative examples are designed to convey the intended depth of a general outcome.

**Numbering System**

In the General Outcomes and Specific Outcomes section (pages 30–59), the specific outcomes are numbered sequentially within each strand. Cross-referencing between this section and the illustrative example section (pages 61–190) has been done. For example, $\underset{(C2-6)}{PR53.}$ is the 53rd specific outcome in the Patterns and Relations strand. It can be cross-referenced to the illustrative example section as the 6th specific outcome in Common Cluster 2.

# K–12 GENERAL OUTCOMES—Number Strand

| Substrand | K | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Number Concepts**<br>*Students will:*<br>• use numbers to describe quantities<br>• represent numbers in multiple ways. | Describe, orally, and compare quantities from 0 to 10, using number words in daily experiences. | Recognize and apply whole numbers from 0 to 100, and explore halves, in familiar settings. | Recognize and apply whole numbers up to 1000, and explore fractions (halves, thirds and quarters). | Develop a number sense for whole numbers 0 to 1000, and explore fractions (fifths and tenths). | Demonstrate a number sense for whole numbers 0 to 10 000, and explore proper fractions. | Demonstrate a number sense for whole numbers, 0 to 100 000, and explore proper fractions and decimals. |
| **Number Operations**<br>*Students will:*<br>• demonstrate an understanding of and proficiency with calculations<br>• decide which arithmetic operation or operations can be used to solve a problem and then solve the problem. | Demonstrate awareness of addition and subtraction. | Apply informal methods of addition and subtraction on whole numbers where the maximum sum is 18. | Apply a variety of addition and subtraction strategies on whole numbers to 100, and use these operations in solving problems.<br><br>Use an appropriate calculation strategy or technology to solve problems. | Apply an arithmetic operation (addition, subtraction, multiplication or division) on whole numbers, and illustrate its use in creating and solving problems.<br><br>Use and justify an appropriate calculation strategy or technology to solve problems. | Apply arithmetic operations on whole numbers, and illustrate their use in creating and solving problems.<br><br>Use and justify an appropriate calculation strategy or technology to solve problems.<br><br>Demonstrate an understanding of addition and subtraction of decimals. | Apply arithmetic operations on whole numbers and decimals, and illustrate their use in creating and solving problems. |

| 6 | 7 | 8 | 9 | 10–12 |
|---|---|---|---|---|
| Develop a number sense for decimals and common fractions, explore integers, and show number sense for whole numbers. | Demonstrate a number sense for decimals and integers, including whole numbers. | Demonstrate a number sense for rational numbers, including common fractions, integers and whole numbers. | Explain and illustrate the structure and the interrelationship of the sets of numbers within the rational number system.<br><br>Develop a number sense of powers with integral exponents and rational bases. | Analyze the numerical data in a table for trends, patterns and interrelationships.<br>Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system. |
| Apply arithmetic operations on whole numbers and decimals in solving problems. | Apply arithmetic operations on decimals and integers, and illustrate their use in solving problems.<br><br>Illustrate the use of rates, ratios, percentages and decimals in solving problems. | Apply arithmetic operations on rational numbers to solve problems.<br><br>Apply the concepts of rate, ratio, percentage and proportion to solve problems in meaningful contexts. | Use a scientific calculator or a computer to solve problems involving rational numbers.<br><br>Explain how exponents can be used to bring meaning to large and small numbers, and use calculators or computers to perform calculations involving these numbers. | Use basic arithmetic operations on real numbers to solve problems.<br>Describe and apply arithmetic operations on tables to solve problems, using technology as required.<br>Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.<br>Solve consumer problems, using arithmetic operations.<br>Describe and apply operations on matrices to solve problems, using technology as required.<br>Design or use a spreadsheet to make and justify financial decisions. |

## K–12 GENERAL OUTCOMES—Patterns and Relations Strand

| Substrand | K | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Patterns**<br>*Students will:*<br>• use patterns to describe the world and to solve problems. | Identify and create patterns arising from daily experiences. | Identify, create and compare patterns arising from daily experiences in the classroom. | Identify, create, describe and translate numerical and non-numerical patterns arising from daily experiences in the school and on the playground. | Investigate, establish and communicate rules for numerical and non-numerical patterns, including those found in the home, and use these rules to make predictions. | Investigate, establish and communicate rules for, and predictions from, numerical and non-numerical patterns, including those found in the community. | Construct, extend and summarize patterns, including those found in nature, using rules, charts, mental mathematics and calculators. |
| **Variables and Equations**<br>*Students will:*<br>• represent algebraic expressions in multiple ways. | | | | | | |
| **Relations and Functions**<br>*Students will:*<br>• use algebraic and graphical models to generalize patterns, make predictions and solve problems. | | | | | | |

| 6 | 7 | 8 | 9 | 10–12 |
|---|---|---|---|---|
| Use relationships to summarize, generalize and extend patterns, including those found in music and art. | Express patterns, including those used in business and industry, in terms of variables, and use expressions containing variables to make predictions. | Use patterns, variables and expressions, together with their graphs, to solve problems. | Generalize, design and justify mathematical procedures, using appropriate patterns, models and technology. | Generate and analyze number patterns.<br>Apply the principles of mathematical reasoning to solve problems and to justify solutions.<br>Generate and analyze cyclic, recursive and fractal patterns.<br>Generate and analyze exponential patterns. |
| Use informal and concrete representations of equality and operations on equality to solve problems. | Use variables and equations to express, summarize and apply relationships as problem-solving tools in a restricted range of contexts. | Solve and verify one-step and two-step linear equations with rational number solutions. | Solve and verify linear equations and inequalities in one variable.<br><br>Generalize arithmetic operations from the set of rational numbers to the set of polynomials. | Generalize operations on polynomials to include rational expressions.<br>Represent and analyze situations that involve expressions, equations and inequalities.<br>Use linear programming to solve optimization problems.<br>Solve exponential, logarithmic and trigonometric equations and identities. |
| | | | | Examine the nature of relations with an emphasis on functions.<br>Represent data, using linear function models.<br>Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.<br>Represent and analyze exponential and logarithmic functions, using technology as appropriate.<br>Represent and analyze trigonometric functions, using technology as appropriate. |

# K–12 GENERAL OUTCOMES—Shape and Space Strand

| Substrand | K | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Measurement**<br>*Students will:*<br>• describe and compare everyday phenomena, using either direct or indirect measurement. | Demonstrate awareness of measurement. | Estimate, measure and compare, using whole numbers and nonstandard units of measure. | Estimate, measure and compare, using standard units for length and primarily nonstandard units for other measures. | Estimate, measure and compare, using whole numbers and primarily standard units of measure. | Estimate, measure and compare, using decimal numbers and standard units of measure. | Use measurement concepts, appropriate tools and results of measurements to solve problems in everyday contexts. |
| **3-D Objects and 2-D Shapes**<br>*Students will:*<br>• describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them. | Sort, classify and build real-world objects. | Explore and classify 3-D objects and 2-D shapes, according to their properties. | Name, describe and construct a variety of 3-D objects and 2-D shapes. | Describe, classify, construct and relate 3-D objects and 2-D shapes. | Describe, classify, construct and relate 3-D objects and 2-D shapes, using mathematical vocabulary. | Use visualization of 3-D objects and 2-D shapes to solve problems related to spatial relations. |
| **Transformations**<br>*Students will:*<br>• perform, analyze and create transformations. | Describe, orally, the position of 3-D objects. | Describe, orally, the relative position of 3-D objects and 2-D shapes. | Apply positional language, orally and in writing, to communicate motion. | Use numbers and direction words to describe the relative positions of objects in one dimension, using everyday contexts. | Use numbers and direction words to describe the relative positions of objects in two dimensions, using everyday contexts. | Describe motion in terms of a slide, a turn or a flip.<br><br>Use coordinates to describe the positions of objects in two dimensions. |

| 6 | 7 | 8 | 9 | 10–12 |
|---|---|---|---|---|
| Solve problems involving perimeter, area, surface area, volume and angle measurement. | Solve problems involving the properties of circles and their connections with angles and time zones. | Apply indirect measurement procedures to solve problems.<br><br>Generalize measurement patterns and procedures, and solve problems involving area, perimeter, surface area and volume. | Use trigonometric ratios to solve problems involving a right triangle.<br><br>Describe the effects of dimension changes in related 2-D shapes and 3-D objects in solving problems involving area, perimeter, surface area and volume. | Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.<br>Solve problems involving triangles, including those found in 3-D and 2-D applications.<br>Use measuring devices to make estimates and to perform calculations in solving problems.<br>Analyze objects, shapes and processes to solve cost and design problems. |
| Use visualization and symmetry to solve problems involving classification and sketching. | Link angle measures to the properties of parallel lines. | Link angle measures and the properties of parallel lines to the classification and properties of quadrilaterals. | Specify conditions under which triangles may be similar or congruent, and use these conditions to solve problems.<br><br>Use spatial problem solving in building, describing and analyzing geometric shapes. | Solve coordinate geometry problems involving lines and line segments.<br>Solve coordinate geometry problems involving lines and line segments, and justify the solutions.<br>Develop and apply the geometric properties of circles and polygons to solve problems.<br>Solve problems involving polygons and vectors, including both 3-D and 2-D applications.<br>Classify conic sections, using their shapes and equations. |
| Create patterns and designs that incorporate symmetry, tessellations, translations and reflections. | Create and analyze patterns and designs, using congruence, symmetry, translation, rotation and reflection. | Create and analyze design problems and architectural patterns, using the properties of scaling, proportion and networks. | Apply coordinate geometry and pattern recognition to predict the effects of translations, rotations, reflections and dilatations on 1-D lines and 2-D shapes. | Perform, analyze and create transformations of functions and relations that are described by equations or graphs. |

## K–12 GENERAL OUTCOMES—Statistics and Probability Strand

| Substrand | K | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Data Analysis**<br>*Students will:*<br>• collect, display and analyze data to make predictions about a population. | Collect and organize, with assistance, data based on first-hand information. | Collect, organize and describe, with guidance, data based on first-hand information. | Collect, display and describe data, independently, based on first-hand information. | Collect first- and second-hand data, display the results in more than one way, and interpret the data to make predictions. | Collect first- and second-hand data, assess and validate the collection process, and graph the data. | Develop and implement a plan for the collection, display and interpretation of data to answer a question. |
| **Chance and Uncertainty**<br>*Students will:*<br>• use experimental or theoretical probability to represent and solve problems involving uncertainty. | | Describe concepts of chance and chance events, using ordinary vocabulary. | Use simple experiments, designed by others, to illustrate chance. | Use simple probability experiments, designed by others, to explain outcomes. | Design and use simple probability experiments to explain outcomes. | Predict outcomes, conduct experiments and communicate the probability of single events. |

| 6 | 7 | 8 | 9 | 10–12 |
|---|---|---|---|---|
| Develop and implement a plan for the collection, display and analysis of data gathered from appropriate samples. | Develop and implement a plan for the collection, display and analysis of data, using measures of variability and central tendency. | Develop and implement a plan for the collection, display and analysis of data, using technology, as required.<br><br>Evaluate and use measures of central tendency and variability. | Collect and analyze experimental results expressed in two variables, using technology, as required. | Implement and analyze sampling procedures, and draw appropriate inferences from the data collected.<br>Apply line-fitting and correlation techniques to analyze experimental results.<br>Analyze graphs or charts of given situations to derive specific information. |
| Use numbers to communicate the probability of single events from experiments and models. | Create and solve problems, using probability. | Compare theoretical and experimental probability of independent events. | Explain the use of probability and statistics in the solution of complex problems. | Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events.<br>Use normal and binomial probability distributions to solve problems involving uncertainty.<br>Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.<br>Model the probability of a compound event, and solve problems based on the combining of simpler probabilities. |

# GRADES 10–12 GENERAL AND SPECIFIC OUTCOMES

## Grade 9

**General Outcome**

Explain and illustrate the structure and the interrelationship of the sets of numbers within the rational number system.

**Specific Outcomes**

1. Give examples of numbers that satisfy the conditions of natural, whole, integral and rational numbers, and show that these numbers comprise the rational number system.
   [C, CN, PS, R]

2. Describe, orally and in writing, whether or not a number is rational.
   [C, R]

3. Give examples of situations where answers would involve the positive (principal) square root, or both positive and negative square roots of a number.
   [C, CN, PS, R]

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Number (Number Concepts)
*Students will:*
- use numbers to describe quantities
- represent numbers in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Analyze the numerical data in a table for trends, patterns and interrelationships. | N1. (C1–1) Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data). [C, CN] |
| | N2. (C1–2) Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data). [C, CN] |
| Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system. | N3. (C1–3) Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V] |
| | N4. (C1–4) Use approximate representations of irrational numbers. [R, T] |

## Grade 9

**General Outcome**

Develop a number sense of powers with integral exponents and rational bases.

**Specific Outcomes**

4. Illustrate power, base, coefficient and exponent, using rational numbers or variables as bases or coefficients. [R, V]

*(continued)*

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Number (Number Concepts)
*Students will:*

- use numbers to describe quantities
- represent numbers in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| | |

## Grade 9

*(continued)*

5. Explain and apply the exponent laws for powers with integral exponents.

$x^m \bullet x^n = x^{m+n}$

$x^m \div x^n = x^{m-n}$

$\left(x^m\right)^n = x^{mn}$

$(xy)^m = x^m y^m$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n},\ y \neq 0$

$x^0 = 1,\ x \neq 0$

$x^{-n} = \frac{1}{x^n},\ x \neq 0$

[PS, R]

6. Determine the value of powers with integral exponents, using the exponent laws.
[PS, R]

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Number (Number Concepts)
*Students will:*

- use numbers to describe quantities
- represent numbers in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| | |

## Grade 9

**General Outcome**

Use a scientific calculator or a computer to solve problems involving rational numbers.

**Specific Outcomes**

7. Document and explain the calculator keying sequences used to perform calculations involving rational numbers. [C, PS, T]

8. Solve problems, using rational numbers in meaningful contexts. [CN, PS]

## Grades 10–12
## Strand: Number (Number Operations)

*Students will:*

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

**[C]** Communication **[PS]** Problem Solving
**[CN]** Connections **[R]** Reasoning
**[E]** Estimation and Mental Mathematics **[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Use basic arithmetic operations on real numbers to solve problems. | N5. (C1–5) Communicate a set of instructions used to solve an arithmetic problem. [C] |
| | N6. (C1–6) Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T] |
| Describe and apply arithmetic operations on tables to solve problems, using technology as required. | N7. (C1–7) Create and modify tables from both recursive and nonrecursive situations. [PS, T, V] |
| | N8. (C1–8) Use and modify a spreadsheet template to model recursive situations. [PS, T, V] |
| | N9. (A2–1) Solve problems involving combinations of tables, using:<br>• addition or subtraction of two tables<br>• multiplication of a table by a real number<br>• spreadsheet functions and templates.<br>[PS, T, V] |
| Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems. | N10. (P1–1) Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E] |
| | N11. (P2–1) Perform operations on irrational numbers of monomial and binomial form, using exact values. [E] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Explain how exponents can be used to bring meaning to large and small numbers, and use calculators or computers to perform calculations involving these numbers.

**Specific Outcomes**

9. Understand and use the exponent laws to simplify expressions with variable bases and evaluate expressions with numerical bases.
   [PS, R]

10. Use a calculator to perform calculations involving scientific notation and exponent laws.
    [PS, R, T]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Number (Number Operations)

*Students will:*

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Solve consumer problems, using arithmetic operations. | N12. (C4–1) Solve consumer problems, including:<br>• wages earned in various situations<br>• property taxation<br>• exchange rates<br>• unit prices.<br>[CN, E, PS, R, T]<br><br>N13. (C4–2) Reconcile financial statements including:<br>• cheque books with bank statements<br>• cash register tallies with daily receipts.<br>[CN, PS, T]<br><br>N14. (C4–3) Solve budget problems, using graphs and tables to communicate solutions.<br>[C, PS, T, V]<br><br>N15. (C4–4) Plot and describe data of exponential form, using appropriate scales.<br>[C, T, V]<br><br>N16. (C4–5) Solve investment and credit problems involving simple and compound interest.<br>[CN, PS, T] |

Grades 10–12
Strand: Number (Number Operations)
*Students will:*

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes | |
|---|---|---|
| Describe and apply operations on matrices to solve problems, using technology as required. | N17. (A6–1) | Show an understanding of matrices and perform the operations of addition, scalar multiplication and matrix multiplication. [C, T] |
| | N18. (A6–2) | Solve problems, using the operations of addition, subtraction, scalar multiplication and matrix multiplication on matrices. [PS, R, T, V] |
| | N19. (A6–3) | Use matrices and matrix operations to model and to solve consumer, network and schedule problems. [C, CN, PS, R, T, V] |
| Design or use a spreadsheet to make and justify financial decisions. | N20. (A8–1) | Design or modify a financial spreadsheet template to allow users to input their own variables. [C, PS, T] |
| | N21. (A8–2) | Use spreadsheets to analyze renting or buying an increasing asset (home) under different sets of circumstances. [C, PS, T] |
| | N22. (A8–3) | Use spreadsheets to analyze leasing or buying a decreasing asset (vehicle, computer) under different sets of circumstances. [C, PS, T] |
| | N23. (A8–4) | Use spreadsheet(s) to analyze an investment or life insurance portfolio, applying such concepts as capital gains, interest rate, inflation rate, risk, total rate of return and after-tax rate of return. [C, PS, T] |
| | N24. (A8–5) | Analyze car or house insurance needs and premiums, using such concepts as loss, probability of loss, compulsory coverage, optional coverage, deductible and claims record. [CN, E, R, T] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Generalize, design and justify mathematical procedures, using appropriate patterns, models and technology.

**Specific Outcomes**

1. Use logic and divergent thinking to present mathematical arguments in solving problems.
   [C, PS, R]

2. Model situations that can be represented by first-degree expressions.
   [CN, PS]

3. Write equivalent forms of algebraic expressions, or equations, with rational coefficients.
   [C, CN, R]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Patterns and Relations (Patterns)

*Students will:*

- use patterns to describe the world and to solve problems.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Generate and analyze number patterns. | PR1. (P2–2) Generate number patterns exhibiting arithmetic growth. [E, R] |
| | PR2. (P2–3) Use expressions to represent general terms and sums for arithmetic growth, and apply these expressions to solve problems. [CN, PS, R, T] |
| | PR3. (P2–4) Relate arithmetic sequences to linear functions defined over the natural numbers. [CN] |
| | PR4. (P2–5) Generate number patterns exhibiting geometric growth. [E, R] |
| Apply the principles of mathematical reasoning to solve problems and to justify solutions. | PR5. (P5–1) Differentiate between inductive and deductive reasoning. [CN, R] |
| | PR6. (P5–2) Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems. [C, PS, R, V] |
| | PR7. (P5–3) Use examples and counterexamples to analyze conjectures. [CN, R] |
| | PR8. (P5–4) Distinguish between an “if–then” proposition, its converse and its contrapositive. [CN, R] |
| | PR9. (P5–5) Prove assertions in a variety of settings, using direct and indirect reasoning. [R] |

Grades 10–12
Strand: Patterns and Relations (Patterns)
*Students will:*

- use patterns to describe the world and to solve problems.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Generate and analyze cyclic, recursive and fractal patterns. | PR10. (A7–1) From cyclic data produce a periodic graph. [C, PS, V]<br><br>PR11. (A7–2) Predict results from graphs that represent periodic events. [E, R, V]<br><br>PR12. (A7–3) Describe periodic events, including sinusoidal curves, using correct terminology. [C, V]<br><br>PR13. (A7–4) Collect sinusoidal data; sketch the graph of the data; and, using degrees, represent the data with an equation of the form:<br>• $y = a \sin(kt) + c$<br>**OR**<br>• $y = a \cos(kt) + c$.<br>[CN, PS, T, V]<br><br>PR14. (A7–5) Develop sinusoidal equations, using degrees, to represent periodic behaviour. [CN, PS, T]<br><br>PR15. (A7–6) Use technology to generate and graph finite or infinite sequences whose recursive definition may or may not be given. [PS, T, V]<br><br>PR16. (A7–7) Identify sequences that appear to be:<br>• divergent<br>• convergent<br>• oscillating<br>• static.<br>[C, V] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Patterns)
*Students will:*

- use patterns to describe the world and to solve problems.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Generate and analyze exponential patterns. | PR17. (A7–8) Construct a fractal pattern by repeatedly applying a procedure to a geometric figure. [CN, R, V]<br><br>PR18. (A7–9) Use the concept of self-similarity to compare and/or predict the perimeters, areas and volumes of fractal patterns. [CN, R, V]<br><br>PR19. (P6–1) Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems. [CN, R, T]<br><br>PR20. (P6–2) Connect geometric sequences to exponential functions over the natural numbers. [E, R, V]<br><br>PR21. (P6–3) Estimate values of expressions for infinite geometric processes. [PS, R, T] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Solve and verify linear equations and inequalities in one variable.

**Specific Outcomes**

4. Illustrate the solution process for a first-degree, single-variable equation, using concrete materials or diagrams. [PS, R, V]

5. Solve and verify first-degree, single-variable equations of forms, such as:
   - $ax = b + cx$
   - $a(x + b) = c$
   - $ax + b = cx + d$
   - $a(bx + c) = d(ex + f)$
   - $\frac{a}{x} = b$

   where $a$, $b$, $c$, $d$, $e$ and $f$ are all rational numbers (with a focus on integers), and use equations of this type to model and solve problem situations. [C, PS, V]

*(continued)*

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Variables and Equations)
*Students will:*
- represent algebraic expressions in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Generalize operations on polynomials to include rational expressions. | PR22. (P1–2) Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]<br><br>PR23. (P1–3) Find the product of polynomials. [E, R]<br><br>PR24. (P1–4) Divide a polynomial by a binomial, and express the result in the forms:<br>• $\frac{P}{D} = Q + \frac{R}{D}$<br>• $P = DQ + R$<br>• $P(x) = D(x)Q(x) + R$.<br>[E, R]<br><br>PR25. (P1–5) Determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored. [PS, R]<br><br>PR26. (P1–6) Determine the nonpermissible values for the variable in rational expressions. [C, CN]<br><br>PR27. (P1–7) Perform the operations of addition, subtraction, multiplication and division on rational expressions. [E, R]<br><br>PR28. (P1–8) Find and verify the solutions of rational equations. [CN, PS] |

## Grade 9

*(continued)*

6. Solve, algebraically, first-degree inequalities in one variable, display the solutions on a number line and test the solutions.
[PS, R, V]

**General Outcome**

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

**Specific Outcomes**

7. Identify constant terms, coefficients and variables in polynomial expressions.
[C]
8. Evaluate polynomial expressions, given the value(s) of the variable(s).
[E]
9. Represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams.
[C, R, V]

*(continued)*

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Patterns and Relations (Variables and Equations)

*Students will:*

- represent algebraic expressions in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Represent and analyze situations that involve expressions, equations and inequalities. | PR29. (C5–1) Graph linear inequalities, in two variables. [PS, V]<br><br>PR30. (C5–2) Solve systems of linear equations, in two variables:<br>• algebraically (elimination and substitution)<br>• graphically.<br>[CN, PS, T, V]<br><br>PR31. (C5–3) Solve nonlinear equations, using a graphing tool. [CN, T, V]<br><br>PR32. (P3–1) Solve nonlinear equations:<br>• by factoring<br>• graphically.<br>[CN, T, V]<br><br>PR33. (P3–2) Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. [E, PS, T]<br><br>PR34. (P3–3) Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]<br><br>PR35. (P3–4) Solve systems of linear equations, in three variables:<br>• algebraically<br>• with technology.<br>[CN, PS, T, V] |

**Grade 9**

*(continued)*

10. Perform the operations of addition and subtraction on polynomial expressions.
[R]

11. Represent multiplication, division and factoring of monomials, binomials, and trinomials of the form $x^2+bx+c$, using concrete materials and diagrams.
[R, V]

12. Find the product of two monomials, a monomial and a polynomial, and two binomials.
[R]

13. Determine equivalent forms of algebraic expressions by identifying common factors and factoring trinomials of the form $x^2+bx+c$.
[PS, R]

14. Find the quotient when a polynomial is divided by a monomial.
[R]

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Variables and Equations)
*Students will:*
- represent algebraic expressions in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Use linear programming to solve optimization problems. | PR36. (A5–1) Solve, graphically, systems of linear inequalities, in two variables, using technology. [CN, PS, T, V] |
| | PR37. (A5–2) Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V] |
| | PR38. (A5–3) Apply linear programming to find optimal solutions to decision-making problems. [C, PS, R, T, V] |
| Solve exponential, logarithmic and trigonometric equations and identities. | PR39. (P6–4) Solve exponential equations having bases that are powers of one another. [E, R] |
| | PR40. (P6–5) Solve and verify exponential and logarithmic equations and identities. [R] |
| | PR41. (P8–1) Distinguish between degree and radian measure, and solve problems, using both. [CN, E] |
| | PR42. (P8–2) Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$. [CN, E] |
| | PR43. (P8–3) Solve first and second degree trigonometric equations over a domain of length $2\pi$:<br>• algebraically<br>• graphically.<br>[PS, T] |

Grades 10–12
Strand: Patterns and Relations (Variables and Equations)
*Students will:*

- represent algebraic expressions in multiple ways.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| | PR44. (P8–4) Determine the general solutions to trigonometric equations where the domain is the set of real numbers. [PS, T]<br><br>PR45. (P8–5) Verify trigonometric identities:<br>• numerically for any particular case<br>• algebraically for general cases<br>• graphically.<br>[PS, R, T, V]<br><br>PR46. (P8–6) Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions.<br>[R, T] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Relations and Functions)
*Students will:*
- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

| | | | |
|---|---|---|---|
| **[C]** | Communication | **[PS]** | Problem Solving |
| **[CN]** | Connections | **[R]** | Reasoning |
| **[E]** | Estimation and Mental Mathematics | **[T]** | Technology |
| | | **[V]** | Visualization |

| General Outcomes | Specific Outcomes |
|---|---|
| Examine the nature of relations with an emphasis on functions. | PR47. (C1–9) Plot linear and nonlinear data, using appropriate scales. [C, V]<br><br>PR48. (C2–1) Represent data, using function models. [CN, PS, V]<br><br>PR49. (C2–2) Use a graphing tool to draw the graph of a function from its equation. [C, T, V]<br><br>PR50. (C2–3) Describe a function in terms of:<br>• ordered pairs<br>• a rule, in word or equation form<br>• a graph.<br>[C, CN, V]<br><br>PR51. (C2–4) Use function notation to evaluate and represent functions. [C, PS]<br><br>PR52. (C2–5) Determine the domain and range of a relation from its graph. [PS, V]<br><br>PR53. (C2–6) Determine the following characteristics of the graph of a linear function, given its equation:<br>• intercepts<br>• slope<br>• domain<br>• range.<br>[PS, V] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Relations and Functions)

*Students will:*

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Represent data, using linear function models. | PR54. (P4–1) Perform operations on functions and compositions of functions. [CN, E, PS]<br><br>PR55. (P4–2) Determine the inverse of a function. [CN, R, V]<br><br>PR56. (C2–7) Use direct variation and arithmetic sequences as applications of linear functions. [CN, PS, V] |
| Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate. | PR57. (C5–4) Determine the following characteristics of the graph of a quadratic function:<br>• vertex<br>• domain and range<br>• axis of symmetry<br>• intercepts.<br>[C, PS, T, V]<br><br>PR58. (P4–3) Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]<br><br>PR59. (P4–4) Model real-world situations, using quadratic functions. [CN, PS]<br><br>PR60. (P4–5) Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:<br>• factoring<br>• the quadratic formula<br>• graphing.<br>[CN, E, T, V] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Relations and Functions)
*Students will:*

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

| | | | |
|---|---|---|---|
| **[C]** | Communication | **[PS]** | Problem Solving |
| **[CN]** | Connections | **[R]** | Reasoning |
| **[E]** | Estimation and Mental Mathematics | **[T]** | Technology |
| | | **[V]** | Visualization |

| General Outcomes | Specific Outcomes |
|---|---|
| Represent and analyze exponential and logarithmic functions, using technology as appropriate. | PR61. (P4–6) Determine the character of the real and non-real roots of a quadratic equation, using:<br>• the discriminant in the quadratic formula<br>• graphing.<br>[C, R, T, V] |
| | PR62. (P4–7) Describe, graph and analyze polynomial and rational functions, using technology.<br>[C, R, T, V] |
| | PR63. (P4–8) Formulate and apply strategies to solve absolute value equations, radical equations, rational equations and inequalities.<br>[CN, R, V] |
| | PR64. (P6–6) Graph and analyze an exponential function, using technology.<br>[R, T, V] |
| | PR65. (P6–7) Model, graph and apply exponential functions to solve problems.<br>[PS, T, V] |
| | PR66. (P6–8) Change functions from exponential form to logarithmic form and vice versa.<br>[CN] |
| | PR67. (P6–9) Use logarithms to model practical problems.<br>[CN, PS, V] |
| | PR68. (P6–10) Explain the relationship between the laws of logarithms and the laws of exponents.<br>[C, T] |
| | PR69. (P6–11) Graph and analyze logarithmic functions with and without technology.<br>[R, T, V] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Patterns and Relations (Relations and Functions)

*Students will:*

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

| | | | |
|---|---|---|---|
| **[C]** | Communication | **[PS]** | Problem Solving |
| **[CN]** | Connections | **[R]** | Reasoning |
| **[E]** | Estimation and Mental Mathematics | **[T]** | Technology |
| | | **[V]** | Visualization |

| General Outcomes | Specific Outcomes |
|---|---|
| Represent and analyze trigonometric functions, using technology as appropriate. | PR70. (P8–7) Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position. [PS, R, V]<br><br>PR71. (P8–8) Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for:<br>• amplitude, if defined<br>• period<br>• domain and range<br>• asymptotes, if any<br>• behaviour under transformations.<br>[CN, T, V]<br><br>PR72. (P8–9) Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for:<br>• period<br>• domain and range<br>• asymptotes<br>• behaviour under transformations.<br>[CN, T, V]<br><br>PR73. (P8–10) Use trigonometric functions to model and solve problems. [PS, R, V] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Use trigonometric ratios to solve problems involving a right triangle.

**Specific Outcomes**

1. Explain the meaning of sine, cosine and tangent ratios in right triangles.
   [C]
2. Demonstrate the use of trigonometric ratios (sine, cosine and tangent) in solving right triangles.
   [PS]
3. Calculate an unknown side or an unknown angle in a right triangle, using appropriate technology.
   [PS, T]
4. Model and then solve given problem situations involving only one right triangle.
   [PS, T, V]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Shape and Space (Measurement)

*Students will:*

- describe and compare everyday phenomena, using either direct or indirect measurement.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. | SS1. (C3–1) Calculate the volume and surface area of a sphere, using formulas that are provided. [CN, PS, V]<br><br>SS2. (C3–2) Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects. [CN, PS, R, V]<br><br>SS3. (A3–1) Enlarge or reduce a dimensioned object, according to a specified scale. [C, CN, PS, V] |
| Solve problems involving triangles, including those found in 3-D and 2-D applications. | SS4. (C3–3) Solve problems involving two right triangles. [CN, PS, V]<br><br>SS5. (C3–4) Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]<br><br>SS6. (C3–5) Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]<br><br>SS7. (P3–5) Solve problems involving ambiguous case triangles in 3-D and 2-D. [CN, PS, R, T] |

## Grade 9

**General Outcome**

Describe the effects of dimension changes in related 2-D shapes and 3-D objects in solving problems involving area, perimeter, surface area and volume.

**Specific Outcomes**

5. Relate expressions for volumes of pyramids to volumes of prisms, and volumes of cones to volumes of cylinders.
   [CN, R]

6. Calculate and apply the rate of volume to surface area to solve design problems in three dimensions.
   [PS, T, V]

7. Calculate and apply the rate of area to perimeter to solve design problems in two dimensions.
   [PS, T, V]

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Shape and Space (Measurement)
*Students will:*

- describe and compare everyday phenomena, using either direct or indirect measurement.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Use measuring devices to make estimates and to perform calculations in solving problems. | SS8. (A1–1) Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes. [E, PS, T]<br><br>SS9. (A1–2) Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy. [C, R]<br><br>SS10. (A1–3) Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]<br><br>SS11. (A1–4) Interpret drawings, and use the information to solve problems. [C, PS, V]<br><br>SS12. (A3–2) Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes. [PS, R, V]<br><br>SS13. (A3–3) Solve problems involving percentage error when input variables are expressed with percentage errors. [PS, R, V]<br><br>SS14. (A3–4) Design an appropriate measuring process or device to solve a problem. [E, PS, V] |

Grades 10–12
Strand: Shape and Space (Measurement)
*Students will:*

- describe and compare everyday phenomena, using either direct or indirect measurement.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Analyze objects, shapes and processes to solve cost and design problems. | SS15. (A9–1) Use dimensions and unit prices to solve problems involving perimeter, area and volume. [E, PS, V]<br><br>SS16. (A9–2) Solve problems involving estimation and costing for objects, shapes or processes when a design is given. [C, E, PS]<br><br>SS17. (A9–3) Design an object, shape, layout or process within a specified budget. [PS, R, V]<br><br>SS18. (A9–4) Use simplified models to estimate the solutions to complex measurement problems. [E, V] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Specify conditions under which triangles may be similar or congruent, and use these conditions to solve problems.

**Specific Outcomes**

8. Recognize when, and explain why, two triangles are similar, and use the properties of similar triangles to solve problems. [C, PS, R, T]

9. Recognize when, and explain why, two triangles are congruent, and use the properties of congruent triangles to solve problems. [C, CN, PS, R, T]

10. Relate congruence to similarity in the context of triangles. [CN, R]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Shape and Space (3-D Objects and 2-D Shapes)

*Students will:*

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Solve coordinate geometry problems involving lines and line segments. | SS19. (C1–10) Solve problems involving distances between points in the coordinate plane. [PS, V]<br><br>SS20. (C1–11) Solve problems involving midpoints of line segments. [PS]<br><br>SS21. (C1–12) Solve problems involving rise, run and slope of line segments. [PS, V]<br><br>SS22. (C1–13) Determine the equation of a line, given information that uniquely determines the line. [PS, V]<br><br>SS23. (C1–14) Solve problems using slopes of:<br>• parallel lines<br>• perpendicular lines.<br>[CN, PS, V] |
| Solve coordinate geometry problems involving lines and line segments, and justify the solutions. | SS24. (P3–6) Solve problems involving distances between points and lines. [CN, PS, R]<br><br>SS25. (P3–7) Verify and prove assertions in plane geometry, using coordinate geometry. [C, R, V] |

## Grade 9

**General Outcome**

Use spatial problem solving in building, describing and analyzing geometric shapes.

**Specific Outcomes**

11. Draw the plan and elevations of a 3-D object from sketches and models.
    [C, R, T, V]

12. Sketch or build a 3-D object, given its plan and elevation views.
    [C, PS, T, V]

13. Recognize and draw the locus of points in solving practical problems.
    [PS, T, V]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Shape and Space (3-D Objects and 2-D Shapes)

*Students will:*

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Develop and apply the geometric properties of circles and polygons to solve problems. | SS26. (C5–5) Use technology and measurement to confirm and apply the following properties to particular cases:<br>• the perpendicular from the centre of a circle to a chord bisects the chord<br>• the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc<br>• the inscribed angles subtended by the same arc are congruent<br>• the angle inscribed in a semicircle is a right angle<br>• the opposite angles of a cyclic quadrilateral are supplementary<br>• a tangent to a circle is perpendicular to the radius at the point of tangency<br>• the tangent segments to a circle, from any external point, are congruent<br>• the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord<br>• the sum of the interior angles of an $n$-sided polygon is $(2n - 4)$ right angles.<br>[PS, R, T, V]<br><br>SS27. (A3–5) Use properties of circles and polygons to solve design and layout problems.<br>[CN, PS, V]<br><br>SS28. (P5–6) Prove the following general properties, using established concepts and theorems:<br>• the perpendicular bisector of a chord contains the centre of the circle<br>• the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle)<br>• the inscribed angles subtended by the same arc are congruent<br>• the angle inscribed in a semicircle is a right angle<br>• the opposite angles of a cyclic quadrilateral are supplementary<br>• a tangent to a circle is perpendicular to the radius at the point of tangency<br>• the tangent segments to a circle from any external point are congruent<br>• the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord<br>• the sum of the interior angles of an $n$-sided polygon is $(2n - 4)$ right angles.<br>[C, R, V] |

Grades 10–12
Strand: Shape and Space (3-D Objects and 2-D Shapes)
*Students will:*
- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Solve problems involving polygons and vectors, including both 3-D and 2-D applications. | SS29. (P5–7) Solve problems, using a variety of circle properties, and justify the solution strategy used. [PS, R, V]<br><br>SS30. (A6–4) Use and give 3-D and 2-D examples of vector terminology and notation, including:<br>• vector (direction, magnitude)<br>• scalar<br>• unit vector<br>• collinear vectors<br>• opposite vectors<br>• parallel vectors<br>• resultant vectors.<br>[C, CN]<br><br>SS31. (A6–5) Assign meaning to the multiplication of a vector by a scalar. [CN]<br><br>SS32. (A6–6) Perform vector additions and subtractions, using triangle or parallelogram methods. [V]<br><br>SS33. (A6–7) Determine the magnitude and direction of a resultant vector, using triangle, parallelogram or component methods. [CN, T, V]<br><br>SS34. (A6–8) Use vector diagrams and trigonometry to analyze and solve practical problems in 3-D and 2-D. [CN, PS, V] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Shape and Space (3-D Objects and 2-D Shapes)

*Students will:*

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Classify conic sections, using their shapes and equations. | SS35. (P9–1) Classify conic sections according to shape. [C, R, V]<br><br>SS36. (P9–2) Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only). [CN, T, V]<br><br>SS37. (P9–3) Convert a given equation of a conic section from general to standard form and vice versa. [R, T] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Apply coordinate geometry and pattern recognition to predict the effects of translations, rotations, reflections and dilatations on 1-D lines and 2-D shapes.

**Specific Outcomes**

14. Draw the image of a 2-D shape as a result of:
    - a single transformation
    - a dilatation
    - combinations of translations and/or reflections.

    [PS, T, V]
15. Identify the single transformation that connects a shape with its image. [R]
16. Demonstrate that a triangle and its dilatation image are similar. [R]
17. Demonstrate the congruence of a triangle with its:
    - translation image
    - rotation image
    - reflection image.

    [R]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Shape and Space (Transformations)

*Students will:*

- perform, analyze and create transformations.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Perform, analyze and create transformations of functions and relations that are described by equations or graphs. | SS38. (P9–4) Describe how various translations of functions affect graphs and their related equations:<br>• $y = f(x - h)$<br>• $y - k = f(x)$.<br>[C, T, V]<br><br>SS39. (P9–5) Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations:<br>• $y = af(x)$<br>• $y = f(kx)$.<br>[C, T, V]<br><br>SS40. (P9–6) Describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations:<br>• $y = f(-x)$<br>• $y = -f(x)$<br>• $y = f^{-1}(x)$.<br>[C, T, V]<br><br>SS41. (P9–7) Using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$.<br>[C, T, V]<br><br>SS42. (P9–8) Using the graph and/or the equation of $f(x)$, describe and sketch $\lvert f(x)\rvert$.<br>[C, T, V]<br><br>SS43. (P9–9) Describe and perform single transformations and combinations of transformations on functions and relations.<br>[C, T, V] |

## Grade 9

**General Outcome**

Collect and analyze experimental results expressed in two variables, using technology, as required.

**Specific Outcomes**

1. Design, conduct and report on an experiment to investigate a relationship between two variables. [C, CN, PS]
2. Create scatterplots for discrete and continuous variables. [C, V]
3. Interpret a scatterplot to determine if there is an apparent relationship. [E, R]
4. Determine the lines of best fit from a scatterplot for an apparent linear relationship by:
   - inspection
   - using technology (equations are not expected).

   [E, PS, T]
5. Draw and justify conclusions from the line of best fit. [C, R]
6. Assess the strengths, weaknesses and biases of samples and data collection methods. [C, R, T]
7. Critique ways in which statistical information and conclusions are presented by the media and other sources. [C, CN]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Statistics and Probability (Data Analysis)

*Students will:*

- collect, display and analyze data to make predictions about a population.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Implement and analyze sampling procedures, and draw appropriate inferences from the data collected. | SP1. (C3–6) Choose, justify and apply sampling techniques that will result in an appropriate, unbiased sample from a given population. [C, PS, R]<br><br>SP2. (C3–7) Defend or oppose inferences and generalizations about populations, based on data from samples. [C, PS, R] |
| Apply line-fitting and correlation techniques to analyze experimental results. | SP3. (A2–2) Determine the equation of a line of best fit, using:<br>• estimate of slope and one point<br>• median–median method<br>• least squares method with technology.<br>[CN, PS, T, V]<br><br>SP4. (A2–3) Use technological devices to determine the correlation coefficient *r*. [T]<br><br>SP5. (A2–4) Interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots. [C, R, V] |

Grades 10–12
Strand: Statistics and Probability (Data Analysis)
*Students will:*

- collect, display and analyze data to make predictions about a population.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Analyze graphs or charts of given situations to derive specific information. | SP6. (A4–1) Extract information from given graphs of discrete or continuous data, using:<br>• time series<br>• glyphs (custom pictorial representations)<br>• continuous data<br>• contour lines.<br>[C, CN, E, PS, R, V]<br><br>SP7. (A4–2) Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]<br><br>SP8. (A4–3) Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation.<br>[C, CN, T, V] |

(C) COMMON
(A) APPLIED
(P) PURE

## Grade 9

**General Outcome**

Explain the use of probability and statistics in the solution of complex problems.

**Specific Outcomes**

8. Recognize that decisions based on probability may be a combination of theoretical calculations, experimental results and subjective judgements.
   [PS, R]

9. Demonstrate an understanding of the role of probability and statistics in society.
   [C, CN]

10. Solve problems involving the probability of independent events.
    [PS, T]

(C) COMMON
(A) APPLIED
(P) PURE

## Grades 10–12
## Strand: Statistics and Probability (Chance and Uncertainty)

*Students will:*

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events.<br><br>Use normal and binomial probability distributions to solve problems involving uncertainty. | SP9. (P2–6) Connect probabilities to calculated expected gains or losses. [CN, PS, R, V]<br><br>SP10. (P2–7) Solve decision-making problems involving expected values, and communicate the solutions. [C, PS, R]<br><br>SP11. (C6–1) Find the population standard deviation of a data set or a probability distribution, using technology. [CN, E, T, V]<br><br>SP12. (C6–2) Use *z*-scores and *z*-score tables to solve problems. [PS, R, T, V]<br><br>SP13. (C6–3) Use the normal distribution and the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS] |

Grades 10–12
Strand: Statistics and Probability (Chance and Uncertainty)

*Students will:*

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations. | SP14. (C6–4) Solve pathway problems, interpreting and applying any constraints. [PS, R]<br><br>SP15. (C6–5) Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]<br><br>SP16. (P7–1) Determine the number of permutations of *n* different objects taken *r* at a time, and use this to solve problems. [PS, R, V]<br><br>SP17. (P7–2) Determine the number of combinations of *n* different objects taken *r* at a time, and use this to solve problems. [PS, R, V]<br><br>SP18. (P7–3) Determine the number of pathways in a given compound pathway problem. [CN, PS, V]<br><br>SP19. (P7–4) Solve problems, using the binomial theorem where *N* belongs to the set of natural numbers. [CN, E, V] |

(C) COMMON
(A) APPLIED
(P) PURE

Grades 10–12
Strand: Statistics and Probability (Chance and Uncertainty)

*Students will:*

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

**[C]** Communication
**[CN]** Connections
**[E]** Estimation and Mental Mathematics
**[PS]** Problem Solving
**[R]** Reasoning
**[T]** Technology
**[V]** Visualization

| General Outcomes | Specific Outcomes |
|---|---|
| Model the probability of a compound event, and solve problems based on the combining of simpler probabilities. | SP20. (C6–6) Construct a sample space for two or three events. [PS, R, V]<br><br>SP21. (C6–7) Classify events as independent or dependent. [C]<br><br>SP22. (C6–8) Solve problems, using the probabilities of mutually exclusive and complementary events. [CN, PS, R]<br><br>SP23. (P7–5) Determine the conditional probability of two events (Bayes' law). [E, PS, R]<br><br>SP24. (P7–6) Solve probability problems involving permutations, combinations and conditional probability. [E, PS, R]<br><br>SP25. (P7–7) Solve probability problems, using the binomial distribution as applied to small samples. [PS, R, T] |

(C) COMMON
(A) APPLIED
(P) PURE

# VI. GENERAL OUTCOMES, AND SPECIFIC OUTCOMES WITH ILLUSTRATIVE EXAMPLES, ORGANIZED BY CLUSTER

*Cluster*

This section elaborates on the general outcomes and specific outcomes by providing illustrative examples, by cluster, for the Grade 10–12 program.

Each specific outcome is coded for mathematical processes, using the codes listed on the top of each page from page 62 to page 190.

## Clusters in the Grade 10–12 Program

There are 24 clusters identified, each representing 20 to 25 hours of instructional time for an average student taking the cluster.

Common clusters, numbered C1 to C6, include the mathematics expected of all students completing a K to 12 mathematics program.

Applied clusters, numbered A1 to A9, emphasize applications of mathematics rather than precise mathematical theory. The approaches used are primarily numerical and geometrical.

Pure clusters, numbered P1 to P9, place more emphasis on precise mathematical theory. The approaches used are primarily algebraic and graphical.

The order of the clusters is intended to indicate a sequence that might be used to construct courses and programs of study.

## Coding for Illustrative Examples (IEs)

The illustrative examples (IEs) listed on the following pages are organized by cluster and have been correlated to specific outcomes (SOs).

## Numbering System

The illustrative examples are numbered sequentially within each cluster by specific outcome. The specific outcomes are cross-referenced to the General Outcomes and Specific Outcomes section (pages 30 to 59). For example, $\underset{(PR53)}{C2-6.}$ is the 6th specific outcome in Common Cluster 2 and the 53rd specific outcome in the Patterns and Relations strand.