


Cluster Pure P1

Strand: Number (Number Operations)
Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.	P1-1. (N10) Explain and apply the exponent laws for powers of numbers and for variables with rational exponents. [C, E]	1.1 Find the exact value of $\left(\frac{8}{27}\right)^{\left(-\frac{2}{3}\right)}$. 1.2 Write the number expression $7^{\left(\frac{2}{3}\right)}$, using radicals. 1.3 Simplify $\left(\sqrt[5]{x^3}\right)\left(\sqrt[3]{x^2}\right)$. 1.4 Show $\left(\sqrt[3]{-8}\right)x = -2x$. 1.5 Write an equivalent expression for $\sqrt[3]{2\sqrt{3x^5}}$, using exponents. 1.6 Prove that $\sqrt{2}$ is an irrational number. 1.7 The 5×5 geoboard shown in the diagram can be used to construct squares whose areas are whole numbers. The sides of the squares can be constructed by joining dots horizontally, vertically or diagonally. What whole number areas can be constructed? Justify your answers with appropriate drawings and calculations. <div style="text-align: center; margin-top: 20px;">  </div>

Cluster Pure P1

Strand: Patterns and Relations (Variables and Equations)
Students will:

- represent algebraic expressions in multiple ways.

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|------|--------------------------------------|------|-----------------|
| [C] | Communication | [PS] | Problem Solving |
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Mental Mathematics | [T] | Technology |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Generalize operations on polynomials to include rational expressions.</p> <p><i>(continued)</i></p>	<p>P1-2. Factor polynomial expressions of the form $ax^2 + bx + c$, and $a^2x^2 - b^2y^2$. [E]</p> <p>P1-3. Find the product of polynomials. [E, R]</p>	<p>2.1 Factor: a) $5x^2 + 6x - 8$ b) $6x^2 - x - 2$.</p> <p>2.2 Factor $4x^2 + 20x + 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the trinomial?</p> <p>2.3 Factor $4x^2 - 25$. a) Compare the two factors. b) For this special product, what is the relationship between the coefficients of the terms of the factors and the coefficients of the terms of the binomial?</p> <p>2.4 For which integral values of k can $4x^2 + kx + 3$ be factored over the set of rational numbers?</p> <p>2.5 Factor $(x + b)^2 + 6(x + b) + 8$.</p> <p>2.6 Factor $6x^4 - x^2 - 2$.</p> <p>3.1 Find the product and simplify: a) $(3x - 4)(2x^2 + 3x + 1)$ b) $(2x - y)^3$.</p>

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[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P1–4. (PR24) Divide a polynomial by a binomial, and express the result in the forms:</p> <ul style="list-style-type: none"> $\frac{P}{D} = Q + \frac{R}{D}$ $P = DQ + R$ $P(x) = D(x)Q(x) + R.$ <p>[E, R]</p> <p>P1–5. (PR25) Determine equivalent forms of simple rational expressions with polynomial numerators, and denominators that are monomials, binomials or trinomials that can be factored.</p> <p>[PS, R]</p>	<p>4.1 Divide $(3x^3 + 2x^2 - 7x + 8)$ by $(x + 2)$.</p> <p>4.2 Divide $(t^2 - 3t - 10)$ by $(t - 3)$.</p> <p>4.3 Divide $(6x^3 - 2x^2 + 7x - 11)$ by $(3x^2 - 2)$.</p> <p>4.4 When the polynomial $P(t) = 4t^4 - 17t^2 - 36t - 20$ is divided by $(2t - 5)$, the remainder is -60. Express the division in the forms:</p> <p>a) $\frac{P(t)}{2t-5} = Q(t) + \frac{R}{2t-5}$</p> <p>b) $P(t) = Q(t)(2t - 5) + R.$</p> <p>5.1 Change each rational expression to its simplest equivalent form:</p> <p>a) $\frac{4x^4 - 6x^3 + 2x^2 - 10x}{2x}$</p> <p>b) $\frac{x^2 - 5x - 6}{x^2 - 36}$</p> <p>c) $\frac{x^2 + 3x}{x^2 + x - 6}$</p> <p>d) $\frac{16x^4 - 81y^4}{(4x^2 + 9y^2)^2 (2x^2 - xy - 3y^2)}.$</p>

Cluster Pure P1

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Mental Mathematics | [T] Technology |
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General Outcomes	Specific Outcomes	Illustrative Examples										
<i>(continued)</i>	P1-6. Determine the nonpermissible values for the variable in rational expressions. (PR26) [C, CN]	6.1 For what value(s) of x are each of the following not defined? Explain your conclusion in each case. <table border="0" style="width: 100%;"> <tr> <td style="width: 50%;">a) $\frac{3}{x}$</td> <td style="width: 50%;">e) $\frac{5x}{x^2 - 3x - 4}$</td> </tr> <tr> <td>b) $\frac{-2}{x+1}$</td> <td>f) $\frac{5x+y}{3x-y}$</td> </tr> <tr> <td>c) $\frac{4}{3x-4}$</td> <td>g) $\frac{7x^2 - 6xy + 3y^2}{4x^2 - 9y^2}$</td> </tr> <tr> <td>d) $\frac{2x+1}{x^2 - 4}$</td> <td>h) $\frac{2}{x^3}$</td> </tr> <tr> <td></td> <td>i) $\frac{5}{(x^3 - 1)}$</td> </tr> </table>	a) $\frac{3}{x}$	e) $\frac{5x}{x^2 - 3x - 4}$	b) $\frac{-2}{x+1}$	f) $\frac{5x+y}{3x-y}$	c) $\frac{4}{3x-4}$	g) $\frac{7x^2 - 6xy + 3y^2}{4x^2 - 9y^2}$	d) $\frac{2x+1}{x^2 - 4}$	h) $\frac{2}{x^3}$		i) $\frac{5}{(x^3 - 1)}$
a) $\frac{3}{x}$	e) $\frac{5x}{x^2 - 3x - 4}$											
b) $\frac{-2}{x+1}$	f) $\frac{5x+y}{3x-y}$											
c) $\frac{4}{3x-4}$	g) $\frac{7x^2 - 6xy + 3y^2}{4x^2 - 9y^2}$											
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Cluster Pure P1

Strand: Patterns and Relations (Variables and Equations)
Students will:

- represent algebraic expressions in multiple ways.

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| [C] Communication | [PS] Problem Solving |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P1-7. (PR27) Perform the operations of addition, subtraction, multiplication and division on rational expressions. [E, R]</p>	<p>7.1 For each expression perform the indicated operations, and identify any nonpermissible values.</p> <p>a) $\left(\frac{1}{x}\right) + \left(\frac{3}{2x}\right)$</p> <p>b) $\left(\frac{4}{x+1}\right) - \left(\frac{1}{x-2}\right)$</p> <p>c) $\left(\frac{2x+1}{x-1}\right) + \left(\frac{x-1}{x^2-x-2}\right)$</p> <p>d) $\left(\frac{x^2+2x+1}{x-5}\right) \left(\frac{x^2-25}{x^2+6x+5}\right)$</p> <p>e) $\left(\frac{3x^2+10x+3}{x^2-9}\right) \div \left(\frac{3x+1}{x-3}\right)$</p> <p>f) $\frac{3}{\left(\frac{2}{x}\right)}$</p> <p>g) $\frac{\left(\frac{2x+6}{x+1}\right)}{\left(\frac{x+3}{x^2-1}\right)}$</p> <p>h) $\frac{\left(\frac{1}{x}+3\right)}{\left(\frac{1}{x}-3\right)}$</p>

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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P1–8. (PR28) Find and verify the solutions of rational equations. [CN, PS]</p>	<p>8.1 Solve for x, checking for any nonpermissible values.</p> <p>a) $\frac{2}{x} = -3$</p> <p>b) $\frac{4}{x} + \frac{3}{2x} = \frac{11}{4}$</p> <p>c) $\frac{5}{x-1} - \frac{2}{x+1} = 2$</p> <p>d) $\frac{2x+1}{x+3} - \frac{x-2}{x+1} = 5$</p> <p>e) $\frac{3}{x^2-25} + \frac{2}{x+5} = \frac{4}{x-5}$</p> <p>f) $\frac{4}{x-5} + 6 = \frac{4}{x-5}$</p> <p>8.2 The average speed of an airplane is five times as fast as the average speed of a passenger train. To travel 400 km, the train requires 4 hours more than the airplane. Find the average speeds of the train and the airplane.</p>

Cluster Pure P2

Strand: Number (Number Operations)
Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

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|------|--------------------------------------|------|-----------------|
| [C] | Communication | [PS] | Problem Solving |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.</p>	<p>P2-1. (N11) Perform operations on irrational numbers of monomial and binomial form, using exact values. [E]</p>	<p>1.1 Show that $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$.</p> <p>1.2 Find an equivalent form of $\left(\frac{3}{\sqrt{5} - \sqrt{2}}\right)$ that has a whole number as its denominator.</p> <p>1.3 Arrange the following in order from least to greatest: $7, 2\sqrt{13}, 3\sqrt{6}, 4\sqrt{5}, 5\sqrt{2}$. Do not use decimal approximations.</p> <p>1.4 Find the exact value of $\sqrt[3]{128} + 4(\sqrt[3]{16})$.</p> <p>1.5 Find an equivalent form of $(3\sqrt{5} + 4\sqrt{2})(4\sqrt{5} - 3\sqrt{2})$.</p> <p>1.6 An equilateral triangle is inscribed in a circle. If the area of the circle is 36π, find the exact area of the equilateral triangle.</p>

Cluster Pure P2

Strand: Statistics and Probability (Chance and Uncertainty)
Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

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|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Make and analyze decisions, using expected gains and losses, based on the probabilities of simple events.</p> <p><i>(continued)</i></p>	<p>P2–6. (SP9) Connect probabilities to calculated expected gains or losses. [CN, PS, R, V]</p>	<p>6.1 A business person is preparing a proposal for a computer contract worth \$12 000. This person estimates that it would cost \$1500 to prepare the proposal, and the probability of receiving the contract is estimated to be 0.20. Find this business person’s expected gain.</p> <p>6.2 The Khan family is considering moving from Calgary to Hamilton. In Calgary, Ali earns \$46 000 and Kareema earns \$34 000. Based on the family’s research, if they move, Ali has an estimated probability of 0.85 of finding a job that pays \$53 000, and an estimated probability of 0.12 of finding a job that pays \$33 000. Otherwise he would be unemployed, receiving \$17 000. Kareema has an estimated probability of 0.65 of finding a job that pays \$62 000, and an estimated probability of 0.12 of finding a job that pays \$33 000. Otherwise she would be unemployed, receiving \$11 000. What is the expected gain in salary, if the Khans move to Hamilton?</p> <p>6.3 Sherry takes a 100-item multiple-choice examination. Each item has four possible choices. She knows 68 of the answers and guesses randomly at the other 32. Calculate her expected number of correct answers.</p>

Cluster Pure P2

Strand: Statistics and Probability (Chance and Uncertainty)
Students will:

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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P2-7. (SP10) Solve decision-making problems involving expected values, and communicate the solutions. [C, PS, R]</p>	<p>7.1 Dave and Tony are playing toss-up with two coins. Dave wins one point, if both coins are heads or both are tails. Tony wins one point, if the two coins are different. After 100 tosses, what are the two players' expected scores? Is this a fair game?</p> <p>7.2 Joe paid \$5 to throw a pair of dice. He wins the sum of the numbers appearing on the top faces of the dice, unless a six appears on either die; then he wins nothing.</p> <p>a) Is this a fair game? b) What difference would it make if the six were changed to a one? c) Justify your answers by analyzing the sample space for this dice throw.</p> <p>7.3 Obtain collision damage figures for inexperienced drivers and for experienced drivers from an insurance company, and then calculate a fair insurance premium for \$1 000 000 liability, \$250 deductible collision and \$100 deductible comprehensive theft/glass coverage. Do the calculation twice, once for each type of driver.</p> <p>What change in premium would be fair, if the deductible for collision were raised to \$1000?</p> <p>7.4 At what point is it worth it to drop collision coverage on an older vehicle? Show a strategy, and explain the supporting calculations.</p> <p>7.5 Explain why it is reasonable to insure a house against fire damage, where the probability of collecting is 0.003, but it is not reasonable for a bank, using current interest rates, to make a loan that has a 90% probability of getting repaid.</p> <p>7.6 The growing of grapes for <i>Eiswein</i> involves harvesting the grapes as late as possible in October. As each day passes, the grapes become more valuable, but there is a greater risk of a frost killing the grapes and reducing their value. For a particular year, the value of the grape juice is \$2.00/L on October 1, and the value of the juice increases by \$0.15/L per day for every day in October. The probability of a killer frost is 0.03 for any particular day in October. After a killer frost, the value of the juice is \$1.50/L. On what day does the risk of frost damage outweigh the gain from extra maturing time?</p>

Cluster Pure P3

Strand: Patterns and Relations (Variables and Equations)
Students will:

- represent algebraic expressions in multiple ways.

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|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Represent and analyze situations that involve expressions, equations and inequalities.</p> <p><i>(continued)</i></p>	<p>P3-1. Solve nonlinear equations: (PR32)</p> <ul style="list-style-type: none"> by factoring graphically. <p>[CN, T, V]</p> <p>P3-2. Use the Remainder Theorem to evaluate polynomial expressions and the Factor Theorem to determine factors of polynomials. (PR33)</p> <p>[E, PS, T]</p>	<p>1.1 Solve by factoring:</p> <ol style="list-style-type: none"> $x^2 - 2x = 24$ $x^3 = 1$ $2x^2 + 9x - 5 = 0$ $7x^2 + 4x - 11 = 0$. <p>1.2 Solve each of the above graphically. For example, $x^2 - 2x = 24$ can be solved by graphing $y = x^2 - 2x$ and $y = 24$ and using the points of intersection to determine the solution.</p> <p>1.3 Solve $3x^2 + 1 = 10x - 2$ graphically in two different ways. Is there one way that gives more reliable results? Explain your procedures and the results obtained.</p> <p>2.1 The polynomial $P(x) = 4x^3 + bx^2 + cx + 11$ has a remainder of -7 when divided by $(x + 2)$ and a remainder of 14 when divided by $(x - 1)$. Find the values of b and c.</p> <p>2.2 Factor $x^3 - 2x^2 - 5x + 6$.</p>

Cluster Pure P3

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

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|--|----------------------|
| [C] Communication | [PS] Problem Solving |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P3–3. (PR34) Determine the solution to a system of nonlinear equations, using technology as appropriate. [PS, T, V]</p>	<p>3.1 Find the solutions to the following system: $y = x^2$ $y = 8 - x^2$.</p> <p>3.2 Graphically, find the solution set to the following system: $y = 3x + 2$ and $y = 2^x$.</p> <p>How do you know that the solution set is complete?</p> <p>3.3 The world’s population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people. The population growth can be modelled by the equation $P_1 = 5(1.02)^n$, with the food production being modelled by $P_2 = 0.2n + 6$. The variable n is the number of years after 1987.</p> <p>a) When does $P_1 = P_2$?</p> <p>b) If $P_1 > P_2$ is true, when does this happen, and how is this inequality interpreted?</p>

Cluster Pure P3

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication

[CN] Connections

[E] Estimation and
Mental Mathematics

[PS] Problem Solving

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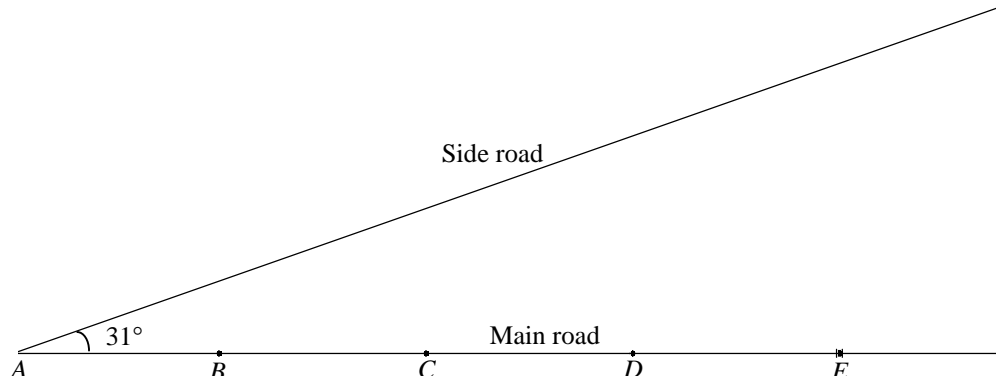
General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P3–4. Solve systems of linear equations, in three variables: (PR35)</p> <ul style="list-style-type: none"> • algebraically • with technology. <p>[CN, PS, T, V]</p>	<p>4.1 Determine the solution to the following system: $2x + y - z = 3$ $x + 2y + z = 0$ $3x - y - 2z = 11.$</p> <p>4.2 The total revenue R is a quadratic function of the price p of books sold. So $R = ap^2 + bp + c$. Find the values of a, b and c, if the revenue is \$6000 at a price of \$30, \$6000 at a price of \$40 and \$5000 at a price of \$50.</p>

Cluster Pure P3

Strand: Shape and Space (Measurement)
Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

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| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
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Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve problems involving triangles, including those found in 3-D and 2-D applications.</p>	<p>P3-5. (SS7) Solve problems involving ambiguous case triangles in 3-D and 2-D. [CN, PS, R, T]</p>	<p>5.1 An 11 cm long line AB is drawn at an angle of 44° to a horizontal line AE. A circle with centre B and a radius of 9 cm is drawn, cutting the horizontal line at points C and D. Calculate the length of the chord CD.</p> <p>5.2 The line segment of equation $y = 2.4x$, passes through $A(0, 0)$ and $C(5, 12)$, has a length of 13 and makes an angle of 67.3° with the horizontal x-axis.</p> <p>a) What points are located with $CB = 10$ and AB horizontal? b) Check your answer by determining the intersection points of the circle $(x - 5)^2 + (y - 12)^2 = 100$ and the line $y = 0$. c) Use a suitable diagram to explain why the answers to a) and b) are the same.</p> <p>5.3</p>  <p>Streetlights A, B, C, D and E are placed 50 m apart on the main road, as indicated on the diagram. The light from a streetlight can travel 24 m. Determine the furthest point on the side road that is lighted and the length of side road that is illuminated by both streetlight C and streetlight D.</p>

Cluster Pure P3

Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve coordinate geometry problems involving lines and line segments, and justify the solutions.</p>	<p>P3-6. Solve problems involving distances between points and lines. [CN, PS, R]</p> <p>P3-7. Verify and prove assertions in plane geometry, using coordinate geometry. [C, R, V]</p>	<p>6.1 Determine the shortest distance from (3, 4) to the line $2x - 5y = 7$.</p> <p>6.2 The lines $y = 3x + 1$ and $y = 3x - 9$ are parallel. Determine the vertical distance between the two lines, the horizontal distance between the two lines and the shortest distance between the two lines.</p> <p>7.1 Given $A = (-1, 3)$, $B = (0, 5)$ and $C = (-2, 6)$:</p> <ol style="list-style-type: none"> Verify that ABC is a right-angled triangle. Is ABC isosceles? Justify your assertion. If M is the midpoint of AB and N is the midpoint of AC, prove that MN is parallel to BC. Find a point D so that $ABCD$ is a parallelogram. Prove that $ABCD$ is not a rectangle. <p>7.2 Use coordinate geometry to prove that:</p> <ol style="list-style-type: none"> the diagonals of any parallelogram bisect one another if ABC is any triangle, with M as the midpoint of AB and N as the midpoint of AC, then MN is parallel to BC and is half its length. <p>7.3 Use coordinate geometry to divide the line segment with end points $A(4, 7)$ and $B(-3, 8)$ into five congruent parts.</p>

Cluster Pure P4

Strand: Patterns and Relations (Relations and Functions)
Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

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| [CN] Connections | [R] Reasoning |
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Mental Mathematics | [T] Technology |
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General Outcomes	Specific Outcomes	Illustrative Examples
<p>Examine the nature of relations with an emphasis on functions.</p> <p><i>(continued)</i></p>	<p>P4-1. Perform operations on functions and compositions of functions. [CN, E, PS]</p>	<p>1.1 If $f(x) = 3x + 2$ and $g(x) = x^2$, find:</p> <ol style="list-style-type: none"> $3f(x)$ $f(x) \cdot g(x)$ $f(x) + g(x)$ $f(g(x))$ $f(f(x))$. <p>1.2 A ball thrown in the air has a velocity given by $v(t) = 49 - 9.8t$. The kinetic energy function $K(v)$ is given by $K(v) = 0.4v^2$. Express the ball's kinetic energy as a function $K(t)$ of time.</p>

Cluster Pure P4

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication

[CN] Connections

[E] Estimation and

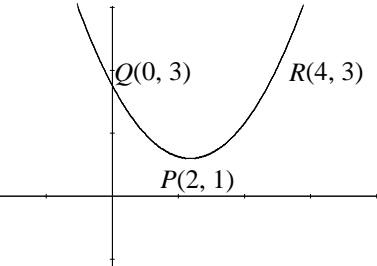
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<i>(continued)</i>	P4-2. Determine the inverse of a function. (PR55) [CN, R, V]	2.1 Graph the inverse of $y = \frac{x}{(x-1)}$, and determine the equation, domain and range of the inverse. 2.2 Sketch the inverse of the following.  2.3 Sketch the inverse of $f(x) = x^2$. 2.4 If $f(x) = 2x - 1$ and $g(x) = \frac{x+1}{2}$, find $f(g(x))$ and $g(f(x))$, and show that the functions $f(x)$ and $g(x)$ are inverses of each other. 2.5 Determine the domain and range for each of the functions in illustrative examples 2.2 and 2.3. 2.6 Is the inverse of $f(x) = 2x - 5$ a function?

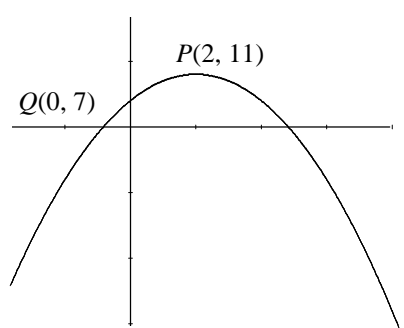
Cluster Pure P4

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

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| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.</p> <p><i>(continued)</i></p>	<p>P4–3. (PR58) Connect algebraic and graphical transformations of quadratic functions, using completing the square as required. [CN, T, V]</p>	<p>3.1 Graph $f(x) = 2x^2 + 5x - 7$.</p> <p>3.2 Give a list of events or situations that might be described by a quadratic, parabolic, shape.</p> <p>3.3 Given the graph of $y = x^2$, sketch $y = -2(x - 3)^2 - 4$.</p> <p>3.4 Given the graph of $y = x^2$, what is the equation for the transformed graph shown here?</p>  <p>3.5 Rewrite the equation of $f(x) = 2x^2 - 12x + 13$ in the form $f(x) = a(x - p)^2 + q$, and graph the function.</p>
	<p>P4–4. (PR59) Model real-world situations, using quadratic functions. [CN, PS]</p>	<p>4.1 Computer software programs are sold to students for \$20 each, and 300 students are willing to buy them at that price. For every \$5 increase in price, there are 30 fewer students willing to buy the software. What is the maximum revenue?</p> <p>4.2 What is the maximum rectangular area that can be enclosed by 120 m of fencing, if one of the sides of the rectangle is an existing wall?</p>

Cluster Pure P4

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication

[CN] Connections

[E] Estimation and

Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P4–5. (PR60) Solve quadratic equations, and relate the solutions to the zeros of a corresponding quadratic function, using:</p> <ul style="list-style-type: none"> • factoring • the quadratic formula • graphing. <p>[CN, E, T, V]</p>	<p>5.1 Solve $3x^2 - 5x + 2 = 0$ algebraically and by graphing the corresponding function $f(x) = 3x^2 - 5x + 2$.</p> <p>5.2 When bicycles are sold for \$280 each, a cycle store can sell 80 in a season. For every \$10 increase in the price, the number sold drops by 3.</p> <ol style="list-style-type: none"> Represent the sales revenue as a quadratic function of either the number sold or the price. What is the number sold, and the price, if the total sales revenue is exactly \$20 000? What is the range of prices that will give a sales revenue that exceeds \$15 000? <p>5.3 Write a quadratic equation whose roots are $\frac{3}{2}$ and $-\frac{1}{4}$. Is this equation unique?</p>

Cluster Pure P4

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P4-6. (PR61) Determine the character of the real and non-real roots of a quadratic equation, using:</p> <ul style="list-style-type: none"> • the discriminant in the quadratic formula • graphing. <p>[C, R, T, V]</p>	<p>6.1 If $3x^2 - mx + 2 = 0$ can be factored, what values of m are possible?</p> <p>6.2 Discuss the implications of a negative discriminant when describing the zeros of a quadratic function.</p> <p>6.3 Given $3x^2 - mx + 3 = 0$:</p> <ol style="list-style-type: none"> For what value(s) of m would one root be double the other? For what values of m would the roots not be real? <p>6.4 The profit y for publishing a book is given by the equation $y = -5x^2 + 400x - 3000$, where x is the selling price per book.</p> <ol style="list-style-type: none"> Is it possible to set a selling price that will earn a total profit of \$6000? Explain your solution with reference to appropriate equations and graphs. What range of selling prices allow the publisher to make a profit on this book?

Cluster Pure P4

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

[C] Communication

[CN] Connections

[E] Estimation and

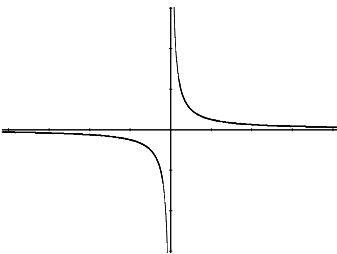
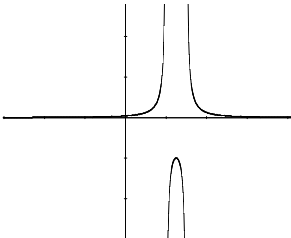
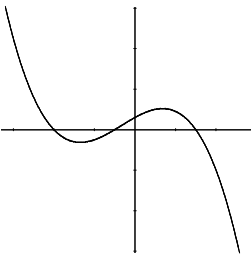
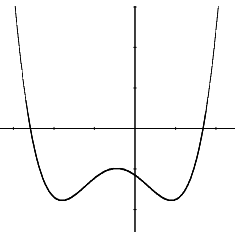
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P4-7. (PR62) Describe, graph and analyze polynomial and rational functions, using technology. [C, R, T, V]</p> <p style="text-align: right;"><i>(continued)</i></p>	<p>7.1 Determine if each of the following examples is a rational function, a polynomial function or some other type of function, and justify your conclusion.</p> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p>a) $y = x^2 - 3x + \sqrt{7}$</p> <p>b) $y = (x - 5)^{-1}$</p> <p>c) $y = \frac{1}{5}x^4 + 3x^3 - 12x - 0.75$</p> </div> <div style="width: 45%;"> <p>d) $y = \sqrt{7x^5} + x^2$</p> <p>e) $y = 2^x - 9$</p> <p>f) $y = \frac{3x - 7}{x^2 - 5x + 6}$</p> </div> </div> <p>7.2 Examine the following graphs. Which could be graphs of rational functions, and which could be graphs of polynomial functions?</p> <div style="display: flex; justify-content: space-around;"> <div style="width: 45%;"> <p>a) </p> <p>c) </p> </div> <div style="width: 45%;"> <p>b) </p> <p>d) </p> </div> </div>

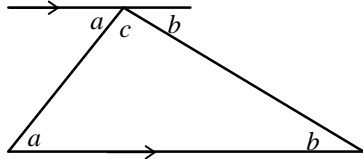
Cluster Pure P5

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

- | | | | |
|------|--------------------------------------|------|-----------------|
| [C] | Communication | [PS] | Problem Solving |
| [CN] | Connections | [R] | Reasoning |
| [E] | Estimation and
Mental Mathematics | [T] | Technology |
| | | [V] | Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Apply the principles of mathematical reasoning to solve problems and to justify solutions.</p> <p>(continued)</p>	<p>P5–1. Differentiate between inductive and deductive reasoning. [CN, R]</p> <p>P5–2. Explain and apply connecting words, such as “and”, “or” and “not”, to solve problems. [C, PS, R, V]</p>	<p>1.1 Find, inductively, the sum of the angles of a triangle, by:</p> <ol style="list-style-type: none"> constructing triangles and tearing the corners off putting the torn corners together to form a straight line. <p>1.2 Show, deductively, that the sum of the measures a, b and c is 180°, by:</p> <ol style="list-style-type: none"> drawing a triangle using one side as a base and drawing a parallel line segment on the opposite vertex knowing that $a = a$, $b = b$, and c is included in both; $\therefore a + c + b = 180^\circ$.  <p>2.1 Each member of a sports club plays at least one of the following sports: soccer, rugby or tennis. The following information is given:</p> <ol style="list-style-type: none"> 163 play tennis; 36 play tennis and rugby; 13 play tennis and soccer 6 play all three sports; 11 play soccer and rugby; 208 play rugby or tennis 98 play soccer or rugby. <p>Use this information to determine the number of members in the club.</p> <p>2.2 On a number line, indicate the location of the sets corresponding to the following:</p> <ol style="list-style-type: none"> $x < 2$ or $x > 5$ $x < 2$ and $x > 5$ $x < 5$ or $x > 2$ $x < 5$ and not $x > 2$. <p>2.3 The phrase “A or B” can be used in ordinary speech in inclusive and exclusive senses, depending on whether “A and B” is included or excluded.</p> <ol style="list-style-type: none"> Give a practical example of each sense of “A or B”. Show the relationship between the inclusive and the exclusive sense of “A or B” on appropriate Venn diagrams. Mathematicians and logicians use the inclusive sense of “A or B”. Justify this choice.

Cluster Pure P5

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P5–3. (PR7) Use examples and counterexamples to analyze conjectures. [CN, R]</p>	<p>3.1 Rajiv concluded that whenever he added two prime numbers the sum was always even. Find a counterexample to prove that Rajiv’s conjecture is false.</p> <p>3.2 A science text states that water boils at 100°C. Find a counterexample.</p> <p>3.3 Mary used her graphing calculator to graph $y = x^x$. She found the screen to be blank for $x < 0$ and made a conjecture that x^x is undefined when $x < 0$. Find an example to show that Mary’s conjecture is reasonable. Find a counterexample to show that Mary’s conjecture is false.</p> <p>3.4 The functions $f(x) = \frac{x^2 - 49}{x - 7}$ and $g(x) = x + 7$ are closely related.</p> <p>a) Explain the similarities and the differences between $f(x)$ and $g(x)$.</p> <p>b) How do the graphs of $f(x)$ and $g(x)$ differ from one another?</p>

Cluster Pure P5

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C] Communication

[CN] Connections

[E] Estimation and

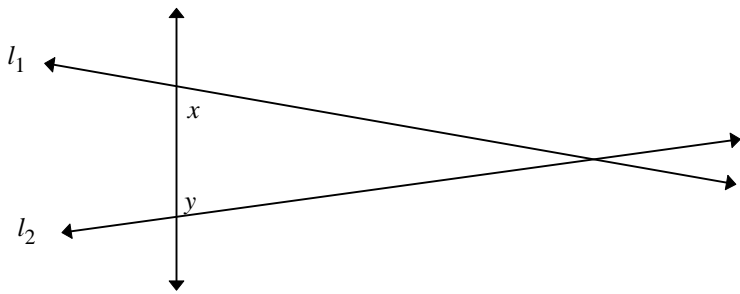
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P5-4. (PR8) Distinguish between an “if-then” proposition, its converse and its contrapositive. [CN, R]</p> <p>P5-5. (PR9) Prove assertions in a variety of settings, using direct and indirect reasoning. [R]</p>	<p>4.1 Change the statement “Multiples of 3 are always multiples of 6” into “if-then” form, and write the converse and contrapositive of the “if-then” statement. Decide on the truth of all three propositions.</p> <p>4.2 Create a true proposition whose converse and contrapositive are both true.</p> <p>5.1 Angle ABC is obtuse, and AD is the median of BC. If AD is not an altitude, prove that ABC is a scalene triangle.</p> <p>5.2 Prove that the medians of a triangle cannot bisect each other.</p> <p>5.3 In the diagram below, show: a) $x + y < 180^\circ$ b) if $x + y = 180^\circ$, lines l_1 and l_2 are parallel.</p> <div style="text-align: center;">  </div> <p>5.4 Prove that the difference of squares of two odd numbers is always divisible by 4.</p>

Cluster Pure P5

Strand: Shape and Space (3-D Objects and 2-D Shapes)
Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Develop and apply the geometric properties of circles and polygons to solve problems.</p> <p><i>(continued)</i></p>	<p>P5–6. (SS28) Prove the following general properties, using established concepts and theorems:</p> <ul style="list-style-type: none"> the perpendicular bisector of a chord contains the centre of the circle the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc (for the case when the centre of the circle is in the interior of the inscribed angle) the inscribed angles subtended by the same arc are congruent the angle inscribed in a semicircle is a right angle the opposite angles of a cyclic quadrilateral are supplementary a tangent to a circle is perpendicular to the radius at the point of tangency the tangent segments to a circle from any external point are congruent the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord the sum of the interior angles of an n-sided polygon is $(2n - 4)$ right angles. <p>[C, R, V]</p>	<p>6.1 a) For what values of c does the line $y = c$ touch the circle $x^2 + y^2 = r^2$? b) Use the result from part a) to show that the tangent to a circle is perpendicular to the radius at the point of tangency.</p> <p>6.2 Show that the angle inscribed in a semicircle is a right angle.</p> <p>6.3 The chord AB is one side of a regular polygon of n sides. The polygon is inscribed in a circle. If D is any other vertex of the polygon, prove that the magnitude of angle ADB is $\frac{180^\circ}{n}$.</p>

Cluster Pure P5

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication

[CN] Connections

[E] Estimation and

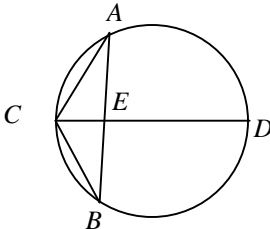
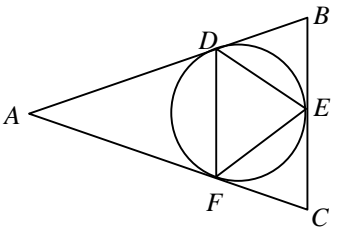
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P5-7. Solve problems, using a variety of circle properties, and justify the solution strategy used. (SS29) [PS, R, V]</p>	<p>7.1 If diameter CD is perpendicular to chord AB at E, prove that triangle ABC is isosceles.</p> <div style="text-align: center;">  </div> <p>7.2 Determine the measure of $\angle BAC$, if $\angle DEF = 60^\circ$ and $\angle EFC = 70^\circ$. Provide a reason for each step in the solution strategy.</p> <div style="text-align: center;">  </div> <p>7.3 A chain on a bicycle connects two gear wheels of diameters 9 cm and 19 cm respectively. The centres of the gear wheels are 87 cm apart. Find the minimum length of the chain.</p>

Cluster Pure P6

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Generate and analyze exponential patterns.</p> <p><i>(continued)</i></p>	<p>P6-1. (PR19) Derive and apply expressions to represent general terms and sums for geometric growth and to solve problems. [CN, R, T]</p>	<p>1.1 Determine the n^{th} term and the sum of the first n terms of the geometric sequence whose first three terms are 2, 6 and 18.</p> <p>1.2 Mathematicians use sigma notation as a way to write the sum of a series. For example: $\sum_{k=1}^5 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5$. Use sigma notation to write the series $5 - 15 + 45 - \dots + 3645$.</p> <p>1.3 Suppose that a principal of P dollars is invested at an annual interest rate r that is compounded annually. The amount A after t years is given by $A = P(1 + r)^t$.</p> <p>a) Find the number of years for the amount to double, if \$2000 is invested at a rate of 7.5%, compounded annually. b) If the interest rate were 7.25% per annum, compounded semi-annually, how would the doubling period change? c) What would be the doubling period, if the rate were 7% per annum, compounded daily?</p> <p>1.4 For the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$, find the sum of 20 terms.</p> <p>1.5 The time needed for an investment to double in value can be estimated using the rule of 72, which states that $n = \frac{72}{i}$ where i is the annual percentage interest rate and n the number of years.</p> <p>a) Compare the rule of 72 doubling time with the exact doubling time for the following interest rates:</p> <ul style="list-style-type: none"> • 4% per annum, compounded annually • 8% per annum, compounded annually • 24% per annum, compounded annually. <p>b) What general conclusion can be drawn as to the accuracy of rule of 72 calculations?</p>

Cluster Pure P6

Strand: Patterns and Relations (Patterns)

Students will:

- use patterns to describe the world and to solve problems.

[C] Communication

[CN] Connections

[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P6–2. Connect geometric sequences to exponential functions over the natural numbers. [E, R, V] PR20</p> <p>P6–3. Estimate values of expressions for infinite geometric processes. [PS, R, T] PR21</p>	<p>2.1 The world’s population grows by 2% per year. The world food production can sustain an additional 200 million people per year. In 1987 the population was 5 billion, and food production could sustain 6 billion people.</p> <p>a) Calculate the population in 1998, 2009, 2019. b) Calculate the population that food production could sustain in 1998, 2009, 2019. c) When will the population exceed the food supply?</p> <p>2.2 The following is a school trip telephoning tree.</p> <div style="text-align: center;"> <pre> graph TD L1[Level 1, teacher] --- L2a[Level 2, students] L1 --- L2b[Level 2, students] L2a --- L3a[Level 3, students] L2a --- L3b[Level 3, students] L2b --- L3c[Level 3, students] L2b --- L3d[Level 3, students] </pre> </div> <p>a) At what level are 64 students contacted? b) How many are contacted at the 8th level? c) By the 8th level how many students, in total, have been contacted? d) By the nth level how many students, in total, have been contacted? e) If there are 300 students in total, by what level will all have been contacted?</p> <p>3.1 For the infinite series $2 + \frac{2}{5} + \frac{2}{25} + \dots$, estimate the sum to four decimal places.</p> <p>3.2 An oil well produces 25 000 barrels of oil during its first month of production. If its production drops by 5% each month, estimate the total production before the well runs dry.</p>

Cluster Pure P6

Strand: Patterns and Relations (Variables and Equations)
Students will:

- represent algebraic expressions in multiple ways.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve exponential, logarithmic and trigonometric equations and identities.</p>	<p>P6-4. (PR39) Solve exponential equations having bases that are powers of one another. [E, R]</p> <p>P6-5. (PR40) Solve and verify exponential and logarithmic equations and identities. [R]</p>	<p>4.1 Solve for x: $3^{(4x-1)} = 27^{2x}$.</p> <p>4.2 A string of ones and zeros is the binary representation of a number. If this number is converted to the base-16 hexadecimal representation, it is 9 digits shorter. As well, the decimal and hexadecimal representations have the same number of digits.</p> <p>a) How many digits are there in the binary representation of the original number?</p> <p>b) Between what two decimal numbers does the original number lie?</p> <p>5.1 Solve for x: $\log_2(x-2) + \log_2(x) = \log_2(3)$.</p> <p>5.2 Solve for x: $2 \times 3^x = 5^{(x-1)}$.</p> <p>5.3 Solve for x, checking for any extraneous solutions: $\log_5(3x+1) + \log_5(x-3) = 3$.</p> <p>5.4 The pH of an acid is given by $\text{pH} = -\log_{10}[\text{H}^+]$, where $[\text{H}^+]$ is the hydrogen ion concentration in moles per litre. What is the hydrogen ion concentration of a weak vinegar solution of $\text{pH} = 3.1$?</p> <p>5.5 Joe has \$50 000 invested at an interest rate of 7% per annum, compounded monthly. Laura has \$40 000 invested at 9.5% per annum, compounded annually. After how many years will the two investments be equal in value?</p> <p>5.6 Verify the identity $\log_a\left(\frac{1}{x}\right) = -\log_a x$ for any base a and any positive value of x.</p>

Cluster Pure P6

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P6–10. Explain the relationship between the laws of logarithms and the laws of exponents. [C, T]</p> <p>P6–11. Graph and analyze logarithmic functions with and without technology. [R, T, V]</p>	<p>10.1 Explain how the exponent law $a^x \times a^y = a^{(x+y)}$ is related to the logarithmic law $\log_a(MN) = \log_a M + \log_a N$.</p> <p>10.2 Use a calculator to find $\log_5 8$, and justify your procedure.</p> <p>11.1 Graph $y = \log_{10} x$ and $y = \log_2 x$ on the same set of coordinate axes. What is the likely position of the graph of $y = \log_5 x$?</p> <p>11.2 Analyze the graph of $y = \log_{10} (2x + 3)$. Identify the domain, range, asymptotes and intercepts.</p>

Cluster Pure P7

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

- | | | | |
|------|--------------------------------------|------|-----------------|
| [C] | Communication | [PS] | Problem Solving |
| [CN] | Connections | [R] | Reasoning |
| [E] | Estimation and
Mental Mathematics | [T] | Technology |
| | | [V] | Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.</p> <p><i>(continued)</i></p>	<p>P7-1. (SP16) Determine the number of permutations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]</p> <p>P7-2. (SP17) Determine the number of combinations of n different objects taken r at a time, and use this to solve problems. [PS, R, V]</p>	<p>1.1 List all possible permutations of the letters in the word bold.</p> <p>1.2 Calculate the number of ways that an executive consisting of four people (president, vice-president, treasurer and secretary) can be selected from a group of 20 people.</p> <p>1.3 Explain the meaning of ${}_8P_3$. Why does ${}_3P_8$ not make sense?</p> <p>1.4 Develop and solve a problem where ${}_8P_3$ would be applicable.</p> <p>1.5 Solve ${}_nP_2 = 30$.</p> <p>1.6 On a 12-question multiple-choice test, three answers are A, three are B, three are C and three are D. How many different answer keys are possible?</p> <p>2.1 From a group of five student representatives, three will be chosen to work on the dance committee.</p> <p>a) List all possible committees.</p> <p>b) Calculate ${}_5C_3$, and compare to the answer in part a).</p> <p>c) If the committee had to have a chairperson, would it still be a combination? Why or why not?</p> <p>d) How many committees of three, with a chairperson, can be chosen from a group of 10 student representatives?</p> <p>2.2 Show that ${}_nC_k = {}_nC_{(n-k)}$, using two different methods. Verify the truth of this assertion for the special case with $n = 10$ and $k = 3$.</p> <p>2.3 How many diagonals are there in a regular polygon with 20 sides? What is the general formula for the number of diagonals in an n-sided polygon?</p>

Cluster Pure P7

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

[C] Communication

[CN] Connections

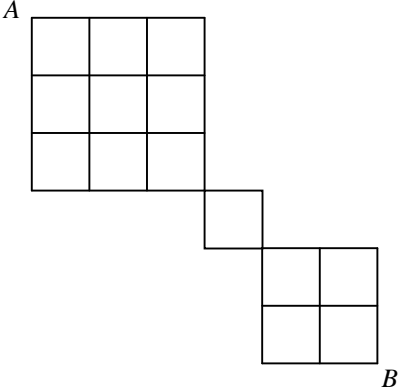
[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<i>(continued)</i>	<p>P7-3. Determine the number of pathways in a given compound pathway problem. [CN, PS, V]</p>	<p>3.1 Student <i>A</i> wants to visit Student <i>B</i>. Roads are shown as lines on the grid. Only south and east travel directions can be used.</p>  <p>a) How many different paths can <i>A</i> take to get to <i>B</i>, if <i>A</i> has to travel along the lines that represent the roads? b) Change the middle square to a 2×2 grid, and repeat the question.</p>

Cluster Pure P7

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<i>(continued)</i>	<p>P7-4. Solve problems, using the binomial theorem where N belongs to the set of natural numbers. [CN, E, V]</p>	<p>4.1 Expand $(x + y)^7$, using the binomial theorem.</p> <p>4.2 Find the 11th term of the expansion of $(x - 2)^{13}$.</p> <p>4.3 Investigate the sample space for flipping 1 coin, 2 coins, 3 coins, 4 coins . . . , and make an organized list. Relate this organized list to Pascal's triangle and the binomial theorem.</p> <p>4.4 Given a set of four elements, list the different proper and improper subsets, and organize them. How is this related to Pascal's triangle? How many subsets are there in total?</p>

Cluster Pure P7

Strand: Statistics and Probability (Chance and Uncertainty)

Students will:

- use experimental or theoretical probability to represent and solve problems involving uncertainty.

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| [C] | Communication | [PS] | Problem Solving |
| [CN] | Connections | [R] | Reasoning |
| [E] | Estimation and
Mental Mathematics | [T] | Technology |
| | | [V] | Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.</p>	<p>P7–5. (SP23) Determine the conditional probability of two events (Bayes’ law). [E, PS, R]</p> <p>P7–6. (SP24) Solve probability problems involving permutations, combinations and conditional probability. [E, PS, R]</p> <p>P7–7. (SP25) Solve probability problems, using the binomial distribution as applied to small samples. [PS, R, T]</p>	<p>5.1 In a particular country, the probability of a child being a girl is 0.510. A family of five children is known to have at least two girls. What is the probability of this family having exactly four girls?</p> <p>5.2 It is known that 10% of a population has a certain disease. For a patient without the disease, a blood test for the disease gives a correct diagnosis 95% of the time. For a patient with the disease, the test gives a correct diagnosis 99% of the time. What is the probability that a person whose blood test shows the disease actually has the disease?</p> <p>6.1 Five books, each of a different colour, and including one red and one green book, are placed on a shelf. What is the probability of the red book being at one end and the green book at the other?</p> <p>6.2 What is the probability of holding all four aces in a five-card hand dealt from a standard 52-card deck?</p> <p>6.3 A shootout consists of teams A and B taking alternate shots on goal. The first team to score wins. Team A has a probability of 0.3 of scoring with any one shot. Team B has a probability of 0.4 of scoring with any one shot.</p> <p>a) If Team A shoots first, what is the probability of Team B winning on its first shot?</p> <p>b) If Team A shoots first, what is the probability of Team A winning on its third shot?</p> <p>c) What is the probability of Team A eventually winning?</p> <p>d) If Team B shot first, what is the probability of Team B eventually winning?</p> <p>7.1 A written test for a driver’s licence consists of 10 multiple-choice questions. To pass the test, a driver must answer 9 or 10 questions correctly. What is the probability of passing by guessing, if there are four possible answers to each question?</p> <p>7.2 A family has nine children. Assuming that there is an equal likelihood for male and female births, what is the probability that there are seven boys and two girls?</p> <p>7.3 An 8 km/h crash test was given to a sample of 20 cars. Four cars failed the test because of damaged bumpers. Find a 95% confidence interval for the proportion of cars that would fail this crash test.</p>

Cluster Pure P8

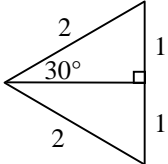
Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication
 [CN] Connections
 [E] Estimation and
 Mental Mathematics

[PS] Problem Solving
 [R] Reasoning
 [T] Technology
 [V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P8–2. Determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90° and 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$. [CN, E]</p> <p>P8–3. Solve first and second degree trigonometric equations over a domain of length 2π: • algebraically • graphically. [PS, T]</p>	<p>2.1 Given an equilateral triangle with a side of 2 units, determine the exact trigonometric ratios of 30°.</p>  <p>2.2 Find the exact values for $\sin \frac{7\pi}{6}$, $\tan \frac{2\pi}{3}$, $\cos \frac{7\pi}{4}$.</p> <p>3.1 Find, algebraically and graphically, the solution to the following trigonometric equations: a) $1 + 2 \cos x = 5 \cos x$; $0 \leq x < 2\pi$. Give solutions in decimal form. b) $\sin^2 x - \sin x = 0$; $0 \leq x < 2\pi$. Give solutions as exact values. c) $\cos 4x = 0.5$; $0 \leq x < 2\pi$. Give solutions as exact values.</p>

Cluster Pure P8

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

[C] Communication

[CN] Connections

[E] Estimation and

Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<i>(continued)</i>	<p>P8-4. Determine the general solutions to trigonometric equations where the domain is the set of real numbers. [PS, T] (PR44)</p> <p>P8-5. Verify trigonometric identities: (PR45)</p> <ul style="list-style-type: none"> numerically for any particular case algebraically for general cases graphically. <p>[PS, R, T, V]</p> <p>P8-6. Use sum, difference and double angle identities for sine and cosine to verify and simplify trigonometric expressions. [R, T] (PR46)</p>	<p>4.1 Sketch the graph of $y = \sin 3x$. Use the graph to find all solutions of $\sin 3x = 0$ in the interval $0 \leq x < 2\pi$.</p> <p>4.2 Use technology to graph $y = x - 2 \sin x$, and use the graph to find all solutions to the equation $2 \sin x = x$. Express answers to a three-decimal place accuracy.</p> <p>4.3 What is the relation between the graphs of $y = \sin x$ and $y = \frac{1}{2}$ and the roots of the equation $0 = 2 \sin x - 1$?</p> <p>4.4 Use technology to solve $\sin 3x = \frac{1}{2}$, and then write the general solution.</p> <p>5.1 a) Verify that $\sin^2 x + \cos^2 x = 1$ for any real number x. b) Use this identity to show that $1 + \tan^2 x = \sec^2 x$ for any real number x, where $\cos x \neq 0$.</p> <p>5.2 Given the identity $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$:</p> <p>a) verify the identity for the particular case when $x = \frac{\pi}{3}$ b) verify for a general angle, using an algebraic approach c) verify, by graphing the left-hand side and the right-hand side of the given identity.</p> <p>6.1 Write $2(\sin 5)(\cos 5)$ in terms of a single trigonometric function.</p> <p>6.2 Graph the function $f(x) = \frac{2 \tan x}{1 + \tan^2 x}$.</p> <p>a) Make a conjecture for the period of the above graph. b) Simplify the expression for $f(x)$ to a single trigonometric function, and then find the period of $f(x)$. c) Compare the solutions to a) and b).</p>

Cluster Pure P8

Strand: Patterns and Relations (Relations and Functions)
Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

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| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Represent and analyze trigonometric functions, using technology as appropriate.</p> <p><i>(continued)</i></p>	<p>P8–7. (PR70) Describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position. [PS, R, V]</p> <p>P8–8. (PR71) Draw (using technology), sketch and analyze the graphs of sine, cosine and tangent functions, for:</p> <ul style="list-style-type: none"> amplitude, if defined period domain and range asymptotes, if any behaviour under transformations. <p>[CN, T, V]</p> <p>P8–9. (PR72) Draw (using technology) and analyze the graphs of secant, cosecant and cotangent functions, for:</p> <ul style="list-style-type: none"> period domain and range asymptotes behaviour under transformations. <p>[CN, T, V]</p>	<p>7.1 Triangle OBA has vertices $O(0, 0)$, $B(4, 0)$ and $A(4, 3)$. The unit circle, centred at $(0, 0)$, intersects OA at point P.</p> <ol style="list-style-type: none"> Use similar triangles to find the coordinates of point P. Use trigonometric ratios to find the sine and cosine of angle AOB. Compare your results in b) to the coordinates of point P. <p>8.1 Using a graphing utility, graph $y = \sin x$ and $y = \cos x$ on the same set of axes.</p> <ol style="list-style-type: none"> What relationship seems to exist between the two? What is the amplitude and period of each graph? <p>8.2 Graph $y = \tan x$ and $y = \tan 2x$. Compare the period, the domain and the range of $y = \tan x$ to those of $y = \tan 2x$.</p> <p>8.3 In the equation $y = A \sin [B(x + C)] + D$; $A = 4$, $B = 3$, $C = \frac{-3\pi}{4}$ and $D = -3$. Compare the graph of this function to the graph of $y = \sin x$ with respect to domain, range, amplitude, period, x and y intercepts, horizontal phase shift and vertical displacement.</p> <p>9.1 Graph and analyze:</p> <ol style="list-style-type: none"> $y = \sec x$ $y = \csc x$ $y = \cot x$. <p>9.2 Compare the domain, range and period of:</p> <ol style="list-style-type: none"> $f(x) = \csc x$ and $g(x) = 5 \csc x$ $f(x) = \cot x$ and $g(x) = \cot 2x$.

Cluster Pure P8

Strand: Patterns and Relations (Relations and Functions)
Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

- | | |
|--|----------------------|
| [C] Communication | [PS] Problem Solving |
| [CN] Connections | [R] Reasoning |
| [E] Estimation and
Mental Mathematics | [T] Technology |
| | [V] Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P8–10. Use trigonometric functions to model and solve problems. (PR73) [PS, R, V]</p>	<p>10.1 For a Saskatchewan town, the latest sunrise time is on December 21, at 09:15. The earliest sunrise time is on June 21, at 03:15. Sunrise times on other dates can be predicted from a sinusoidal equation. Note: There is no Daylight Saving Time in Saskatchewan.</p> <ol style="list-style-type: none"> What is the equation that describes sunrise times? What is the amplitude and period of the equation describing sunrise times? Use the equation to predict the time of sunrise on April 9. What is the average time of sunrise throughout the year? <p>10.2 The depth of water in a harbour is given by the equation $d(t) = -4.5 \cos(0.16\pi t) + 13.7$, where $d(t)$ is the depth, in metres, and t is the time, in hours, after low tide.</p> <ol style="list-style-type: none"> Sketch the graph of $d(t)$. What is the period of the tide, from one high tide to the next? A bulk carrier needs at least 14.5 m of water to dock safely. For how many hours per cycle can the bulk carrier dock safely? <p>10.3 The average daily maximum temperature in Vancouver follows a sinusoidal pattern with a highest value of 23.6°C on July 26, and a lowest value of 4.2°C on January 26.</p> <ol style="list-style-type: none"> Describe this variation with a sine or cosine equation. What is the expected maximum temperature for May 26? How many days will have an expected maximum of 21.0°C or higher? Explain why different equations give the same answers for b) and c).

Cluster Pure P9

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

[C] Communication

[CN] Connections

[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Classify conic sections, using their shapes and equations.</p>	<p>P9-1. (SS35) Classify conic sections according to shape. [C, R, V]</p> <p>P9-2. (SS36) Classify conic sections according to a given equation in general or standard (completed square) form (vertical or horizontal axis of symmetry only). [CN, T, V]</p> <p>P9-3. (SS37) Convert a given equation of a conic section from general to standard form and vice versa. [R, T]</p>	<p>1.1 Visualize the shapes generated from the intersection of a double-napped cone and a plane. For each conic section, describe the relationship between the plane, the central axis of the cone and the cone's generator.</p> <p>2.1 A circle with a radius of 4 units has the equation $x^2 + y^2 - 16 = 0$. What are the values of A, C and F in the general form? What is the radius of the circle $25x^2 + 25y^2 - 100 = 0$?</p> <p>2.2 a) Graph the circle $x^2 + y^2 = 25$. b) Graph $Ax^2 + y^2 = 25$ where $A > 1$. c) Graph $Ax^2 + y^2 = 25$ where $0 < A < 1$. d) Graph $Ax^2 + y^2 = 25$ where $A = 0$. e) Graph $x^2 + Cy^2 = 25$ where $C > 1$. f) Graph $x^2 + Cy^2 = 25$ where $0 < C < 1$. g) Graph $x^2 + Cy^2 = 25$ where $C = 0$. h) Draw a conclusion based on the results found in b) through g).</p> <p>2.3 Graph $2x^2 + y^2 - 12 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph?</p> <p>2.4 Graph $4x^2 - 25y^2 - 100 = 0$, using technology. Graph two other equations of this type, by changing one of the coefficients. What shape is represented by this type of graph?</p> <p>3.1 Convert to standard form: a) $x^2 + y^2 + 6x - 8y = 11$ b) $3x^2 + y^2 + 6x + 4y = 9$.</p> <p>3.2 Convert to general form: a) $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{16} = 1$ b) $\frac{(x+3)^2}{25} - \frac{(y-4)^2}{16} = 1$.</p>

Cluster Pure P9

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C] Communication

[CN] Connections

[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p>Perform, analyze and create transformations of functions and relations that are described by equations or graphs.</p> <p><i>(continued)</i></p>	<p>P9-4. (SS38) Describe how various translations of functions affect graphs and their related equations:</p> <ul style="list-style-type: none"> $y = f(x - h)$ $y - k = f(x)$. <p>[C, T, V]</p> <p>P9-5. (SS39) Describe how various stretches of functions (compressions and expansions) affect graphs and their related equations:</p> <ul style="list-style-type: none"> $y = af(x)$ $y = f(kx)$. <p>[C, T, V]</p>	<p>4.1 Describe how the graph of $y = x^2$ compares to the graph of $y = x^2 - 2$.</p> <p>4.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of:</p> <ol style="list-style-type: none"> $f(x) - 2$ $f(x - 2)$ $f(x - 2) + 1$. <p>5.1 Describe how the graph of $y = x^2$ compares to the graph of:</p> <ol style="list-style-type: none"> $y = 2x^2$ $y = \frac{2}{3}x^2$. <p>5.2 Graph any function $f(x)$. On the same set of coordinate axes, sketch the graph of:</p> <ol style="list-style-type: none"> $2f(x)$ $-2f(x)$ $\frac{2}{3}f(x)$. <p>Discuss the changes.</p> <p>5.3 Given the graph of $f(x) = \sin x$, sketch the graph of:</p> <ol style="list-style-type: none"> $f(2x)$ $\frac{2}{3}f(x)$. <p>5.4 Given the graph of $f(x) = x^3$ and its image under the transformation $g(x) = 3f(x)$, find the equation describing $g(x)$.</p>

Cluster Pure P9

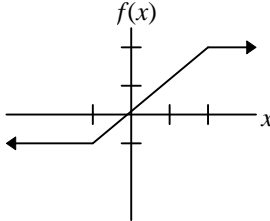
Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

[C] Communication
 [CN] Connections
 [E] Estimation and
 Mental Mathematics

[PS] Problem Solving
 [R] Reasoning
 [T] Technology
 [V] Visualization

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P9-6. Describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations: • $y = f(-x)$ • $y = -f(x)$ • $y = f^{-1}(x)$. [C, T, V]</p> <p>P9-7. Using the graph and/or the equation of $f(x)$, describe and sketch $\frac{1}{f(x)}$. [C, T, V]</p>	<p>6.1 Graph any function $f(x)$. Sketch the graph of: a) $-f(x)$ b) $f(-x)$ c) $f^{-1}(x)$ d) $f^{-1}[f(x)]$.</p> <p>6.2 If $g(x)$ is the reflection of $f(x)$ in the y-axis, write the equation of $g(x)$ in terms of $f(x)$.</p> <p>7.1 Given $f(x) = 2x + 1$, sketch the graph of $f(x)$ and of $\frac{1}{f(x)}$. What happens to the x-intercepts of $f(x)$?</p> <p>7.2 Sketch the graph of $f(x) = \sin x$, and sketch $\frac{1}{\sin x}$.</p> <p>7.3 Sketch $\frac{1}{f(x)}$, if $f(x)$ is shown by the accompanying sketch.</p> <div style="text-align: center;">  </div>

Cluster Pure P9

Strand: Shape and Space (Transformations)

Students will:

- perform, analyze and create transformations.

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|------|--------------------------------------|------|-----------------|
| [C] | Communication | [PS] | Problem Solving |
| [CN] | Connections | [R] | Reasoning |
| [E] | Estimation and
Mental Mathematics | [T] | Technology |
| | | [V] | Visualization |

General Outcomes	Specific Outcomes	Illustrative Examples
<p><i>(continued)</i></p>	<p>P9–8. Using the graph and/or the equation of $f(x)$, describe and sketch $f(x)$. [C, T, V]</p> <p>P9–9. Describe and perform single transformations and combinations of transformations on functions and relations. [C, T, V]</p>	<p>8.1 Given the graph of $f(x) = 2x + 1$, sketch $f(x)$.</p> <p>8.2 Sketch $y = 3 \sin x$. What is the period of this function?</p> <p>8.3 Sketch $f(x) = \frac{1}{ x^2 - 1 }$.</p> <p>8.4 An AC generator has a voltage given by $V = 170 \cos(120\pi t)$, where V is the voltage and t the time in seconds. A simple DC rectifier has voltage output given by $V = 170 \cos(120\pi t)$. Sketch the output graphs for both devices, and describe the similarities and differences.</p> <p>9.1 Given $f(x) = x^2$, sketch the graph of $f(x)$, and sketch the graph of $-2f(x - 1) + 3$.</p> <p>9.2 Determine the equation of the ellipse $x^2 + 4y^2 - 25 = 0$, after each of the following transformations: a) translated two units to the right b) translated three units down c) expanded by a factor of two along the horizontal axis d) expanded by a factor of one quarter along the vertical axis.</p> <p>9.3 Given the circle $x^2 + y^2 = 1$ and its image under a translation described by the ordered pair $(2, -3)$: a) write the equation of the image b) if a point $P(a, b)$ is on the graph of the circle $x^2 + y^2 = 1$ and $P'(a', b')$ is the transformed image of P, what are the coordinates of P' in terms of a and b?</p>